REPORTS

OF THE

COMMITTEE ON ELECTRICAL STANDARDS

APPOINTED BY

THE BRITISH ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE,

REPRINTED BY PERMISSION OF THE COUNCIL.

REVISED BY

SIR W. THOMSON, LL.D., F.R.S., DR. J. P. JOULE, LL.D., F.R.S.,
PROFESSORS J. CLERK MAXWELL, M.A., F.R.S., AND F. JENKIN, F.R.S.

WITH

A REPORT TO THE ROYAL SOCIETY

ON UNITS OF ELECTRICAL RESISTANCE,

BY PROF. F. JENKIN, F.R.S.;

AND

THE CANTOR LECTURES,

DELIVERED BY PROF. JENKIN BEFORE THE ROYAL SOCIETY OF ARTS.

EDITED BY

PROF. FLEEMING JENKIN, F.R.S.

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PREFACE.

THE reissue of the contents of the present volume is made by Sir William Thomson, Dr. J. P. Joule, Prof. J. Clerk Maxwell, and Prof. Fleeming Jenkin, in consequence of the demand for the several Reports, which cannot now be obtained in their original form. As the centimetre is now frequently used instead of the metre as the fundamental unit of length, it has been introduced into the Reports, not to the exclusion of the metre, but along with the metre, all results being expressed in terms of both measures.

REPORTS

OF THE

COMMITTEE

ON

STANDARDS OF ELECTRICAL RESISTANCE.

FIRST REPORT—CAMBRIDGE, OCTOBER 3, 1862.

MEMBERS OF THE COMMITTEE:—Professor A. Williamson, F.R.S., Professor C. Wheatstone, F.R.S., Professor W. Thomson, F.R.S., Professor W. H. Miller, F.R.S., Dr. A. Matthiessen, F.R.S., Mr. F. Jenkin.

THE Committee regret that they are unable this year to submit a final Report to the Association, but they hope that the inherent difficulty and importance of the subject they have to deal with will sufficiently account for the delay.

The Committee considered that two distinct questions were before them, admitting of entirely independent solutions. They had first to determine what would be the most convenient *unit* of resistance, and second what would be the best form and material for the *standard* representing that unit. The meaning of this distinction will be apparent when it is observed that, if the first point were decided by a resolution in favour of a unit based on Professor Weber's or Sir Charles Bright and Mr. Latimer Clark's system, this decision would not affect the question of construction; while, on the other hand, if the second question were decided in favour of any particular arrangement of mercury or gold wire as the best form of standard, this choice would not affect the question of what the absolute magnitude of the unit was to be.

The Committee have arrived at a provisional conclusion as to the first question; and the arguments by which they have been guided in coming to this decision will form the chief subject of the present Report.

They have formed no opinion as to the second question, viz. the best form and material for the standard.

In determining what would be the most convenient unit for all purposes, both practical and purely scientific, the Committee were of opinion that the unit chosen should combine, as far as was possible, the five following qualities.

1. The magnitude of the unit should be such as would lend itself to the more usual electrical measurements, without requiring the use of extravagantly high numbers of cyphers or of a long series of decimals.

2. The unit should bear a definite relation to units which may be adopted for the measurement of electrical quantity, currents, and electromotive force, or, in other words, it should form part of a complete system for electrical measurements.

3. The unit of resistance, in common with the other units of the system, should, so far as is possible, bear a definite relation to the unit of work, the great connecting link between all physical measurements.

4. The unit should be perfectly definite, and should not be liable to require correction or alteration from time to time.

5. The unit should be reproducible with exactitude, in order that, if the original standard were injured, it might be replaced, and also in order that observers who may be unable to obtain copies of the standard may be able to manufacture them without serious error.

The Committee were also of opinion that the unit should be based on the French metrical system, rather than on that now used in this country.

Fortunately no very long use can be pleaded in favour of any of the units of electrical resistance hitherto proposed, and the Committee were therefore at liberty to judge of each proposal by its inherent merits only; and they believe that, by the plan which they propose for adoption, a unit will be obtained combining to a great extent the five qualities enumerated as desirable, although they cannot yet say with certainty how far the fourth quality, that of absolute permanency, can be ensured.

The question of the most *convenient magnitude* was decided by reference to those units which have already found some acceptance. These, omitting for the moment Weber's $\frac{\text{metre}}{\text{second}}$, were found to range between one foot of copper wire weighing one hundred grains (a unit proposed by Professor Wheatstone in 1843) and one mile of copper wire of $\frac{1}{16}$ inch diameter, and weighing consequently about $84\frac{1}{2}$ grains per foot. The smaller units had generally been used by purely scientific observers, and the larger by engineers or practical electricians.

Intermediate between the two lay Dr. Werner Siemens's mercury unit, and the unit adopted by Professor W. Thomson as approximately equal to one hundred millions of absolute $\frac{\text{foot}}{\text{seconds}}$. The former is approximately equal to 371 feet, and the latter to 1217 feet, of pure copper wire $\frac{1}{16}$ inch diameter at 15° C. Both of these units have been adopted in scientific experiments and in practical tests; and it was thought that the absolute magnitude of the unit to be adopted should not differ widely from these resistances.

The importance of the *second quality* required in the unit, that of forming part of a coherent system of electrical measurements, is felt not only by purely scientific investigators, but also by practical electricians, and was indeed ably pointed out in a paper read before this Association in Manchester by Sir Charles Bright and Mr. Latimer Clark.

The Committee has thus found itself in the position of determining not only the unit of resistance, but also the units of current, quantity, and electromotive force. The natural relation between these units are, clearly, that a unit electromotive force maintained between two points of a conductor separated by the unit of resistance shall produce the unit current, and that this current shall in the unit of time convey the unit quantity of electricity.

The first relation is a direct consequence of Ohm's law; and the second was independently chosen by Weber and by the two electricians above named.

Two only of the above units can be arbitrarily chosen; when these are fixed, the others follow from the relations just stated.

Sir Charles Bright and Mr. Latimer Clark propose the electromotive force of a Daniell's cell as one unit, and choose a unit of quantity depending on this electromotive force. Their resistance-unit, although possessing what we

have called the second requisite quality, and superior consequently to many that have been proposed, does not in any way possess the third quality of bearing with its co-units a definite relation to the unit of work, and has therefore been considered inferior to the equally coherent system proposed by Weber many years since, but until lately comparatively little known in this country.

Professor Weber chose arbitrarily the unit of current and the unit of electromotive force, each depending solely on the units of mass, time, and length, and consequently independent of the physical properties of any arbitrary material.

Professor W. Thomson has subsequently pointed out that this system possesses what we have called the third necessary quality, since, when defined in this measure, the unit current of electricity, in passing through a conductor of unit resistance, does a unit of work or its equivalent in a unit of time*.

The entire connexion between the various units of measurement in this system may be summed up as follows.

A battery or rheomotor of unit electromotive force will generate a current of unit strength in a circuit of unit resistance, and in the unit of time will convey a unit quantity of electricity through this circuit, and do a unit of work or its equivalent.

An infinite number of systems might fulfil the above conditions, which leave the absolute magnitude of the units undetermined.

Weber has proposed to fix the series in various ways, of which two only need be mentioned here—first by reference to the force exerted by the current on the pole of a magnet, and secondly by the attraction which equal quantities of electricity exert on one another when placed at the unit distance.

In the first or electro-magnetic system, the unit current is that of which the unit length at a unit distance exerts a unit of force on the unit magnetic pole, the definition of which is dependent on the units of mass, time, and length alone. In the second or electro-static system, the series of units is fixed by the unit of quantity, which Weber defines as that quantity which attracts another equal quantity at the unit distance with the unit force.

Starting from these two distinct definitions, Weber, by the relations defined above, has framed two distinct systems of electrical measurement, and has determined the ratio between the units of the two systems—a matter of great importance in many researches; but the electro-magnetic system is more convenient than the other for dynamic measurements, in which currents, resistances, &c. are chiefly determined from observations conducted with the aid of magnets.

As an illustration of this convenience, we may mention that the common tangent galvanometer affords a ready means of determining the value in electro-magnetic units of any current γ in function of the horizontal component of the earth's magnetism H, the radius of the coil R, its length L, and the deflection δ.

$$\gamma = \text{tang}\,\delta\,\frac{R^2 H}{L}.$$

In this Report, wherever Professor Weber's, or Thomson's, or the absolute system is spoken of, the electro-magnetic system only is to be understood as referred to. The immense value of a coherent system, such as is here described, can only be appreciated by those who seek after quantitative as

* *Vide* "Application of Electrical Effect to the Measurement of Electromotive Force," Phil. Mag. 1851.

distinguished from merely qualitative results. The following elementary examples will illustrate the practical application of the system.

It is well known that the passage of a current through a metal conductor heats that conductor; and if we wish to know how much a given conductor will be heated by a given current in a given time, we have only to multiply the time into the resistance and the square of the current, and divide the product by the mechanical equivalent of the thermal unit. The quotient will express the quantity of heat developed, from which the rise of temperature can be determined with a knowledge of the mass and specific heat of the conductor.

Again, let it be required to find how much zinc must be consumed in a Daniell's cell or battery to maintain a given current through a given resistance. The heat developed by the consumption of a unit of zinc in a Daniell's battery has been determined by Dr. Joule, as also the mechanical equivalent of that heat; and we have only to multiply the square of the current into the resistance, and divide by the mechanical equivalent of that heat, to obtain the quantity of zinc consumed per unit of time.

Again, do we wish to calculate the power which must be used to generate by a magneto-electric machine a given current of (say) the strength known to be required for a given electric light?

Let the resistance of the circuit be determined, and the power required will be simply obtained by multiplying the resistance into the square of the current.

Again, the formula for deducing the quantity of electricity contained in the charge of a Leyden jar or submarine cable from the throw of a galvanometer-needle depends on the relation between the unit expressing the strength of current, the unit of force, and the unit magnet-pole. When these are expressed in the above system, the quantity in electro-magnetic measure is immediately obtained from the ballistic formula. In estimating the value of the various insulators proposed for submarine cables, this measure is of at least equal importance with the measure of the resistance of the conductor and of the insulating sheath; and the unit in which it is to be expressed would be at once settled by the adoption of the general system described.

These five very simple examples of the use of Weber's and Thomson's system might be multiplied without end; but it is hoped that they will suffice to give some idea of the range and importance of the relations on which it depends to those who may hitherto not have had their attention directed to the dynamical theory.

No doubt, if every unit were arbitrarily chosen, the relations would still exist in nature, and by a liberal use of coefficients experimentally determined the answer to all the problems depending on these relations might still be calculated; but the number of these coefficients and the complication resulting from their use would render such an arbitrary choice inexcusable.

A large number of units of resistance have from time to time been proposed, founded simply on some arbitrary length and section or weight of some given material more or less suited for the purpose; but none of these units in any way possessed what we have called the second and third requisite qualities, and could only have been accepted if the unit of resistance had been entirely isolated from all other measurements. We have already shown how far this is from being the case; and the Committee consider that, however suitable mercury or any other material may be for the construction or reproduction of a standard, this furnishes no reason for adopting a foot or a metre length of some arbitrary section or weight of that material.

Nevertheless it was apparent that, although a foot of copper or a metre of

mercury might not be very scientific standards, they produced a perfectly definite idea in the minds of even ignorant men, and might possibly, with certain precautions, be both permanent and reproducible, whereas Weber's unit has no material existence, but is rather an abstraction than an entity. In other words, a metre of mercury or some other arbitrary material might possess what we have called the first, fourth, and fifth requisite qualities, to a high degree, although entirely wanting in the second and third. Weber's system, on the contrary, is found to fulfil the second and third conditions, but is defective in the fourth and fifth; for if the absolute or Weber's unit were adopted *without qualification*, the material standard by which a decimal multiple of convenient magnitude might be practically represented would require continual correction as successive determinations made with more and more skill determined the real value of the absolute unit with greater and greater accuracy. Few defects could be more prejudicial than this continual shifting of the standard. This objection would not be avoided even by a determination made with greater accuracy than is expected at present, and was considered fatal to the *unqualified* adoption of the absolute unit as the standard of resistance.

It then became matter for consideration whether the advantages of the arbitrary material standard and those of the absolute system could not be combined; and the following proposal was made and adopted as the most likely to meet every requirement. It was proposed that a material standard should be prepared in such form and materials as should ensure the most absolute permanency; that this standard should approximate as nearly as possible, in the present state of science, to ten millions of $\frac{\text{metre}}{\text{seconds}}$, but that, instead of being called by that name, it should be known simply as the unit of 1862, or should receive some other simpler name, such as that proposed by Sir Charles Bright and Mr. Latimer Clark in the paper above referred to; that from time to time, as the advance of science renders this possible, the difference between this unit of 1862 and the true ten millions of $\frac{\text{metre}}{\text{seconds}}$ should be ascertained with increased accuracy, in order that the error, resulting from the use of the 1862 unit in dynamical calculations instead of the true absolute unit, may be corrected by those who require these corrections, but that the material standard itself shall under no circumstances be altered in substance or definition.

By this plan the first condition is fulfilled; for the absolute magnitude of this standard will differ by only 2 or 3 per cent. from Dr. Siemens's mercury standard.

The second and third conditions will be fulfilled with such accuracy as science at any time will allow.

The fourth condition, of permanency, will be ensured so far as our knowledge of the electrical qualities of matter will permit; and even the fifth condition, referring to the reproduction, is rendered comparatively easy of accomplishment.

There are two reasons for desiring that a standard should be reproducible: first, in order that if the original be lost or destroyed it may be replaced; second, in order that men unable to obtain copies of the true standard may approximately produce standards of their own. It is indeed hoped that accurate copies of the proposed material standard will soon be everywhere obtainable, and that a man will no more think of producing his own standard than of deducing his foot-rule from a pendulum, or his metre from an arc of the

meridian; and it will be one of the duties of the Committee to facilitate the obtaining of such copies, which can be made with a thousandfold greater accuracy than could be ensured by any of the methods of reproduction hitherto proposed.

It is also hoped that no reproduction of the original standard may ever be necessary. Nevertheless great stress has been lately laid upon this quality, and two methods of reproduction have been described by Dr. Werner Siemens and Dr. Matthiessen respectively; the former uses mercury, and the latter an alloy of gold and silver, for the purpose. Both methods seem susceptible of considerable accuracy. The Committee has not yet decided which of the two is preferable; but their merits have been discussed, from a chemical point of view, in the appended Report C, by Prof. Williamson and Dr. Matthiessen. An interesting letter from Dr. Siemens on the same point will also be found in the Appendix E. This gentleman there advocates the use of a metre of mercury of one square millimetre section at 0° C. as the resistance-unit; but his arguments seem really to bear only on the use of mercury in constructing and reproducing the standard, and would apply as well to any length and section as to those which he has chosen.

When the material 1862 standard has once been made, whether of platinum, gold and alloy, or mercury, or otherwise, the exact dimensions of a column of mercury, or of a wire of gold-silver alloy, corresponding to that standard can be ascertained, published, and used where absolutely necessary for the purpose of reproduction.

It should at the same time be well understood that, whether this reproduction does or does not agree with the original standard, the unit is to be that one original material permanent standard, and no other whatever, and also that a certified copy must always be infinitely preferable to any reproduction.

The reproduction by means of a fresh determination of the absolute unit would never be attempted, inasmuch as it would be costly, difficult, and uncertain; but, as already mentioned, the difference between new absolute determinations and the material standard should from time to time be observed and published.

The question whether the material standard should aim at an approximation to the $\frac{\text{metre}}{\text{second}}$ or $\frac{\text{foot}}{\text{second}}$ was much debated. In favour of the latter it was argued that, so long as in England feet and grains were in general use, the $\frac{\text{metre}}{\text{second}}$ would be anomalous, and would entail complicated reductions in dynamical calculations. In favour of the $\frac{\text{metre}}{\text{second}}$ it was argued that, when new standards were to be established, those should be chosen which might be generally adopted, and that the metre is gaining universal acceptance. Moreover the close accordance between Dr. Siemens's unit and the decimal multiple of the $\frac{\text{metre}}{\text{second}}$ weighed in favour of this unit; so that the question was decided in favour of the metrical system.

In order to carry out the above views, two points of essential importance had to be determined. First, the degree of accuracy with which the material standard could at present be made to correspond with the $\frac{\text{metre}}{\text{second}}$; and second, the degree of permanency which could be ensured in the material standard when made.

The Committee is, unfortunately, not able yet to form any definite opinion upon either of these points.

Resistance-coils, prepared by Professor W. Thomson, have been sent to Professor Weber; and he has, with great kindness, determined their resistance in electro-magnetic units as accurately as he could. It is probable that his determinations are very accurate; nevertheless the Committee did not feel that they would be justified in issuing standards based on these determinations alone. In a matter of this importance, the results of no one man could be accepted without a check. Professor Weber had made some similar determinations with less care some years since, but he has unfortunately not published the difference, if any, between the results of the two determinations. Indirect comparisons between the two determinations show a great discrepancy, amounting perhaps to 7 per cent.; but it is only fair to say that this error may have been due to some error in other steps of the comparison, and not to Professor Weber's determination. Meanwhile it was hoped that a check on Weber's last result would by this time have been obtained by an independent method due to Professor Thomson. Unfortunately, that gentleman and Mr. Fleeming Jenkin, who was requested to assist him, have hitherto been unable to complete their experiments, owing chiefly to their occupation as jurors at the International Exhibition. The apparatus is, however, now nearly complete, and it is hoped will before Christmas give the required determinations.

If Professor Weber's results accord within one per cent. with these new determinations, it is proposed that provisional standards shall be made of German-silver wire in the usual way, and that they should be at once issued to all interested in the subject, without waiting for the construction of the final material standard.

The construction of this standard may possibly be delayed for some considerable time by the laborious experiments which remain to be made on the absolute permanency of various forms and materials. An opinion is very prevalent that the electrical resistances of wires of some, if not all, metals are far from permanent; and since these resistances are well known to vary as the wires are more or less annealed, it is quite conceivable that even the ordinary changes of temperature, or the passage of the electric current, may cause such alterations in the molecular condition of the wire as would alter its resistance. This point is treated at some length in the two Reports B and C, appended, by Professor Williamson and Dr. Matthiessen. The experiments hitherto made have not extended over a sufficient time to establish any very positive results; but, so far as can be judged at present, some, though not all wires do appear to vary in conducting power.

Mercury would be free from the objection that its molecular condition might change; but, on the other hand, it appears from Report C that the mercury itself would require to be continually changed, and that consequently, even if the tube containing it remained unaltered (a condition which could not be absolutely ensured), the standards measured at various times would not really be the same standard. A possibility at least of error would thus occur at each determination, and certainly no two successive determinations would absolutely agree. If, therefore, wires can be found which *are* permanent, they would be preferred to mercury, although, as already said, no conclusion has been come to on this point.

Some further explanation will now be given of the resolutions passed from time to time by the Committee, and appended to this Report.

Dr. Matthiessen was requested to make experiments with the view of

determining an alloy with a minimum variation of resistance due to change of temperature. The object of this research was to find an alloy of which resistance-coils could be made requiring little or no correction for temperature during a series of observations. A preliminary Report on this subject is appended (A), in which the curious results of Dr. Matthiessen's experiments on alloys are alluded to, and, in particular, the following fact connected with the resistance of alloys of two metals is pointed out.

Let us conceive two wires of the two pure metals of equal length, and containing respectively the relative weights of those two metals to be used in the alloy. Let us further conceive these two wires connected side by side, or, as we might say, in multiple arc. Then let the difference be observed in the resistance of this multiple arc when at zero and 100° Cent. This difference will be found almost exactly equal in all cases to the difference which will be observed in the resistance of a wire drawn from the alloy formed of those two metal wires at zero and 100°, although the actual resistance at both temperatures will in most cases be very much greater than that of the hypothetical multiple arc.

In order to obtain a minimum percentage of variation with a change of temperature, it was consequently only necessary to make experiments on those alloys which offer a very high resistance as compared with the mean resistance of their components. The results of a few experiments are given in the Report, but these are only the first of a long series to be undertaken. Hitherto an alloy of platinum and silver is the only one of which the conducting power and variation with temperature are less than that of German silver.

Professor W. Thomson and Dr. Matthiessen were requested to examine the electrical permanency of metals and alloys. A preliminary Report on the subject by Dr. Matthiessen is appended (B), in which he shows that, after four months, one copper and two silver hard-drawn wires have altered, becoming more like annealed wires, but that no decided change has yet been detected in the great majority of the wires.

Several eminent practical electricians were requested to advise the Committee as to the form of coil they considered most suitable for a material standard, and also to furnish a sample coil such as they could recommend. Sir Charles Bright informed the Committee that he was ready to comply with the request. The point is one of considerable importance, respecting which it was thought that practical men might give much valuable information. Coils of wire may be injured by damp, acids, oxidation, stretching and other mechanical alterations. They may be defective from imperfect or uncertain insulation; and they may be inconveniently arranged, so that they do not readily take the temperature of the surrounding medium, or cannot be safely immersed in water or oil baths, as is frequently desirable. No definite conclusion as to the form of coil to be recommended, even for copies, has been arrived at.

It was resolved "That the following gentlemen should be informed of the appointment of the present Committee, and should be requested to furnish suggestions in furtherance of its object:"—

Professor Edlund (Upsala).
Professor T. Fechner (Leipzig).
Dr. Henry (Washington).
Professor Jacobi (St. Petersburg).
Professor G. Kirchhoff (Heidelberg).
Professor G. Matteucci (Turin).
Professor Neumann (Königsberg).
Professor J. C. Poggendorff (Berlin).
M. Pouillet (Paris).
Werner Siemens, Ph.D. (Berlin).
Professor W. E. Weber (Göttingen).

A letter, appended to this Report, was consequently addressed to each of

these gentlemen. Answers have been received from Professor Kirchhoff and Dr. Siemens, which will be found in the Appendix. The resolution arrived at by the Committee to construct a material standard will entirely meet Professor Kirchhoff's views. The Committee have been unable entirely to adopt Dr. Siemens's suggestions; but his statements as to the accuracy with which a standard can be reproduced and preserved by mercury will form the subject of further special investigation, and the Committee will be most happy to take advantage of his kind offers of assistance.

A letter was also received from Sir Charles Bright, containing an ingenious method of maintaining a constant tension or difference of potentials. This point will probably come before the Committee at a later period, when Sir Charles Bright's suggestion will not be lost sight of.

The Committee also received, on the 29th ultimo, after the present Report had been drawn up, a letter from Dr. Esselbach, a well-known electrician, who had charge of the electrical tests of the Malta and Alexandria Cable during its submergence. In this letter Dr. Esselbach arrives at substantially the same conclusions as those recommended by the Committee. Thus, his first conclusion is "to adopt Weber's absolute unit substantially, and to derive from it, by the multiple 10^{10}, the practical unit." This practical unit is precisely that recommended by your Committee. Dr. Esselbach uses the multiple 10^{10}, starting from the $\frac{\text{millimetre}}{\text{second}}$, where your Committee recommend the multiple 10^{7}, starting from the $\frac{\text{metre}}{\text{second}}$: the result is the same.

Dr. Esselbach's next conclusion is also of great practical value. He points out that the electro-magnetic unit of electromotive force, also multiplied by 10^{10}, differs extremely little from that of the common Daniell's cell, and that, without doubt, by proper care such a cell could be constructed as would form a practical unit of electromotive force. This suggestion has the approval of the Committee. Dr. Esselbach next points out that the unit of resistance which he proposes differs very little from Dr. Siemens's mercury unit, which he, like your Committee, considers a great advantage; and the difference is, indeed, less than he supposes. He also proposes to use Weber's absolute unit for the unit of current—a suggestion entirely in accordance with the foregoing Report; and he further points out that this current will be of convenient magnitude for practical purposes. He next approves of the suggestions of Sir Charles Bright and Mr. Latimer Clark with reference to nomenclature and terminology. In the body of the Report he gives some valuable data with reference to the unit of quantity, which he defines in the same manner as your Committee. This result will be analyzed in the Report which Professor W. Thomson and Mr. Fleeming Jenkin will make on the fresh determination of the absolute unit of resistance.

The Committee attach high importance to this communication, showing as it does that a practical electrician had arrived at many of the very same conclusions as the Committee, quite independently and without consultation with any of its members. Dr. Esselbach has omitted to point out, what he no doubt was well aware of, that, if, as he suggests, two equal multiples of the absolute units of resistance and electromotive force are adopted, the practical unit of electromotive force, or Daniell's cell, will, in a circuit of the practical unit of resistance, produce the unit current.

Mr. Fleeming Jenkin was requested to furnish an historical summary of the various standards of resistance, but he has been unable to complete his Report in time for the present meeting.

Professor Williamson and Dr. Matthiessen were requested to put together the facts regarding the composition of the various materials hitherto used for standards of resistance, and the physical changes they were likely to undergo. Wires of pure solid metals, columns of mercury, and wires of alloys have been used for the purpose. The Report of the above gentlemen is appended (C). In it they arrive at the following conclusions:—

Firstly, with reference to pure metals in a solid state, they consider that the preparation of those metals in a state of sufficient purity to ensure a constant specific resistance is exceedingly difficult, as is proved by the great discrepancy in the relative conducting powers obtained by different observers. Electrotype copper is excepted from this remark. They also point out that the influence of annealing on the conducting powers of pure solid metals is very great, and would render their use for the purpose of reproducing a standard very objectionable, inasmuch as it is impossible to ensure that any two wires shall be equally hard or soft. They observe that errors of the same kind might be caused by unseen cavities in the wires, and give examples of the actual occurrence of these cavities. They point out another objection to the use of pure solid metals as standards, in the fact that their resistance varies rapidly with a change of temperature, so that slight errors in a thermometer or its reading would materially affect the results of an experiment.

Secondly, with reference to mercury, they show that it is comparatively easily purified, varies little in resistance with a change of temperature, and can undergo no change analogous to that caused by annealing, but that, on the other hand, measurements of its conducting-power by different observers vary much, that the tube used cannot be kept full of mercury for any length of time, as it would become impure by partial amalgamation with the terminals, and that consequently each time a mercury standard is used it has, practically, to be remade. The accuracy with which *different* observers can reproduce mercury-standards has not been determined.

Thirdly, with reference to alloys, they say that there is better evidence of the independent and accurate reproduction of a standard by a gold-silver alloy of certain proportions than by pure solid metal or by mercury. They point out that annealing and changes of temperature have far less effect on alloys than on pure metals, and that consequently any want of homogeneity or any error in observing the temperature during an experiment is, with alloys, of little consequence, but that, on the other hand, the existence of cavities must be admitted as possible in all solid wires. They are of opinion that the permanence of jewellery affords strong ground for believing that a gold-silver alloy will be quite as permanent as any solid pure metal; and in the course of the Report they point out some curious facts showing that a great change in the molecular condition of some pure metals and alloys may occur without any proportional change in their conducting powers.

Finally, they recommend that practical experiments should be made independently by several gentlemen to determine whether mercury or the gold-silver alloy be really the better means of reproducing a standard.

The main resolution arrived at by the Committee, viz. that a material standard shall be adopted which, at the temperature of 70° Cent., shall approximate to $10^7 \frac{\text{metre}}{\text{seconds}}$, as far as present data allow, has been already fully explained. It was not arrived at until after several meetings had been held, and the merits of the various proposals fully discussed.

This resolution was passed (unanimously) at a meeting when five out of the six members of the Committee were present.

It was at the same time resolved that provisional copies should be distributed at the present meeting. The circumstances have been already explained which have prevented this resolution from being carried into effect.

It was thought desirable that an apparatus should be designed which could be recommended by the Committee for use in copying and multiplying the units to be issued, since it is certain that some of the glaring discrepancies in coils intended to agree must have been due to defective modes of adjustment. Mr. Fleeming Jenkin has consequently designed an apparatus for the purpose, of which a description is appended. Messrs. Elliott Brothers have kindly constructed a couple of these instruments, which may be seen in action by members interested in this subject.

The present Report was drawn up by Mr. Jenkin, and adopted at a meeting of the Committee on the 30th ultimo.

Appendix to Report on Standards of Electrical Resistance.

A. On the variation of the electrical resistance of alloys due to change of temperature, by Dr. Matthiessen, F.R.S.

B. On the electrical permanency of metals and alloys, by Dr. Matthiessen, F.R.S.

C. On the reproduction of electrical standards by chemical means, by Professor Williamson, F.R.S., and Dr. Matthiessen, F.R.S.

D. Professor Kirchhoff's letter.

E. Dr. Siemen's letter.

F. Dr. Esselbach's letter.

G. Circular addressed to foreign men of science.

H. Description of apparatus for copying and multiplying the units of resistance.

Appendix A.—*On the Variation of the Electrical Resistance of Alloys due to Change of Temperature. By* Dr. Matthiessen, *F.R.S.*

It has been shown* that the influence of temperature on the electric conducting power of the metals amounts to 29·3 per cent. on their conducting power between 0° and 100° C.: an exception to this law has been found in iron †, the conducting power of which decreases between those limits 38·2 per cent. It was, therefore, useless to try any of the other pure metals, as they would, in all probability, have decreased by the same amount, as well as from the fact that the metals which would have suited the purpose had already been tried. I therefore turned my attention to the alloys, and, in conjunction with Dr. C. Vogt, have made a long series of experiments respecting the influence of temperature on their electric conducting power. After having determined the conducting power of a few of them at different temperatures, together with the help of the few experiments which have already been made by different observers, it became obvious that the percentage decrement in their conducting power stands in some relation to the fact that, when a solid metal is alloyed with another (with the exception of lead, tin, zinc, and cadmium amongst each other), a lower conducting power is observed than the mean of that of the components ‡. The law which we found to regulate this property was with most alloys the following, viz.:—

* Phil. Trans. 1862, pt. 1.

† Matthiessen and Vogt, unpublished researches.

‡ Assuming that the conducting power or resistance of an alloy is equal to that of parallel wires of the components forming it.

"*The percentage decrement between* 0° *and* 100° *in the conducting power of an alloy in a solid state stands in the same ratio to the mean percentage decrement of the components between* 0° *and* 100° *as the conducting power of the alloy at* 100° *does to the mean conducting power of the components at* 100°;" or, in other words, "*the absolute difference in the observed resistance between* 0° *and* 100° *of an alloy is equal to the absolute difference between the means of the resistance of the component metals between* 0° *and* 100°."

For example, the conducting power of the hard-drawn gold-silver alloy was found equal to 15·03 at 0° (taking silver equal 100 at 0°), and decreases 6·49 per cent. between 0° and 100°. The mean decrement of the components between 0° and 100° being 29·3 per cent., the conducting power of the alloy is 14·05 at 100°, and that of the mean of the components is 62·58 at 100°. If we now calculate the percentage decrement in the conducting power of the alloy between 0° and 100° from the above data, we find it equal to 6·58 per cent., and by experiments it was found equal to 6·49 per cent. Or, taking the resistance of silver at 0°=100, and that of gold at 0°=128·3, we find the resistance of the alloy at 0°=665·3, and at 100°=711·7, and that calculated from a mean of the volumes of its components at 0°=113·2, and at 100°=159·8; therefore the absolute difference between the observed resistance at 0° and 100° is 46·4, and that between the calculated at 0° and 100°=46·8.

Knowing already, from my experiments on the electric conducting power of alloys*, that when two metals are alloyed together in any proportion, if the alloy is merely a solution of the two metals in one another, its conducting power may be approximatively foretold, and that, from the above law, it is necessary that if the conducting power of an alloy should vary between the limits of 0° and 100° to a minimum extent, the alloy itself must have a minimum conducting power as compared with that calculated from its components,—I at once foresaw that it would be useless, as was afterwards proved by the research made in conjunction with Dr. Vogt, to make any experiments with the two metal-alloys, which may be looked upon as a solution of one metal in the other, as no practical alloy would be found which would vary in its conducting power between 0° and 100° to a small extent. It must also be borne in mind that the alloy sought for must be a ductile one, capable of being drawn into wire, not too soft, as would easily be damaged by covering and winding, easily produced, and cheap in price. Bearing this in mind, we turned our attention to some three metal-alloys, thinking that we had some chance there of obtaining a good result; for it is well known that the conducting power of German-silver wire varies in such a slight extent between 0° and 100°.

It also appeared worth while to experiment with some of those alloys which may perhaps be considered chemical combinations, or to contain such, as, for instance, platinum and silver; and, on account of their other physical properties, the platinum-iridium alloys were also experimented with.

In the following Table I give the results obtained in conjunction with Dr. Vogt. The unit here taken for comparison is that of a hard-drawn silver wire at 0°. The normal wires were made of German silver, and in order to obtain their values in terms of hard-drawn silver, they were compared with the gold-silver alloy. In these experiments it was thought better first to use those pure metals which are easily obtained, so as to learn something regarding the manner in which the three metal-alloys behave, and

* Phil. Trans. 1860, p. 161.

then try some alloys made of the cheaper commercial metals. As will be seen by the Table, only the first part has been as yet carried out.

TABLE.

(With each series, the formula deduced from the observations for the correction of the conducting power of the alloy for temperature is given, when λ is equal to the conducting power at the temperature t C.)

Composition of alloy.	Weight.	Length 532 mm.; diameter 0·625 mm.	
		T.	Conducting power. Found.
(1) Gold	58·3	9·0	11·956
Copper	26·5	53·5	11·674
Silver	15·2	100·0	11·438
Made from pure metals. Hard-drawn.			

$$\lambda=12{\cdot}017-0{\cdot}0069033t+0{\cdot}0000111t^2.$$

This alloy was taken, as Karmarsch states it is the hardest and most elastic of all the gold-silver-copper alloys.

		Length 341·5 mm.; diameter 0·618 mm.	
		T.	Conducting power. Found.
(2) Gold	66·5	10·95	10·5637
Silver	18·1	33·52	10·4341
Copper	15·4	55·15	10·3130
Made of pure metals.		78·35	10·1846
Hard-drawn.		97·52	10·0852

$$\lambda=10{\cdot}6220-0{\cdot}0056248t+0{\cdot}0000009863t^2.$$

This alloy was tried, as it corresponded to equal volumes of gold-copper and gold-silver, and these again correspond to an alloy possessing the lowest conducting power of any of those made of gold-copper or gold-silver.

Composition of alloy.	Weight.	Length 764 mm.; diameter 0·553 mm.	
		T.	Conducting power. Found.
(3) Copper	78·3	11·0	45·591
Silver	14·3	55·5	40·333
Gold	7·4	100·0	37·560
Made from pure metals. Hard-drawn.			

$$\lambda=44{\cdot}472-0{\cdot}081525t+0{\cdot}0003240t^2.$$

This alloy was taken to see the effect such a combination would have.

		Length 244 mm.; diameter 0·682 mm.	
		T.	Conducting power. Found.
(4) Platinum	66·6	12·0	4·506
Iridium	33·4	56·0	4·384
Commercial alloy.		100·0	4·271
Hard-drawn.			

$$\lambda=4{\cdot}541-0{\cdot}0029307t+0{\cdot}000002724t^2.$$

This alloy was tried, as it possesses very great elasticity and does not become softer on annealing. On account of these properties, as well as its permanency

in air (not oxidizing on its surface), it would serve exceedingly well for making springs and contacts for electric and telegraphic apparatus.

(5) Silver .... 95·0
Platinum ...... 5·0
Made from pure silver and commercially pure platinum.
Hard-drawn.

Length 381·5 mm.; diameter 0·451 mm.

T.	Conducting power. Found.
12·0	31·173
56·0	29·550
100·0	28·068

$$\lambda = 31{\cdot}640 - 0{\cdot}039363t + 0{\cdot}00003642t^2.$$

This and the following two alloys were taken, as they probably contain chemical combinations.

(6) Silver .... 90·2
Platinum ...... 9·8
The metals employed were the same as in No. 5.
Hard-drawn.

Length 708 mm.; diameter 0·26 mm.

T.	Conducting power. Found.
9·0	17·920
54·5	17·319
100·0	16·767

$$\lambda = 18{\cdot}045 - 0{\cdot}013960t + 0{\cdot}00001183t^2.$$

(7) Silver .... 66·6
Platinum ...... 33·4
Commercial alloy.
Hard-drawn.

Length 169 mm.; diameter 0·408 mm.

T.	Conducting power. Found.
8·270	6·6850
54·00	6·5826
99·90	6·4987

$$\lambda = 6{\cdot}7032 - 0{\cdot}0022167t + 0{\cdot}000001394t^2.$$

In the following Table I have given the results in such a manner that they may be easily compared.

TABLE.

	Conducting power at 0°.	Percentage variation in conducting power between 0° and 100°.
Pure iron	....	38·2
Other pure metals in a solid state	....	29·3
Alloy 3	44·5	15·5
„ 5	31·6	11·3
„ 6	18·0	7·1
„ Gold-silver*	15·0	6·5
„ 4	4·5	5·9
„ 2	10·6	5·2
„ 1	12·0	4·8
„ German silver†	7·8	4·4
„ 7	6·7	3·1

The method and apparatus employed for the above determinations, together with the precautions taken to ensure correct results, have already been described‡. We have made only three observations between 0° and 100°,

* Phil. Mag. Feb. 1861. † Phil. Trans. 1862, pt. 1. ‡ Ibid.

for it was found that they gave almost exactly the same formulæ for the correction of the conducting power for temperature as if we had taken seven or more observations between 0° and 100°. Each of the above values for the conducting power, at those temperatures, is the mean of three or more observations. It was easy to obtain the desired temperatures as a mean of several observations, after very little practice. I have no doubt that, in the course of our experiments, we shall be able to find an alloy the conducting power of which will decrease between 0° and 100° even less than that of silver-platinum. The experiments are being continued, and I hope, before the next meeting of the Association, to be able to lay before you results which will throw more light on the subject, as well as to propose an alloy with a minimum variation in its conducting power due to change of temperature, which may be made commercially in a cheap manner of the common commercial metals, and possessing those properties which are essential that it should have.

Appendix B.—*On the Electrical Permanency of Metals and Alloys.* *By* Dr. Matthiessen, *F.R.S.*

Having, in conjunction with Prof. Thomson, been requested by your Committee to make some experiments on this subject, we thought it advisable for one of us to undertake some preliminary experiments in which all possible disturbing causes were isolated. The chief of these are:—oxidation by the oxygen of the air, as well as by acids produced by the oxidation of the oil or grease with which a wire is almost always covered when drawn, as the holes in the draw-plates are generally oiled or greased; stretching during the process of covering and winding; and after being wound on the bobbin, elongation by expansion or contraction, owing to variations of temperature, &c. These, I think, have been obviated in the following manner:—The wires were carefully wound round a glass tube in order to bring them into a smaller compass, and after taking them off, they were placed inside wide glass tubes, and soldered to two thick copper wires, these having been previously passed through corks which fitted into the ends of the glass tube; through each of the corks a small glass tube passed, drawn out in the middle to enable it to be drawn off easily, and sealed hermetically by a lamp. The wire being soldered to the thick copper connectors, and the corks fitted into the tube, dry carbonic-acid gas was led through it for the space of about six hours, for the purpose of drying it perfectly, as well as of displacing the air contained in it; after which the small glass tubes were melted off at the points, when they had been previously drawn out. Tin caps, filled with melted marine glue, were then fitted over the corks and the ends of the tube, to prevent diffusion of the carbonic acid and air through the corks. The whole of the tin caps outside, as well as those parts of the copper-wire connectors which dipped in water of the bath in which they were placed whilst being tested, were covered with a thick coating of marine glue.

The wires experimented with were as follows:—

1. Silver: hard-drawn .... 2. Silver: annealed	Cut from the same piece; pure.
3. Silver: hard-drawn .... 4. Silver: annealed	Cut from the same piece, but different from 1 and 2; pure.
5. Copper: hard-drawn ...... 6. Copper: annealed..........	Cut from the same piece; pure.

7. Copper: hard-drawn ...... 8. Copper: annealed	Cut from the same piece, but different from 5 and 6; pure.
9. Gold: hard-drawn .... 10. Gold: annealed	Cut from the same piece; pure.
11. Gold: hard-drawn .... 12. Gold: annealed	Cut from the same piece, but different from 9 and 10; pure.
13. Platinum: hard-drawn .... 14. Platinum: hard-drawn	Cut from the same piece; commercial.
15. Gold-silver alloy: hard-drawn 16. Gold-silver alloy: hard-drawn	Cut from same piece. Made by Messrs. Johnson and Matthews.
17. German silver: annealed .... 18. German silver: annealed .... 19. German silver: annealed	Cut from the same piece. No. 19 arranged with longer connectors, and used as normal wire with which the rest were compared.

The reason why duplicates were made in each case was that, in case any of them should by any cause get damaged, the experiments might be continued with the duplicate. When being tested, they were placed in a large bath containing from 40 to 50 litres of water. By testing the wires at 20° it was found easy to keep that temperature in the bath, during the experiments, to 0°·1 or 0°·2.

Up to the present time, that is to say, four months since they were first tested, the conducting power of the wires 1, 3, and 5 has altered, owing to becoming, in all probability, partially annealed. Wire 8 has also altered materially, having decreased in conducting power 3·5 per cent.: this decrement may be possibly due to bad soldering. The differences found with the other wires are so very small, that it is impossible to say whether they have altered or not; for 0°·1 or 0°·2 will account for them. It was, therefore, thought better to wait for another two or four months before giving an opinion as to whether they alter or not; for as the wires are in tubes and only surrounded by carbonic acid, we can never be absolutely sure that the wire has exactly the same temperature as the bath, more especially when it is considered that each time the connexion with the battery is made the wire becomes somewhat heated.

If, two or four months hence, they still show no difference in their conducting powers, it is proposed to expose the one set to variations of temperature such as may occur (for instance, from 0° to 40°), and then, should no change occur in their conducting powers, to lead a weak current through them, say, for a month; for it has been asserted that a current passing through wire causes a permanent change in its conducting power.

If, after these experiments the conducting power of the wires remains unaltered, the different forms of resistance-coils, made from those wires which have shown themselves permanent will then be tested in order to prove which is the best form of coil for the British-Association unit.

Appendix C.—*On the Reproduction of Electrical Standards by Chemical Means. By* Professor Williamson, *F.R.S., and* Dr. Matthiessen, *F.R.S.*

In the following Report we have discussed, more especially from a chemical point of view, the relative merits of the different propositions which have been made to reproduce standards of electric resistance, and have treated them under three heads:—

I. *Those reproduced by a given length and section or weight, at a given temperature, of a pure metal in a solid state.*
II. *Those reproduced by a given length and section or weight, at a given temperature, of a pure metal in a liquid state.*
III. *Those reproduced by a given length and section or weight, at a given temperature, of an alloy.*

The points on which we shall speak will be:—

1. *On their preparation in a state of purity.*
2. *On their homogeneity and their molecular condition.*
3. *On the effect of annealing on their conducting power.*
4. *On the influence of temperature on their conducting power.*

I. *Those reproduced by a given length and section or weight, at a given temperature, of a pure metal in a solid state.*

As type of this class we have chosen copper, for it has been more extensively used as unit of electric resistance, both by scientific as well as by practical men, than any other metal or alloy; but what we are about to say regarding copper will hold good in almost every case for all pure metals in a solid state.

1. *On its preparation in a state of purity.*—As traces of foreign metals materially affect the conducting power of most pure metals, it is of the utmost importance that those used for the reproduction of units of electric resistance should be absolutely chemically pure. The difficulty in obtaining absolutely pure metals even by chemists is very great. Thus, for instance, Becquerel* found the conducting power of pure gold at 0° equal to 68·9, compared with that of pure silver at 0° equal to 100; whereas, under the same circumstance, Matthiessen and Von Bose† found it equal to 77·9,—showing a difference of about 12 per cent. in the values observed for the conducting power of gold, prepared pure by different chemists. This difference may be due to the silver not being pure, or to all of them being more or less pure. Now when we consider that these standards are required by electricians and other physicists who have little or no acquaintance with chemical manipulation, and that the cost of the preparation of absolutely pure metals by scientific chemists would be very expensive on account of the time and trouble they require, we think that this fact alone constitutes a very serious drawback to their use as a means for the reproduction of standards of electric resistance. From the experience which one of us has had on this subject, it is more than probable that if pure metals be prepared by different chemists in the ordinary way of business, variations in their conducting power would be found equal to several per cent. Thus, copper supplied as *pure* by a well-known assayer had a conducting power equal to 92, whereas pure copper conducts at the same temperature 100‡. Again, the *pure* gold of the assayer conducts only 65·5, whereas pure gold at the same temperature would have a conducting power equal to 73§. In order to show that the conducting power of commercial metals varies to a great extent, we give in the following Table (X.) the values found for that of the different coppers of commerce; and it will be evident from it, that to take a given length and weight or section of a commercial metal as unit, as has often been done, is very wrong, and can only lead to great discrepancies between the results of different observers.

* Ann. de Chim. et de Phys. (1846) t. xvii. p. 242.
† Phil. Trans. 1862, pt. 1.
‡ Proceedings of the Royal Society, vol. xi. p. 126.
§ Phil. Trans. 1860, p. 178.

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TABLE X.*

(All the wires were annealed.)

	Conducting power.
Pure copper	100·0 at 15·5
Lake Superior native, not fused	98·8 at 15·5
Ditto, fused, as it comes in commerce	92·6 at 15·0
Burra Burra	88·7 at 14·0
Best selected	81·3 at 14·2
Bright copper wire	72·2 at 15·7
Tough copper	71·0 at 17·3
Demidoff	59·3 at 12·7
Rio Tinto	14·2 at 14·8

Similar variations will be found with most other metals, and we shall give examples of these further on.

2. *On its homogeneity and its molecular condition.*—It is well known that the wires of some metals require much more care in drawing than in others: thus, copper and silver, if not annealed often enough during the process of drawing, will often become quite brittle, and break off short when bent. Now, if the fracture be closely observed, it will be seen that the wire is hollow; in fact, wherever it is broken, cavities will be found, and sometimes of a millimetre or two in length; so that such a wire may almost be regarded as a tube with a very fine bore. The reason of this is simply that in not annealing the wire often enough, the internal part of it becomes hard and brittle, whilst the outside remains annealed, from the heat evolved by its passage through the holes of the draw-plates; after a time, however, the inside, being very brittle, will give way, whilst the outside is still strong enough to bear the force used in drawing it through the draw-plates. These places in the wires are easily discovered on drawing the wire finer; for then at these points the wire slightly collapses, owing to the quicker elongation of the weak points by the force used in drawing. Silver and copper are the only metals which have been experimented with in this manner; we are therefore unable to say whether it may occur with the other metals. However, although no such wires could be used for experiments, yet what has been shown possible to occur to such a marked extent when purposely trying to obtain such results, may occur to some slight extent, especially when great care is not used, and when the wires are drawn by different persons. This may explain why, with some metals and alloys of the same preparation, conducting powers are often obtained which vary several per cent. For instance, W. Thomson† found the conducting power of several alloys of copper which he had had made and tested to alter considerably on being drawn finer; some of them were faulty from the cause we have just mentioned, and, on their being drawn finer, these places showed themselves and were then cut away.

It has also been shown‡ that when copper wire is heated to 100° for several days, a permanent alteration takes place in its conducting power: thus, with the first wire experimented on, it increased almost to the same extent as if it had been annealed. With the second wire the increment was not nearly so large as with the first, and with the third it hardly altered at all. That this is not due to one or the other of the wires being faulty in the just-mentioned manner is proved,

1st, By the close agreement in the conducting powers.

* Report of the Government Submarine Cable Committee, p. 335.

† Proceedings of the Royal Society, vol. xi. p. 126. ‡ Phil. Trans. 1862, pt. 1.

2nd, By the close agreement between the differences in the values found for the conducting powers of the hard-drawn and annealed wires. They were—

	1st wire at 0°.	2nd wire at 0°.	3rd wire at 0°.
Hard-drawn	99·5	100·0	100·3
Annealed............	101·8	102·1	102·2

The values given for the hard-drawn wires are those which were observed before the wire was heated at all.

3rd, That the same occurs with pressed wires: thus, with bismuth it was found that the pieces of the same wire behaved differently; wire 1 showing, after 1 day's heating to 100°, an increment in the conducting power of 16 per cent., whereas wire 2 increased, although a piece from the same length of wire, 9 per cent.

Again, take the case of tellurium, and taking the conducting power of each bar at first equal to 100, we find that the conducting power of bar 1 had decreased after 13 days' heating to 4, where it then remained constant, that of bar 2 after 32 days became constant at 19, and that of bar 3 after 33 days at 6.

The cause of these marked changes in the conducting power must therefore be looked for in the molecular arrangement of the wires or bars employed. In the case of copper, they may be, and probably are, due to the partial annealing of the wires; for we find that wire 1, although the conducting power increased after having been kept at 100° for several days almost to the same extent as if it had been annealed, yet, on annealing it, it only gained as follows (the results obtained with wires 2 and 3 are added):—

	1st wire at 0°.	2nd wire at 0°.	3rd wire at 0°.
Hard-drawn	99·5	100·0	100·3
After being kept several days at 100°	101·6	101·6	100·6
After annealing	101·8	102·1	102·2

The above shows that, in all probability, the annealing plays here a part, but not the whole, in the change; for otherwise why do the wires behave differently? This point will be fully discussed in another Report which will be laid before your Committee, and in which it will be shown where the hard-drawn wires become partially annealed, and annealed wires partially hard-drawn, by age.

It is a curious fact that a change in the molecular arrangement of the particles of wire of some metals which may be considered homogeneous has very little effect on its electric conducting power. Thus pure cadmium*, which when cold is exceedingly ductile, becomes quite brittle and crystalline at about 80°, and returns again to its ductile condition on cooling, shows no marked changed in its conducting power at that temperature; in fact, it behaves as if no such change had taken place. Again, when iron wire is heated in a current of ammonia it becomes perfectly brittle and crystalline, without altering its conducting power to any marked extent.

That a wire which changes its molecular condition in becoming crystalline does not necessarily materially alter in its conducting power, is an important as well as a very interesting point, and has also been proved in the case of German silver.

* Phil. Trans. 1862, pt. 1.

3. *On the effect of annealing on the conducting power.*—When hard-drawn wires of silver, copper, gold, &c. are heated to redness and cooled slowly, they become much softer, and on testing their conducting powers they will be found to have increased thus:—

	Silver.	Copper.	Gold.	According to
Taking the hard-drawn wire as	100·0	100·0	100·0	
The annealed will be..	107·0	102·6	101·6	Becquerel*.
Ditto	109·0	102·3	102·0	Matthiessen and Von Bose†.
Ditto	110·0	106·0	—	Siemens‡.

Now there is a certain difficulty in drawing a wire which is hard-drawn; and if annealed wires be used for the reproduction of standards, the molecular condition, or perhaps the process of annealing, has an influence on the increment of the conducting power. Thus, according to Siemens §, the difference in the conducting power between hard-drawn and annealed silver varies between 12·6 and 8 per cent., and that of copper between 6 and —0·5 per cent.; according to Matthiessen and Von Bose ‖, that of silver varies between 10 and 6 per cent., and that of copper between 2·6 and 2 per cent.

Again, the annealed wires of pure metals are so soft that they would easily get damaged in covering them with silk or winding them on the bobbins, so that in using them the utmost care would have to be employed in order to prevent their getting injured.

4. *On the influence of temperature on the electric conducting power.*—It has been shown that the conducting power of most pure metals decreases, between 0° and 100°, 29·3 per cent.: pure iron has been found to form an exception to this law, its conducting power decreasing between those temperatures 38·2 per cent. If pure metals be therefore used as standards, very accurate thermometers are necessary, as an error of 0·1° in comparing two standards would cause an error in the resistance of about 0·04 per cent. Now there is great difficulty in obtaining normal thermometers; and we must bear in mind that supposing the zero-point of the thermometer is correct today, we are not at all justified in assuming that it will be so in six months time; so that we ought to redetermine the zero-point of the thermometer before using it for the above purpose. Again, it has been proved that the influence of temperature on the conducting power of wires of the same metal is not always the same¶. Thus, for the conducting power of annealed copper wires the following values were found:—

°	No. 1.	No. 3.
0	100·0	100·0
20	92·8	92·4
40	86·3	85·6
60	80·4	79·6
80	75·1	74·4
100	70·5	70·0

showing therefore that if standards of pure metals be used, the influence of temperature on the conducting power of each would have to be ascertained.

* Ann. de Chim. et de Phys. 1846, t. xvii. p. 242.

† Phil. Trans. 1862, pt. 1.

‡ Phil. Mag. Jan. 1861.

§ Phil. Mag. Jan. 1861.

‖ Matthiessen and Vogt's unpublished researches.

¶ Phil. Trans. 1862, part 1.

It must also be borne in mind that it is not at all easy to maintain a standard, even in a bath of oil or water at a given temperature, for any length of time.

II. *Those reproduced by a given length and section or weight, at a given temperature, of a pure metal in a liquid state.*

The only metal which has been proposed to be used in a liquid state for the reproduction of units of resistance is mercury. We shall only have to speak of its preparation in a state of purity, and on the influence of temperature on its conducting power. For a tube, carefully filled with mercury, will certainly form a homogeneous column, and its molecular condition will always be the same at ordinary temperatures.

On its preparation in a pure state.—Although this metal is one of the most easily purified, yet the use of it as a standard is open to the same objections, although in a less degree, as have been advanced against the use of pure metals in a solid state when speaking of their preparation. We there stated that metals prepared by different chemists conducted differently. Now although the same manipulator may obtain concordant results in purifying metals from different sources, yet that by no means proves that the results of different observers purifying the same metal would show the same concordance. Thus we find that the values obtained by one experimenter* for the resistance of mercury, determined in six different tubes, varied 1·6 per cent. This difference, he says, is not greater than was to be expected. The resistances found were as follows:—

Tubes	I.	II.	III.	IV.	V.	VI.
Experiment ..	1016·52	427·28	555·38	217·73	194·70	1142·3
Calculated	1025·54	427·28	555·87	216·01	193·56	1148·9

Again, the values found for the conducting power of different preparations of pure hard-drawn gold, by the same observer†, were found equal to

78·0 at 0°	78·2 at 0°	76·8 at 0°
79·5 at 0°	78·3 at 0°	76·7 at 0°
77·0 at 0°	78·0 at 0°	77·3 at 0°

These values agree together as well as might be expected, considering that 0·01 per cent. impurity would cause these differences. Now the values obtained by different observers vary between the numbers 59 and 78.

If we now take the case of copper, the values found by the same experimenters‡ for different preparations of the pure hard-drawn metal were—

99·9 at 0°	99·4 at 0°	99·8 at 0°
101·0 at 0°	99·4 at 0°	100·3 at 0°
99·8 at 0°	99·9 at 0°	100·0 at 0°
99·9 at 0°		

They were drawn by themselves, and all, with one exception, electrotype copper.

It is well known how differently the so-called *pure* copper conducts when

* Phil. Mag. Jan. 1861. The same experimenter (Dr. Siemens) states, however, in a later paper (Pogg. Ann. cxiii. p. 95), that he is able to reproduce standards of resistance by means of mercury with an accuracy equal to 0·05 per cent., but does not indicate what other precautions he takes (see remarks on the above, Phil. Mag. Sept. 1861).

† Phil. Trans. 1862, p. 12.

‡ Phil. Trans. 1862, p. 9.

prepared by different experimenters. In the following Table, in order to show these facts more clearly, we have given the conducting powers of the metals, taking that of silver equal 100 at 0°. Silver, copper, gold, and platinum were hard-drawn. All values given, except where the contrary is mentioned, have been reduced to 0°.

	Siemens.	Lenz.	Becquerel.	Matthiessen.
Silver*	100	100	100	100
Copper	96·9	73·4	95·3	99·9
Gold	......	58·5	66·9	78·0
Cadmium	......	......	26·3	23·7
Zinc	......	......	25·7	29·0
Tin	......	22·6	15·0	12·3
Iron	......	13·0	13·1	14·4 at 20·4
Lead	......	10·7	8·8	8·3
Platinum	14·2	10·4	8·6	10·5 at 20·7
Mercury	1·72	3·42 at 18·9	1·86	1·65

If, now, mercury be taken as unit, we find the following values :—

	Siemens.	Lenz.	Becquerel.	Matthiessen.
Silver	58·20	29·24	53·76	60·60
Copper	56·40	21·46	51·23	60·55
Gold	......	17·10	37·04	47·27
Cadmium	......	......	14·14	14·42
Zinc	......	......	13·82	17·70
Tin	......	6·59	8·10	7·45
Iron	......	3·80	7·04	8·72 at 20·4
Lead	......	3·12	4·73	5·03
Platinum	8·25	3·04	4·62	6·36 at 20·7
Mercury	1·00	1·00 at 18·7	1·00	1·00

A glance at the foregoing Tables will suffice to show how badly Lenz's series agrees with the rest when mercury is taken as unit; and, in fact, we obtain more concordant results if, in the above series, we take any other metal as unit. These facts therefore seem to indicate that mercury is not yet proved to be a safe means of reproducing standards of electric resistance.

The influence of temperature on the conducting power of mercury, between 0° and 100°, is, comparatively speaking, small, being only 8·3 per cent., whereas that of the metals in a solid state decreases between those limits 29·3 per cent. This property would, of course, render the use of very accurate thermometers unnecessary; for 1° would only cause a difference in the conducting power of about 0·08 per cent., and therefore 0·1 only 0·008 per cent., so that an error of 1 or 2 tenths of a degree might almost be overlooked.

A fact has just come to our knowledge through Mr. Jenkin. He informs us that, having to make a report on the electrical apparatus in the International Exhibition, he tested, amongst other things, several resistance-coils. Now he found two sets of coils made by the same firm, the one exhibited in the Prussian, the other in the English department. Both were said to be multiples of the mercury unit proposed by Siemens†, and their resistances determined by comparing a coil in each set with that of a tube filled with mercury. Taking each set by itself and comparing the coils in it with one another in

* This and the following Table have been copied from a paper published in the Phil. Mag. for Sept. 1861.

† *Phil. Mag.* Feb. 1861.

the proper combination, they were found to be perfect; in fact, the adjustment of them was perfectly accurate. When, however, Mr. Jenkin compared coils of the two sets with each other, instead of being equal, they were found to show a difference of 1·2 per cent.*

III. *Those reproduced by a given length and section or weight, at a given temperature, of an alloy.*

The alloy on which we have to speak is that composed of two parts by weight of gold and one of silver. The reason why this alloy was proposed is that the use of (say) 1 per cent. more or less gold does not materially alter its conducting power.

1. *On its preparation.*—It has been shown that the alloy may be made of commercially pure metals and have the same conducting power as that made from chemically pure ones; for the maximum differences in the conducting power between those made in different parts of the world are not greater than those of a pure metal, either in a solid or liquid state, prepared by the same experimenter. But it may be urged that part of the differences obtained by different observers are due to the different methods employed in determining their conducting powers, and therefore had the conducting power of these alloys been determined by different persons, much greater differences would have been found. In answer to this, we give, in the following Table, the determination of the conducting power of several alloys by Thomson and Matthiessen†, independently of one another. The alloys were made by Messrs. Johnson and Matthey.

Alloy.	Thomson.	Matthiessen.
1	100·0	100·05
2	95·8	95·0
3	102·9	102·7
4	100·8	99·1
5	98·1	97·7
6	89·9	92·7
7	80·6	80·06

Pure copper.	Thomson.	Matthiessen.
1	107·0	107·2
2	107·5	105·9
3	108·7	106·9
4	107·7	108·1

The differences here, with the exception of alloy 6 and copper 2, may be due to the temperature at which the observations were made not being in both cases the same; for 2 or 3 degrees' difference will account for them. The Table, however, shows that different observers do obtain the same values for the conducting power of the same wires.

The values obtained for the conducting power of the gold-silver alloy, made by different persons, of different gold and silver, are given in the following Table—

* This discrepancy may perhaps be attributed to some inaccuracy in the reproduction of the mercury standard.

† Proceedings of the Royal Society, Feb. 1861.

Alloy.	Hard-drawn.	Annealed.
1	100·3	100·6
2	100·2	100·7
3	98·8	99·2
4	....	100·2
5	100·4	100·7
6	99·7	99·8
7	100·3	100·8
8	100·1	100·4

which shows, therefore, that the alloy may be prepared in a commercial way, and still have a conducting power which varies less than that of a pure metal prepared at different times by the same experimenter. If we look at the hard-drawn series, we find five out of the seven wires tested agree together exceedingly well, the greatest difference being only 0·3 per cent. These five alloys were made, three in London, by scientific chemists, one in Frankfort-on-the-Maine, and one in Brussels. Those which agree least with the others were made in New York (No. 3) and by a well-known assayer in London (No. 6).

2. *On its homogeneity and its molecular condition.*—If the wires of the alloy made and drawn by different persons were not homogeneous, the values obtained for the conducting power could not have agreed so well together. It has been already mentioned that some of the alloys determined by Thomson, when redrawn, were found to have a different conducting power*.

Alloy.	Conducting power of wire as received from the wire-drawer.	Conducting power after being re-drawn.
1	100·0	100·0
2	100·7	95·8
3	103·9	102·9
4	94·6	100·8
5	96·0	98·1
6	92·0	89·9
7	74·7	86·0
Pure copper.	100·0	98·6

Of course, here again, some of these differences are due to the temperature in each case not being the same; but the differences found with the alloys 2, 4, and 6 were undoubtedly due to faulty wires. It was for this reason that care was taken to have the alloy drawn by different persons, in order to see if this would influence the results obtained with them, as well as to ascertain whether the wires would show the same faults as silver and copper does when not carefully drawn. It has been argued that the molecular condition of all alloys is liable to undergo a change by age, and that, therefore, alloys are not fit to be used as standards. Thus it is well known that brass and German silver become brittle and crystalline by age, and that the same may occur with the gold-silver alloy; but on looking at the composition of the alloy, it will be found to have nearly the same as that of the gold chains of commerce. Now, we do not know of a single instance where such a chain, even after years of use, becomes brittle or crystalline; so that we think it more than possible that the alloy will not change its molecular condition by age. It must also be remembered that even when German silver becomes

* Proceedings of the Royal Society, Feb. 1861.

brittle, it does not materially alter in its conducting power. The same has already been proved, and mentioned in this Report, to be the case with iron and cadmium.

3. *On the effect of annealing on the conducting power of the alloy.*—When the alloy is heated to redness and cooled slowly, its conducting power was found to have increased only 0·3 per cent.—this value being the mean of eight wires annealed in different ways,—proving, therefore, that if the wires may be only partially hard-drawn, it will make but little difference in the conducting power.

4. *On the influence of temperature on the conducting power of the alloy.*—When wires of this alloy are heated from 0° to 100°, a decrement in the conducting power, amounting to 6·5 per cent., will be found. The same arguments may, therefore, be put forward in favour of the use of the alloy as a standard, as were done in the case of mercury when speaking of this property.

To sum up, therefore, the arguments in favour of and against the use of the three propositions made to reproduce standards of electric resistance, we find in favour of a pure metal in a solid state:—

1. That it appears that all descriptions of electrotype copper, when carefully drawn, have the same conducting power.

Against it:—

1. That their preparation, with the exception of the electrotype copper in a state of purity, is exceedingly difficult; so that independent persons preparing the same metal find, on comparing the conducting powers obtained for them, that they vary several per cent.

2. That the influence of annealing on their conducting powers is so great that differences may occur simply because the wires are partially hard-drawn.

3. That the influence of temperature on their conducting power is very great; so that slight errors in thermometers, or in the reading of them off, would materially affect the result.

In favour of using mercury as a means of reproducing standards the following may be said:—

1. That no molecular change can take place in the metal, nor can any alteration occur in its conducting power, on account of annealing; for its temper is always the same.

2. That the influence of temperature has only a small effect upon its conducting power.

And against it:—

1. That there is a difficulty in obtaining absolutely pure mercury; so that the results obtained by different observers show great variations.

2. That the standard tube cannot be kept full of mercury for any length of time, owing to the diffusion of impure metal, arising from the amalgamated terminals into the narrow tube; so that each time the standard has to be used, it must practically be remade.

3. If the tube be broken during the process of cleaning or otherwise, it is not yet certain with what exactitude the standard could be reproduced.

4. It is doubtful whether the resistance of a tube filled with mercury to-day will have the same resistance if filled a year hence; for we have no proof if the dimensions of the tube will not alter by being kept. It is well known that the bulbs of thermometers are liable to change, and are continually changing, in capacity.

In favour of the gold-silver alloy may be said:—

1. That this material, when prepared and drawn by different persons, was

found not to vary in its conducting power more than 1·6 per cent.; whereas the variations found with the metals in a solid state, prepared and drawn by different persons, amount to several per cent., and those found for mercury by different observers amount also in all cases to several per cent.

2. That the homogeneity and molecular condititiion of this alloy are always the same.

3. That the effect of annealing on the conducting power is very small, being only 0·3 per cent.; so that if a wire be partially hard-drawn, its conducting power will not suffer to any appreciable extent.

4. That the influence of temperature on its conducting power between 0° and 100°, viz. a reduction of 6·5 per cent., is smaller than either that of the metals in a solid state, viz. 29·3 per cent., or that of mercury, viz. 8·3 per cent.

And against it:—

That the conducting power may alter by age, as the physical properties of alloys are more likely to change than those of metals.

From the foregoing statements, based on facts at present known, it would appear that the best method of reproducing standards, for those who are unable to procure copies of the British Association unit of electrical resistance, is that they should make, or have made, a certain amount of the gold-silver alloy (as described in the Phil. Mag., Feb. 1861), by two or three different persons, in order to ensure a correct result, and take a given length and section or weight of it, at a given temperature, which has been found equal in resistance to the British-Association unit. We would recommend, in order further to test what we have stated in the foregoing Report, that three or more scientific men and electricians be requested to compare the resistances of pure mercury (obtained by them from the best sources they are able) and of the gold-silver alloy (made in the manner described in the Phil. Mag.) with a German-silver standard supplied to them by your Committee. If this be done, results would be obtained which would put an end to many disputes on the subject, as well as decide which of the above means is practically the best for reproducing standards of electrical resistance where no copies of the British-Association unit can be obtained.

Appendix D.—Professor Kirchhoff's *Letter.*

To Fleeming Jenkin, Esq.

Heidelberg, June 8, 1862.

Dear Sir,—I have the honour to acknowledge the receipt of your letter of the 31st of May, in which you inform me of the labours of the Committee appointed by the British Association, to try and bring about the general introduction of one unit of electrical resistance. I gladly respond to the invitation to express my view on the manner in which the desired object might be best attained.

To define the unit of resistance by the resistance of a wire of given dimensions of a pure metal appears to me impossible, for the reasons which have been urged by the Committee; hence, of the three proposals discussed by the Committee, there only remains two for our consideration.

1. To adopt the unit proposed by Weber; or, 2. To establish, as unit of resistance, the resistance of a column of pure mercury of given dimensions and at a given temperature.

I do not think that to these a third of equal value can be added; for to define the unit of resistance by the thermal action of an electrical current would certainly never answer the purpose, because this thermal action cannot be measured with the necessary accuracy, and the resistance of any wire which is to be permanently kept cannot be fixed as unit; for the resistance of any wire for a given temperature certainly undergoes changes if electrical currents are transmitted through it, and it is exposed to fluctuations of temperature.

Of the above two units, the first recommends itself by coming up more satisfactorily to the demands of science; the second, as I think, by being capable for the present of being practically carried out with greater accuracy. But is it really necessary to decide for one and against the other of these two units? I think not. If the ratio between them is established with the accuracy which is now attainable, there can, I think, arise no more confusion from their simultaneous use, than from the practice of expressing lengths sometimes in metres and sometimes in millimetres. You say, "It is proposed that the unit adopted shall be represented by one particular standard, constructed of very permanent materials, laid up in a national repository;" and further, "The Committee will probably endeavour to devise some plan by which copies of the actual material standard adopted may be easily procured at a reasonable cost." This plan, the execution of which I consider highly desirable, might evidently be realized in all its essential points without its being necessary to give the preference to one of these units over the other: it would only be necessary to measure the resistance of the normal standard in *both* units, and to add to each copy its resistance expressed in *both* units.

In choosing the metal or the alloy of which the normal standard and the copies are to be made, care must undoubtedly *first* be taken that the resistance is as unalterable as possible for *one* temperature. It is undoubtedly desirable that the resistance shall not vary rapidly with the temperature. This is, however, not very important, provided that the temperature of the wire can be accurately observed at any moment. To satisfy this condition, the wires must not be coiled upon cylinders, but fastened so that, for the greater part of their extent, they lie clear, and hence rapidly assume the temperature of the surrounding air or of the non-conducting liquid in which they may have been immersed.

You request me to point out to you any researches of mine which refer to a unit of electrical resistance. I have to mention a short treatise only, which appeared in vol. lxxvi. of Poggendorff's 'Annalen,' under the title "Determination of the Constants on which the Intensity of Induced Electrical Currents depends," and which formed the answer to an academical prize-question which Professor Neumann, in Königsberg, had proposed in the year 1846. In this treatise a unit of electrical resistance has not been suggested; but in it the resistance of a wire has been measured by the unit (or rather by double the unit), which was afterwards proposed by Weber in his "Electrodynamic Measurements." Professor Weber has subsequently had the kindness to compare the copper wire whose resistance I measured with those whose resistances he himself had determined (Pogg. Ann. vol. lxxxii. p. 360); he thereby found the resistance of my wire about one-seventh greater than I had found it. The reason of this want of agreement consists partly in the imperfection of the instruments which I had used, and partly in the fact that in my experiments the temperature was little above 0° R., while in Weber's experiments it was about 20° R.

Allow me, my dear Sir, to record the very great respect with which I have the honour to be,

Yours very truly,
G. KIRCHHOFF.

APPENDIX E.—Dr. SIEMENS'S *Letter.—Suggestions for the adoption of a Common Unit in measurement of Electrical Resistance.*

To the Committee appointed by the British Association to report on Standards of Electrical Resistance.

GENTLEMEN,—I beg to acknowledge, with thanks, the honour you have done me, in requesting me to furnish you with suggestions in furtherance of your endeavours to procure the adoption of a common unit of electrical resistance.

I proposed in Poggendorf's 'Annalen' (vol. cx. p. 1) to supply this want by the adoption of the conducting power of mercury as unit, and of the resistance which a prism of that metal a metre long and a square millimetre section, at 0° C., opposes to the passage of a current, as unit of resistance.

The method by which I constructed standards in this unit was as follows:—

From the ordinary glass tubes of commerce, pieces were selected whose calibre was found to vary most regularly. After the selected tubes had been ground to the length of a metre, they were carefully cleaned and filled with pure mercury—the temperature being measured. The contents were then weighed, and the values reduced to 0° C. for expansion of glass and metal. The resistances of the tubes were calculated by the formula

$$W=\frac{l^2\sigma}{g}\cdot\frac{1+\sqrt{a}+\frac{1}{\sqrt{a}}}{3},$$

which represents the resistance to a current in the longer axis of a prismatic conductor either in the above unit or in 0·001 unit, according as l is expressed in metres and g in grammes, or l in millimetres and g in milligrammes respectively. $\sigma=13{\cdot}557$, the specific gravity of mercury, at 0° C.

$$\frac{1+\sqrt{a}+\frac{1}{\sqrt{a}}}{3}$$

is the coefficient for conicalness, which in good tubes equals 1 very nearly. a is the ratio of the greatest to the least transverse section of the tube.

All the data therefore necessary for the value of W are exact measures of length and weight. Measurements of the same tube, at different times, gave results corresponding within 0·01 per cent. with each other.

The first objection which is raised against the adoption of mercury as unit, "that the tubes cannot be made of uniform or similar wires, and that the standard once broken is lost for ever," is clearly untenable, since the tubes are not required to be uniform, and the breakage of the standard involves only the necessity of a new tube, and the determinations of length and weight anew, to put the operator in possession of a new standard, whose agreement with the broken one will depend solely on his own handiness in manipulating. Every standard, of whatever material, is liable to injury; but the breakage of a glass is infinitely to be preferred to the treacherous results of a bruised wire.

Mercury is, of all metals, that which is best suited to supply a reproducible standard.

In the first place, it is procurable *pure* in sufficient quantities. I heated for some hours samples of commercial mercury under sulphuric acid containing a few drops of nitric acid, and found their conducting powers afterwards to be precisely the same as that of a quantity of chemically pure mercury reduced from the oxide.

Secondly, mercury has always the same molecular structure, and has therefore, at the same temperature, always the same resistance.

From these two grounds it is possible to couple with this unit a geometrical conception which is indispensable in practice.

Thirdly, of all metals capable of being used for resistances, mercury has the lowest conducting power; and of all pure metals capable of the same application, its resistance varies least with variations of temperature.

Having formed such original standards, it only remained to copy them in a convenient form for employment in practice. This I have done,—

1. In mercury contained in glass spirals, and
2. In German-silver wire.

The resistance-bridge which I made use of in these measurements, with a reflecting galvanometer in its circuit, enabled me to attain a precision of within 0·01 per cent.

The mercury spirals, as may be seen by the accompanying drawing*, are provided with cups at their ends, for convenience of filling and for receiving the contacts of the measuring apparatus. They are either of known resistances, approximating only to a multiple of the unit, or may be adjusted to an exact multiple by boring out one of the ends of the tube, which, in this case, must stand up half an inch inside the cup. The resistances of the bridge must then be arranged so that no current passes through the instrument only when the desired resistance in the fourth side is reached. When the spiral is filled, a vulcanized india-rubber ring is put round the cups, and the spiral is suspended in a vessel of ice-water or water kept in circulation by passing a current of air through it, and the temperature measured by a delicate thermometer.

The electrical value of each spiral which I have made has been determined by comparing it with at least two of the straight normal tubes, both being kept during the measurement in ice-water. The greatest differences which I have found between such determinations do not exceed 0·05 per cent., to which limit the copies may be trusted.

In answer to the objection that an admixture takes place between the mercury and the solid metal used for the terminals, I must remark that I have found this occasion really less inconvenience than is generally believed. I kept the copper connexions immersed in the mercury a whole week, but could not perceive the slightest decrease in its resistance. Platinum electrodes of considerable surface might be employed; but I believe that the removal of the copper connexion after each test, and the removal of the old mercury from their surfaces before using them again, are a sufficient safeguard against error arising from this source. Besides, it is easy to fill the spiral with fresh mercury whenever it is suspected to have dissolved any quantity of copper, or even on every occasion when a measurement with it is to be made. Nor does mercury change its resistance in the least by standing in the air. This I have proved by keeping a spiral six months filled without changing the mercury, and found its resistance to be constant.

* The drawings have been omitted, the descriptions being intelligible without them.

The material which I have extensively employed in copying this measure, viz. German silver, may be classed under the same head as the expensive gold-silver alloy of Dr. A. Matthiessen, over which it has, however, the considerable advantages of a greater specific resistance, and that its resistance varies less with temperature variations.

As a preventive against alteration of resistance by the influence of the air, I have usually had the resistances made of this metal covered with a coating of silk and lac.

Intermediate between the resistances to be measured and the measure itself I have introduced resistance-scales. These contain each a series of resistances (multiples of the unit), and are so arranged that each resistance is exact when it stands stopped alone in the circuit. When carefully made, these scales may be depended on to 0·1 per cent.

Being convinced of the sufficiency of the method I have described of reproducing a standard of electrical resistance, I have the honour to suggest to you:—

1st. To recommend the universal adoption of the conducting power of mercury as unit, and of the resistance which a prism of that metal, a metre long and square millimetre section, at 0° C., opposes to a current of electricity as common unit of resistance.

2nd. To have the value of this measure ascertained, with the greatest possible exactness, in absolute units.

3rd. To have copies of this unit constructed in mercury contained in glass spirals for preservation in scientific repositories.

In the event of my suggestions being adopted, the mercury unit should be determined again with the greatest possible care, and with all the help which pure and applied science offers, and copies of it made with equal exactness.

According to a late determination by Weber, the mercury unit is only about $2\frac{1}{2}$ per cent. greater than 10^{10} absolute units, or one mercury unit at $-26°$ C. would equal 10,000,000,000 absolute units.

Since those cases in which the expression of resistances in absolute measure is of advantage in facilitating calculations occur only very seldom, and only in purely scientific exercises, a single determination of the relation of the two measures would be amply sufficient. Should the absolute unit or any multiple of it be adopted as common unit of resistance, there would still be wanted a unit for expressing the conducting powers of bodies; and mercury is indisputably the best calculated for this purpose. And for practical purposes, which in adopting a universal unit should be principally taken into consideration, it is indispensable to define the resistance-measure as a geometrical body of that material which is selected as unit of conducting power. Every other definition would not only burden unnecessarily the calculations which occur in common life, but also confuse our conception of the measure.

The reason why the arbitrary unit proposed by Jacobi (a length of copper only *approximately* defined) found no admittance into general use is to be sought in the fact that it failed to fulfil this condition, and because the conducting power of all solid bodies is too dependent on their molecular structure.

The same objection renders the adoption of the gold-silver alloy proposed by Dr. A. Matthiessen equally incapable.

Another disadvantage in the way of a solid metal unit is the impossibility to solder thick connexions into the ends of a defined length of any wire without altering its resistance.

Should the adoption of the mercury unit be deemed advisable, I would

place at the service of the British Association any further information or assistance in my power.

I have the honour to be, Gentlemen,

Your most obedient Servant,

W. Siemens.

Appendix F.—*Extracts from a Letter addressed to* Professor Williamson *by* Dr. Esselbach.

The two objections against the practical applications of Weber's absolute unit have been sufficiently pointed out as being—

1. Its minuteness; and
2. That the electromotive force of galvanic elements does not allow of variation (as strength of current, tension, and resistance do), but that we have to accept certain constants as nature has fixed them.

I take it for granted that the standard of absolute unit would not lose in authority if a plain multiple of it were adopted. I need not point out that the French metre itself is only a submultiple, $\frac{1}{10,000,000}$th of a natural unit—the earth's quadrant. The multiple of the natural electro-magnetic unit I am about to suggest for practical use is 10^{10}, therefore very simple (which is of no little importance); and it is a multiple which leads us to those standards which are practically used.

M. Bosscha gives the electromotive force of his Daniell's cells in absolute measure as

$$1025{\cdot}80 \,.\, 10^{8},$$

and calculates the one used by Mr. Joule to be

$$1045{\cdot}1 \,.\, 10^{8}.$$

It will therefore be practicable to determine such concentration of sulphuric acid as to make the electromotive force equal to

$$10 \,.\, 10^{10};$$

and I believe the concentration required would be very near what is actually used in telegraphy.

Resistance.—The different copies of Jacobi's étalons are well known to differ as much between each other as Daniell's cells; and if Siemens had done nothing else for galvanometry than to give us copies which agree among themselves within a quarter per cent., the progress is obvious.

Weber's copy of Jacobi's étalon is

$$598 \,.\, 10^{7};$$

and that of M. Bosscha was

$$607 \,.\, 10^{7}$$

in absolute measure.

Other statements (of Kirchhoff and others) give a much smaller value.

In comparing Mr. Siemens's mercury standard with three copies of Jacobi's étalon in his possession, I found two of them agreeing tolerably well with each other, and with a third one copied by my friend Dr. Teddersen, at Leipzig, from the original of M. Leyser, which I took therefore to be the more correct ones. I found the absolute value of Siemens's unit to be

$$\frac{603}{660} \,.\, 10^{10},$$

or

$$1{\cdot}1 \text{ Siemens's unit} = 10^{10}.$$

We should therefore only have to multiply all observations expressed in

Siemens's units by $\frac{10^{10}}{1 \cdot 1}$ to reduce them to absolute measure, and the suggested multiple for the future standard would not be far from 1·1 of Siemens's units, which every one admits to be for metallic conductors a practical unit.

For the resistance of insulating materials the figures become impracticably high; but it would be a matter of professional telegraphy to adopt, in conformity with the system, the 'resistance' 10^{10} and, besides, another 'great resistance' containing 10^{10} 'resistances.'

While the resistance of a mile of copper in an ordinary cable would be (say) 4 R. (four resistances), the insulation-resistance of a mile of cable would be about 0·04 G. R. (great or gutta-percha resistances).

My suggestion would therefore be—

1. To adopt Weber's absolute unit, and to derive from it, by the multiple 10^{10} (or 10,000,000,000), the practical unit.
2. To adopt 10^{10} of Weber's electro-magnetic units as the 'practical absolute unit' for electromotive force and resistance.
   (10 of these units would be exactly 1 Daniell's cell).
3. 1 of these units would be 1·1 of Siemens's units.
4. To allow, besides, a 'practical great unit,' viz. 10^{10} of the 'practical units,' for resistances in order to express the insulation-resistance of cables in convenient figures.
5. To allow also a 'practical small unit' of $\frac{1}{10^{10}}$ absolute units to express insulation-currents and charge-quantities of cables in convenient figures.
6. To adopt, in order to avoid confusion, for such 'practical units' a terminology as proposed by Messrs. Bright and Clark.

London, September 18, 1862.

Appendix G.—*Circular addressed to Foreign Men of Science.*

Sir,—I am requested to inform you that a Committee was appointed by the British Association, which met last year at Manchester, to report on Electrical Standards of Resistance.

The Committee consists of the following gentlemen:—

Professor A. W. Williamson, F.R.S. (University College, London).
Professor Charles Wheatstone, F.R.S. (London).
Professor William Thomson, F.R.S. (Glasgow).
Professor W. H. Miller, F.R.S. (Cambridge).
A. Matthiessen, Ph.D., F.R.S. (London).
Fleeming Jenkin, Esq. (London).

The Committee met on December 6th, 1861, and on April 3rd, 1862. On the latter occasion the following Resolution was passed:—

"*Resolved,*—That the following gentlemen be informed of the appointment of the present Committee, and be requested to furnish suggestions in furtherance of its object.

Professor Edlund (Upsala).
Professor Th. Fechner (Leipzig).
Dr. Henry (Washington).
Professor Jacobi (St. Petersburg).
Professor G. Kirchhoff (Heidelberg).
Professor C. Matteucci (Turin).
Professor Neumann (Königsberg).
Professor J. C. Poggendorff (Berlin).
M. Pouillet (Paris).
Werner Siemens, Ph.D. (Berlin).
Professor W. G. Weber (Göttingen)."

I have, in consequence, the honour of addressing you the present letter.

The Resolutions passed at the two meetings are enclosed, and from them you will gather the general scope of the Committee's inquiry. I add some further explanation as to the objects and intentions of the Committee.

Great inconvenience has been felt from the absence of any generally adopted unit for the measurement of electrical resistance, and it was thought that the influence of the British Association might be successfully exerted to procure the adoption of a common standard. The present time was thought especially favourable, since, although the methods of observation have been brought to great perfection, no local units have as yet taken very deep root.

The units which up to the present time have been considered by the Committee may be classed under three heads:—

1st. A given length and weight or section of wire made of some pure metal, and observed at a given temperature, as originally proposed by Professors Wheatstone, Jacobi, and others.

2nd. Units based on Weber's and Gauss's system of absolute measurement.

3rd. A given length and section of pure mercury at a given temperature.

Whatever basis is adopted for the unit, it is proposed that the unit adopted shall be represented by one particular standard, constructed of very permanent materials, laid up in a national repository; and it has been proposed to use Dr. A. Matthiessen's gold-and-silver alloy for this purpose. The arguments which have been used for and against these systems are as follows:—

In favour of the use of a wire of some pure metal it is said—

That the plan is the simplest possible, and admits of independent observers forming their own standard.

Against this plan it is said—

1st. That even when pure, two apparently similar wires do not resist equally unless their temper or molecular condition be the same—a condition which cannot practically be ensured.

2nd. That there is reason to believe that the resistance of a given wire is not constant even at a constant temperature.

3rd. That the resistance of all pure metals varies very rapidly with the temperature.

4th. That great difficulty is found in obtaining any metal pure, and that the attempt of most persons to reproduce the unit for their own use would be attended with incorrect results. This is evidenced by the different relative results as to the resistance of pure metals published by different observers.

In favour of Weber's units it is urged—

1st. That their use will ensure the adoption of a complete system of corresponding standards for electrical currents, quantities, and tension or difference of potential.

2nd. That their use is essential in the dynamic treatment of any problem connected with electricity; for instance, in determining the heat generated, the force exerted, the work done, and the chemical action required or produced under any given circumstances.

3rd. That their use would be a simple extension of the system already universally adopted in magnetic measurements.

4th. That the unit is independent of the physical properties of any material.

Against the system it is urged that the unit cannot be determined with sufficient accuracy, and that even its approximate reproduction, where copies cannot be obtained, is difficult and expensive.

In favour of the mercury standard the following arguments are used:—

1st. No change can occur in the molecular structure or temper of the

material, and therefore the same tube filled with pure mercury will certainly always conduct alike.

2nd. Change of temperature causes only a slight difference in resistance.

Against this plan it is said—

1st. That tubes cannot be made of uniform or similar wires, and that, therefore, the standard once broken is lost for ever.

2nd. That the standard tube cannot be kept full of pure mercury, owing to the admixture which would take place of the solid metal used for the terminals, so that each time the standard has to be used it has practically to be remade.

3rd. That the attempt, by most observers, to reproduce the unit for their own use would be attended with incorrect results, as is shown by the different results obtained by different observers.

In favour of Dr. Matthiessen's alloy, as compared with wires of pure metal, or with mercury, as a material for the standard, it is said—

1st. That the variations of resistance, corresponding with variations of temperature or temper, are small.

2nd. That a unit expressed in this material can be more readily and certainly reproduced than one expressed by a pure metal, because the presence of slight impurities in the component metals, or a slight change in their proportion, does not sensibly affect the result.

Against this plan it is said that the physical properties of an alloy are more likely to change than those of a pure metal.

Against all the plans for standards, based on an arbitrary length and section of an arbitrary material, the supporters of the absolute units state that the adoption of such an arbitrary standard would lead to great confusion and complication in the measurement of all other electrical properties, and in the expression of the relation of such measurements to those of force, work, heat, &c.

This objection does not, of course, apply to the expression of the absolute unit by means of a wire of pure metal, of an alloy, or by mercury: but it is urged that no observer should ever attempt the reproduction of a standard when a copy of the proposed universal standard can possibly be obtained; and the Committee will probably endeavour to devise some plan by which such copies of the actual material standard adopted may be easily procured at a reasonable cost.

It will be seen, from the resolutions passed, that the Committee are now engaged in investigating the degree of accuracy with which Weber's units can be obtained, and the degree of permanency which may be expected from the use of the metal or alloy forming the material standard expressing these or other units.

The Committee will feel greatly indebted to you if you will afford them the benefit of your valuable advice and experience on the above points, and on any others which may occur to you. They also venture to hope that such a standard may be selected as will give very general satisfaction; and, if approved by you, that you will kindly take an interest in procuring its general adoption.

Personally being charged with the duty of preparing an historical summary of the various units proposed, I shall be grateful if you will favour me with any remarks as to your own labours in this field, or if you could oblige me with references to any papers or works in which the subject is treated.

I am, Sir,

Your obedient Servant,

FLEEMING JENKIN.

Appendix H.—*Description of the Electrical Apparatus arranged by* Mr. Fleeming Jenkin *for the production of exact copies of the Standard of Resistance.*

This apparatus is a simple modification of that generally known as "Wheatstone's bridge." It contains, however, some special arrangements, in virtue of which various practical difficulties are avoided, so that very great accuracy can be ensured with comparative ease. The usual bridge-arrangement is shown in Plate I. fig. 9, where the irregular scrolls, A, C, R, S, represent the four conductors of which the resistance is to be compared; the thick black lines show those portions of the circuit which join the coils with the four corners, U, V, Z, Y, and are supposed to have no sensible resistance in comparison with the coils; finally, the thin lines show connexions, the resistance of which in no way affects the accuracy of the comparison between the four coils. By this arrangement the four conductors, A, C, R, S, are so connected with the galvanometer, G, and the battery, B, that no current passes through the galvanometer when the conductors bear such a relation to one another that the equation $\frac{A}{C}=\frac{S}{R}$ holds good; whereas a current in one or other direction passes so soon as $\frac{A}{C}$ is greater or less than $\frac{S}{R}$*. Thus the direction and strength of the current observed serve as guides by which the resistance of any one of the conductors may be gradually adjusted by shortening or lengthening the wire, until on the completion of the circuit no deflection whatever can be observed on the galvanometer, however delicate it may be, or however powerful the battery used. When this has been done, we may be sure that the above relation exists between the four conductors. In practice, it is seldom desirable to use powerful batteries; the test is made delicate by the use of an extremely sensitive astatic galvanometer.

In speaking of the four conductors, A, C, R, S, which are generally all coils of wire of similar construction, although each fulfilling a distinct function, some difficulty often occurs in explaining readily which coil or conductor is referred to. They can of course be distinguished by letters; but this requires reference to a diagram on every occasion, and the writer has therefore been in the habit of distinguishing the four coils by names drawn from a very obvious analogy existing between this electrical arrangement and the common balance in which one weight is compared with another. The equality between the two weights on either side of a balance, when the index is at zero, depends on the equality of the arms of the balance; and if the arms are unequal, the weights required to bring the index to zero are proportional to the arms (inversely). Let A and C be called the arms of the electrical balance, while S and R are looked on as analogous to the standard weight and mass to be weighed respectively, and let the galvanometer-needle stand for the index of the balance. Then all the above statements, with respect to the weights and arms, hold good for the electrical arrangement (except that the proportion between the electrical arms and weights is direct instead of inverse). The writer therefore calls this arrangement an electric balance—A and C the arms, S the standard, and R the resistance measured†.

* This statement holds good also if the battery and galvanometer wires, as shown in diagram, are interchanged.

† The name of parallelogram, sometimes given to the arrangement, is objectionable, inasmuch as the relation obtaining between the four conductors is not that which exists between the four sides of any parallelogram, except in the one case of equality between all four conductors. The connexions are, however, most easily followed in a drawing when

In the adjustments of resistance-coils or copies of a standard, the object is to produce a second coil, R, exactly *equal* to the first or standard, S; and the arms, A, C, must therefore be absolutely equal before, by this arrangement, an exact copy can be made. Hitherto it has often been the practice to use for the arms, A, C, two coils made as equal as possible, and placed so close as to remain at sensibly equal temperatures; so that the equality between R and S is dependent on the equality between A and C, and cannot be determined with greater accuracy than that between these coils. This limit to the accuracy is a defect for our present purpose, and the writer has, moreover, found it undesirable to depend on the permanent equality of two coils. It is by no means certain that, without very extraordinary precautions, the two arms will remain unaltered in their original equality. A slight molecular change, or a slight chemical action on the surface of the wires, disturbs this equality permanently; and even if the coils are so constructed as to remain really equal at equal temperatures, the accidental passage of a current through one arm, and not through the other, for a very short time, will disturb their accuracy very sensibly for a considerable time. There are various devices by which the equality to be established between R and S may be rendered independent of the absolute equality between A and C, and the writer has adopted a plan, now to be explained with the aid of the diagrams (figs. 7, 8). This plan allows the approximation to equality between R and S to be almost indefinitely increased.

It will be seen that fig. 7 does not differ from fig. 9, except by the addition of a wire, WX, of sensible resistance, between the two coils A and C. The point U is no longer fixed, but can be moved along WX. The arms of the balance are therefore no longer A and C, but A+XU and C+WU. Thus the movable point U affords the means of slightly altering or adjusting the ratio of the two arms. A and C are made as equal as possible, independently of WX, which is a very short wire.

The test is made as follows:—When the standard and coil to be measured have been put in their places as in fig. 7, the point U is moved along the wire WX until the galvanometer-index is not deflected when the circuit is closed. The position of the point U is noted by a scale. R and S are then reversed, so as to occupy the position relatively to A, C shown in fig. 8. The point U is again moved until the galvanometer-needle remains undeflected on the circuit's being closed. The new position of U is again observed by a scale. If the point U does not require to be moved at all, we may be quite sure that R is exactly equal to S, and that $A+XU=C+WU$, since it would be quite impossible that the ratio $\frac{A+XU}{C+WU}$ should be equal to both $\frac{R}{S}$ and $\frac{S}{R}$, unless this ratio were equal to 1. Moreover, if WX be made of the same wire as the coils A and C, and if those coils are formed of about 100 inches of wire, and if the observed positions of U differ by a given distance, x, this length, x, measured in inches, will express very nearly the difference between R and S in a percentage of the whole length of R. Thus, if x be one inch, the standards S and R differ by about one per cent. If the point U, when adjusted in each case, be found nearer R than S, then R is the smaller of the two, and *vice versâ*. The percentage of error in R, thus measured, is not of course strictly accurate, inasmuch as the ratio between the two arms is not exactly

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arranged as the four sides of a quadrilateral figure. Professor Wheatstone's original name of Differential Resistance Measurer does not, as it seems to the writer, sufficiently distinguish this arrangement from other differential methods.

$\frac{101}{100}$; but if WX be not more than three or four inches long, the percentage of error measured in this way is quite sufficiently accurate to allow the new coil to be so exactly adjusted after very few trials, that no greater movement of U than (say) $\frac{1}{10}$th of an inch is required to prevent any deflection on the galvanometer when R and S are reversed. We may then be sure that no greater error than (say) about 0·1 per cent. exists in the equality between the new coil and the standard. Two fresh coils, A_1, C_1, are then taken, containing each about 1000 inches of wire similar to WX, or an equivalent resistance. It will then be found that, to maintain the index at zero when R and S are reversed, U must be moved about ten times as much as before, or (say) one inch. R can then be still further adjusted till U is not moved more than $\frac{1}{10}$th of an inch, when a new degree of approximation to equality, with an error of not more than 0·01 per cent., will have been reached. Then the coils A_1, C_1 are changed for a fresh pair, A_2, C_2, with a resistance equal to about 10,000 inches of the wire WX: one-tenth of an inch on WX will then represent an error of only 0·001 per cent. By a repetition of this process, quite independently of any absolute equality between the pairs A, C, A_1, C_1, A_2, C_2, &c., a gradual approximation to any required extent may be ensured. The delicacy of the galvanometer used, and the nicety of the means available for increasing or diminishing the resistance of R, form the only limits to the approximation. A slight want of equality between any pair of arms will simply bring the point U a little to one side or the other of the centre of WX, as the final adjustment with that pair is made, but will not affect the truth of the comparison between R and S. Each pair must, however, be so nearly equal that the addition of part of the short wire, WX, to one side will be sufficient to correct the other; otherwise the adjustable point U would not bring the index to zero, even when at one end of the wire.

This arrangement, besides rendering us independent of the accuracy of any two arms, has some incidental advantages of considerable practical importance. At each test it gives a measure of the amount by which the new coil to be adjusted must be lengthened or shortened. The test is at first comparatively rough, or adapted to errors of one or two per cent., and only gradually increases in delicacy as the desired equality is more and more nearly approached. It is not necessary that the resistance of WX should remain absolutely constant, since it is only used (numerically) to give a rough approximation to the percentage of error. It is desirable that the battery should remain in circuit as short a time as possible; the circuit is therefore broken between 1 and 2, figs. 7 and 8, by a key, K, with which contact should be only momentarily made, when all the other connexions are complete. The direction of the jerk of the galvanometer-needle to one side or the other need alone be observed; no permanent deflection is required with this arrangement as a guide to the amount of error. This is a considerable advantage, inasmuch as it avoids heating the wires, and saves time. The induction of the coils on themselves might lead to some false indications, unless special precaution were taken against it, as pointed out by Professor W. Thomson*. To avoid this source of error, the galvanometer circuit is broken between 3 and 4, figs. 7 and 8, at K_1, and should only be closed after the battery circuit has been completed at K and equilibrium established throughout all the conductors.

Before passing to a detailed description of the apparatus as actually con-

* *Vide* Phil. Mag. August 1862.

structed, some remarks are required as to the means of making temporary connexions. All connexions which require to be altered may be the means of introducing errors, inasmuch as the points of contact are very apt to offer a sensible but uncertain resistance. In measuring small resistances, the resistance at the common binding-screws is found to create very considerable errors. Binding-screws have therefore to be avoided at all points where an uncertain resistance could cause error. Mercury-cups, made as follows, have been found in practice very suitable for temporary connexions, and have been adopted in the apparatus. The bottom of each cup is a stout copper plate, with its surface well amalgamated, forming one of the two terminals to be joined. A stout copper wire, $\frac{1}{4}$ inch in diameter, with a flat end well amalgamated, forms the other terminal. When the amalgamation is good, and care is taken that the wire shall rest on the plate, this form of connexion offers no sensible resistance. The amalgamated wire is easily kept bright and clean by being dipped from time to time in a solution of chloride of mercury and wiped. The copper plate should also be removed from the cup, cleaned, and re-amalgamated occasionally. All permanent connexions should be soldered.

The apparatus itself, as actually constructed, will now be described (figs. 1 to 6). It consists of a wooden board*, about 12 in. × 7 in., containing the mercury-cups, the adjusting wire, WX, the key, K, and the terminals to which the battery and galvanometer are connected. The letters in the figures 1 to 6 correspond exactly to those used in the diagrams 7 and 8; and the apparent complexity of the connexions can thus be easily disentangled. cc_1, aa_1 are two pairs of mercury-cups, into which the terminal wires on the bobbin, C, A, dip. This bobbin contains the two coils, C and A, forming the arms of the balance. rr_1 and ss_1 are mercury-cups, into which the terminals of the standard and coil to be adjusted are placed. These mercury-cups are so connected with the four cups, d, d_1, f, f_1, that when d is connected with d_1, and f with f_1, by a couple of wires in a small square of wood, D, then A, C, S, and R are connected as in fig. 7; but when D is turned round, so as to connect d with f, and d_1 with f_1, A, C, R, and S are connected as in fig. 8. D is called the commutator. The same end might be effected without a commutator by simply interchanging R and S; but it is frequently inconvenient to do this. All these connexions are made by short stout copper bars, dotted in fig. 2. The wire WX, the sliding brass piece H, carrying a spring for the contact at U (fig. 4), and the scale E, by which the position of H is observed, will be readily understood from the drawing. The sliding piece, H, is connected with the proper points by the helix of copper wire, h, and the screw, I. G G_1 and B B_1 are common binding-screws, to which the wires from the galvanometer and battery are attached. K is the key, by depressing which, first, the battery is thrown into circuit, and then the galvanometer. It consists of three brass springs, 1, 2, 3 (fig. 5), each insulated one from the other, and connected by three screws, 1, 2, 3 (fig. 2), with the necessary points of the arrangement. A fourth terminal, 4 (figs. 2 and 6), is immediately under the free end of the springs, and is armed with a small platinum knob or contact-piece. The three springs are also all armed with platinum contact-pieces, all in a line one above the other (fig. 6). When the finger-piece, T, is pressed down, 1 and 2 are first joined, and then 3 and 4; 3 is insulated from 2 by the vulcanite, Q. All the connexions permanently made, under the board, are shown in fig. 2. Those which have no

* Experience has shown that this board should be made wholly of vulcanite, and not, as shown in the drawing, partly of wood and partly of vulcanite.—F. J., 1872.

sensible resistance are stout copper bars, and form the bottoms of the mercury-cups: those of which the resistance is immaterial are made of wire, insulated by gutta percha, and are simply shown as dotted irregular lines in fig. 2; they will be found, on comparison, to correspond with the thin lines on fig. 7. It will also be found that all those parts shown by thick lines in the diagram are made by thick bars or rods and mercury-cups.

Three sets of arms, C A, C_1 A_1, C_2 A_2, are provided; the shortest pair is first used, and U adjusted by the slide, H, till the galvanometer does not deflect when T is pressed down. The commutator, D, is then turned round, and U adjusted afresh. The coil, R, is then altered according to the two positions of U, and this process repeated, using the second and third pair of arms as required, until the desired approximation between R and S has been obtained. An astatic galvanometer, with a very long coil, will, for most purposes, give the best results; and one or two elements will be found a sufficient battery. The construction of R and S recommended, and the precautions to ensure perfect equality of temperature, will form part of next year's Report.

The apparatus, although specially designed for the production of equal coils, is applicable to ordinary measurements of resistance by comparison with a set of resistance-coils; for this purpose the terminals of the resistance-coils should be put in the place of the standard S, and any conductor of which the resistance is to be measured in the place of R. If a comparison by equality is to be made, the wire WX can be used as already described; it is, however, frequently desirable to make a comparison with one arm tenfold or a hundredfold greater than the other, by which means measurements of resistances can be made ten or a hundred times greater or smaller than could be done if equality alone between R and S were measured; for this purpose the three pairs, A C, A_1 C_1, A_2 C_2, are made exactly decimal multiples one of the other, and then, by taking A and C_1, or A and C_2, &c., in the cups $a\,a_1$ and $c\,c_1$, the required decimal ratio is obtained. The resistance of the wire WX would, however, falsify this ratio, and it is eliminated by a simple copper rod, which is placed for the purpose between the two cups $e\,e_1$, and maintains the whole wire WX at sensibly one potential. The commutator also is useless in measurements of this kind, and should be left untouched in the position shown in fig. 1.

The apparatus exhibited was manufactured for the Committee by Messrs. Elliott Brothers, of London, and gives excellent results.

SECOND REPORT—NEWCASTLE-ON-TYNE, AUGUST 26, 1863.

Members of the Committee:—Professor Wheatstone, F.R.S., Professor Williamson, F.R.S., Mr. C. F. Varley, Professor Thomson, F.R.S., Mr. Balfour Stewart, F.R.S., Mr. C. W. Siemens, F.R.S., Dr. A. Matthiessen, F.R.S., Professor Maxwell, F.R.S., Professor Miller, F.R.S., Dr. Joule, F.R.S., Mr. Fleeming Jenkin, F.R.S., Dr. Esselbach, Sir. C. Bright.

The Committee on Electrical Measurements, appointed in 1862, have not confined their attention to determining the best unit of electrical resistance, the point to which the duties of the Committee of 1861 were nominally restricted, but have viewed this comparatively limited question as one part only

of the much larger subject of general electrical measurement. The Committee, after mature consideration, are of opinion that the system of so-called absolute electrical units, based on purely mechanical measurements, is not only the best system yet proposed, but is the only one consistent with our present knowledge both of the relations existing between the various electrical phenomena and of the connexion between these and the fundamental measurements of time, space, and mass. The only hesitation felt by the Committee was caused by doubts as to the degree of accuracy with which this admirable system could be or had been reduced to practice.

The measurements of voltaic currents, electromotive force, and quantity would offer little difficulty, provided only electrical resistance could be measured in absolute units; and for this purpose it would be sufficient that the resistance of a single standard conductor should be so determined, since copies of this standard could be multiplied at will with any desired precision, and from comparison with these copies the absolute resistance of any circuit whatever could be obtained by methods requiring comparatively little skill and well known to all electricians. The practical adoption of the absolute system was felt therefore to depend on the accuracy with which the absolute resistance of some one standard conductor could be measured; and while doubts existed on this point, it was thought premature to make any extended experiments on the application of the absolute system to voltaic currents, electromotive force, or quantity. The Committee are happy to report that these doubts have been dispelled by the success of the experiments made for the Committee by Professor J. Clerk Maxwell, Mr. Balfour Stewart, and Mr. Fleeming Jenkin, according to the method devised by Professor W. Thomson. These experiments have been actively prosecuted at King's College for the last five months with continually increasing success, as, one by one, successive mechanical and electrical improvements have been introduced, and the various sources of error discovered and eliminated.

The Subcommittee are confident that considerably greater accuracy can yet be obtained by the further removal of slight defects, the importance of which only became apparent when the main difficulties had been overcome. In order, therefore, to secure the best attainable result, and still further to test the accuracy and concordance of the experiments before taking any irrevocable step, the Committee have decided not to issue standard coils at the present Meeting; but the results already obtained leave no room for doubt that the absolute system may be adopted, and that the final standard of resistance may be constructed without any serious delay. Over-haste might eventually entail corrections as inconvenient as those which would follow an arbitrary and unscientific choice of units, and the very experiments made by the Subcommittee prove that the hesitation of many to adopt the absolute units as hitherto determined was well founded. It is certain that resistance-coils purporting to have been constructed from previous absolute determinations do not agree one with another within 7, 8, or even 12 per cent.

Before further alluding to the results obtained by the Subcommittee, it is desirable that the experiments themselves should be understood; and to this end the Committee have thought fit that a full explanation of the meaning of absolute measurement, and of the principles by which absolute electrical units are determined, should form part of the present Report, especially as the only information on the subject now extant is scattered in detached papers by Weber, Thomson, Helmholtz, and others, requiring considerable labour to collect and understand. In order to make this account as clear as possible, it has been thought best to disregard entirely the chronological order of the

discoveries and writings on which the absolute system is founded; and this has rendered it very difficult to refer to the original source of each statement or conclusion. In the Appendix (C) this want is, it is hoped, remedied.

The word "absolute" in the present sense is used as opposed to the word "relative," and by no means implies that the measurement is accurately made, or that the unit employed is of perfect construction; in other words, it does not mean that the measurements or units are absolutely correct, but only that the measurement, instead of being a simple comparison with an arbitrary quantity of the same kind as that measured, is made by reference to certain fundamental units of another kind treated as postulates. An example will make this clearer. When the power exerted by an engine is expressed as equal to the power of so many horses, the measurement is not what is called absolute; it is simply the comparison of one power with another arbitrarily selected, without reference to units of space, mass, or time, although these ideas are necessarily involved in any idea of work. Nor would this measurement be at all more absolute if some particular horse could be found who was always in exactly the same condition and could do exactly the same quantity of work in an hour at all times. The foot-pound, on the other hand, is one derived unit of work, and the power of an engine when expressed in foot-pounds is measured in a kind of absolute measurement, *i. e.* not by reference to another source of power, such as a horse or a man, but by reference to the units of weight and length simply—units which have been long in general use, and may be treated as fundamental. In this illustration, chosen for its simplicity, the unit of force is assumed as fundamental, and as equal to that exerted by gravitation on the unit mass; but this force is itself arbitrarily chosen, and is inconstant, depending on the latitude of the place of the experiment.

In true absolute measurement the unit of force is defined as the force capable of producing the unit velocity in the unit of mass when it has acted on it for the unit of time. Hence this force acting through the unit of space performs the absolute unit of work. In these two definitions, time, mass, and space are alone involved; and the units in which these are measured, *i. e.* the second, gramme, and metre, will alone, in what follows, be considered as fundamental units. Still simpler examples of absolute and non-absolute measurements may be taken from the standards of capacity. The gallon is an arbitrary or non-absolute unit. The cubic foot and the litre or cubic decimetre are absolute units. In fine, the word absolute is intended to convey the idea that the natural connexion between one kind of magnitude and another has been attended to, and that all the units form part of a coherent system. It appears probable that the name of "derived units" would more readily convey the required idea than the word "absolute," or the name of mechanical units might have been adopted; but when a word has once been generally accepted, it is undesirable to introduce a new word to express the same idea. The object or use of the absolute system of units may be expressed by saying that it avoids useless coefficients in passing from one kind of measurement to another. Thus, in calculating the contents of a tank, if the dimensions are in feet, the cubic contents are given in cubic feet, without the introduction of any coefficient or divisor; but to obtain the contents in gallons, the divisor 6·25 is required. If the power of an engine is to be deduced from the pressure on the piston and its speed, it is given in foot-pounds or metre-kilogrammes per second by a simple multiplication; to obtain it in horse-power, the coefficients $\frac{1}{550}$ or $\frac{1}{76}$ must be used. No doubt all the natural relations between the various magnitudes to be measured may be expressed and made use of, how-

ever arbitrary and incoherent the units may be. Nevertheless the introduction of the numerous factors then required in every calculation is a very serious annoyance; and moreover, where the relations between various kinds of measurement are not immediately apparent, the use of the coherent or absolute system will lead much more rapidly to a general knowledge of these relations than the mere publication of formulæ.

The absolute system is, however, not only the best practical system, but it is the only rational system. Every one will readily perceive the absurdity of attempting to teach geometry with a unit of capacity so defined that the contents of a cube should be $6\frac{1}{4}$ times the arithmetical cube of one side, or with a unit of surface of such dimensions that the surface of a rectangle would be equal to 0·000023 times the product of its sides; but geometry so taught would not be one whit more absurd than the science of electricity would become unless the absolute system of units were adopted.

In determining the unit of electrical resistance and the other electrical units, we must simply follow the natural relation existing between the various electrical quantities, and between these and the fundamental units of time, mass, and space. The electrical phenomena susceptible of measurement are four in number—current, electromotive force, resistance, and quantity. The definitions of these need not now be given, but will be found in the Appendix (C. §§ 14, 15, 16, and 17). Their relations one to another are extremely simple, and may be expressed by two equations.

First, by Ohm's law, experimentally determined, we have the equation

$$C=\frac{E}{R}, \quad \ldots\ldots\ldots\ldots \quad (1)$$

where C=current, E=electromotive force, and R=resistance. From this formula it follows that the unit electromotive force must produce the unit current in a circuit of unit resistance; for if units were chosen bearing any other relation to each other, C would be equal to $x\frac{E}{R}$, where x would be a useless and absurd factor, complicating all calculation, and confusing the very simple conception of the relation established by Ohm's law.

Secondly, it has been experimentally proved by Dr. Faraday that the statical quantity of electricity conveyed by any given current is simply proportional to the strength of the current, whether electro-magnetically or electro-chemically measured, and to the time during which it flows; hence, in mathematical language, we have the equation

$$Q=Ct, \quad \ldots\ldots\ldots\ldots \quad (2)$$

where t=time, and Q=quantity. From this equation it follows that the unit of quantity must be the quantity conveyed by the unit current in the unit of time; otherwise we should have $Q=yCt$, where y would be a second useless and absurd coefficient. From equations (1) and (2) it follows that only two of the electrical units could be arbitrarily chosen, even if the natural relation between electrical and mechanical measurements were disregarded. Thus if the electromotive force of a Daniell's cell were taken as the unit of electromotive force, and the resistance of a metre of mercury of one millimetre section at 0° were taken as the unit of resistance, it would follow from equations (1) and (2) that the unit of current must be that which would be produced by the Daniell's cell in a circuit of the above resistance, and the unit of quantity would be the quantity conveyed by that current in a second of time. Such a system would be coherent; and if all mechanical, chemical,

and thermal effects produced by electricity could be neglected, such a system might perhaps be called absolute. But all our knowledge of electricity is derived from the mechanical, chemical, and thermal effects which it produces, and these effects cannot be ignored in a true absolute system. Chemical and thermal effects are, however, now all measured by reference to the mechanical unit of work; and therefore, in forming a coherent electrical system, the chemical and thermal effects may be neglected, and it is only necessary to attend to the connexion between electrical magnitudes and the mechanical units. What, then, are the mechanical effects observed in connexion with electricity? First, it has been proved that whenever a current flows through any circuit it performs work, or produces heat or chemical action equivalent to work. This work or its equivalent was experimentally proved by Dr. Joule to be directly proportional to the square of the current, to the time during which it acts, and to the resistance of the circuit; and it depends on these magnitudes only. In mathematical language this is expressed by the equation

$$W=C^2 Rt, \qquad (3)$$

where W=the work equivalent to all the effects produced in the circuit, and the other letters retain their previous signification. This is the third fundamental equation affecting the four electrical quantities, and represents the most important connexion between them and the mechanical units. From equation (3) it follows (unless another absurd coefficeint be introduced) that the unit current flowing for a unit of time through a circuit of unit resistance will perform a unit of work or its equivalent. If every relation existing between electrical and mechanical measurements were expressed by the three fundamental equations now given, they would still leave the series of units undefined, and one unit might be arbitrarily chosen from which the three other units would be deduced by the three equations; but these three equations by no means exhaust the natural relations between mechanical and electrical measurements. For instance, it is observed that two equal and similar quantities of electricity collected in two points repel one another with a force (F) directly proportional to the quantity Q, and inversely to the square of the distance (d) between the points. This gives the equation

$$F=\frac{Q^2}{d^2}; \qquad (4)$$

from which it would follow that the unit quantity should be that which at a unit distance repels a similar and equal quantity with unit force. The four equations now given are sufficient to measure all electrical phenomena by reference to time, mass, and space only, or, in other words, to determine the four electrical units by reference to mechanical units. Equation (4) at once determines the unit of quantity, which, by equation (2), determines the unit current; the unit of resistance is then determined by equation (3), and the unit electromotive force by equation (1). Here, then, is one absolute or coherent system, starting from an effect produced by electricity when at rest. The units based on these four equations are precisely those called by Weber electrostatical units, although it may be observed that he chose those units without reference to what is here called the third fundamental equation, or, in other words, without reference to the idea of work, introduced into the system by Thomson and Helmholtz*.

The four equations are sufficient to determine the four units, and into this system no new relation can be introduced. The first three equations may,

* *Vide* Appendix C, § 31.

however, be retained, and a distinct absolute system established by substituting some other relation between electrical and mechanical magnitudes than is expressed in equation (4); and, indeed, the electrostatical system just defined is not that which will be found most generally useful. It is based on a statical phenomenon, whereas at present the chief applications of electricity are dynamic, depending on electricity in motion, or on voltaic currents with their accompanying electro-magnetic effects. Now the force exerted on the pole of a magnet by a current in its neighbourhood is a purely mechanical phenomenon. This force (f) is proportional to the magnetic strength (m) of the pole of the magnet, and to the strength of the current C; and if the conductor be at all points equidistant from the pole, or, in other words, be bent in a circle of the radius k round the pole, the force is proportional to the length of the conductor (L); it is also inversely proportional to the square of the distance (k) of the pole from the conductor, and is affected by no other circumstances than those named. Hence we have

$$f=\frac{CLm}{k^2}. \quad \ldots \ldots \ldots \quad (5)$$

From this equation it follows that the unit length of the unit current must produce the unit force on a unit pole at the unit distance. If the equations (1), (2), (3), and (5) are adopted as fundamental, they give a distinct absolute system of units, called by Weber the electro-magnetic units. Equations (4) and (5) are incompatible one with another, if equation (2) be considered fundamental; but the electro-magnetic units have a constant and natural relation to the electrostatic units. It will be seen that in the fundamental equation (5) of the electro-magnetic system, besides the measurement of time, space, and mass, alone entering into the other equations, a fourth measurement (m) of a magnetic pole is required; but this measurement is in itself made in terms of the mechanical units, for the unit pole is simply that which repels another equal pole at unit distance with unit force. Thus in the electro-magnetic as in the electrostatic system all measurements are ultimately referred to the fundamental units of time, space, and mass. The electro-magnetic units are found much the more convenient when dealing, as we have now chiefly occasion to do, with electro-magnetic phenomena.

The relations of the electro-magnetic units one to another and to the mechanical units may be summed up as follows:—The unit current conveys a unit quantity of electricity through the circuit in a unit of time. The unit current in a conductor of unit resistance produces an effect equivalent to the unit of work in the unit of time. The unit current will be produced in a circuit of unit resistance by the unit electromotive force. The unit current flowing through a conductor of unit length will exert the unit force on a unit pole at unit distance. (In the electrostatic system all the above propositions hold good except the last, for which the following must be substituted:—the unit quantity of electricity will repel a similar quantity at the unit distance with unit force.)

It remains to be explained how electrical measurements can be practically made in electro-magnetic units. Of all the magnitudes, currents are the most easily measured, provided the horizontal force (H) of the earth's magnetism be known. Let a length (L) of wire be wound so as to form a circular coil of small section as compared with its radius (k).

Let a short magnet be hung in the centre of the coil placed in the magnetic meridian, as in the ordinary tangent galvanometer, and let the

deflection produced by the current C be called d, then it is easily* proved from the fundamental equation (5) that

$$C=\frac{Hk^2}{L}\tan d. \qquad (6)$$

Thus, where the value of H is known, a tangent galvanometer only is required to determine the magnitude of a current in electro-magnetic absolute measurement, although neither the resistance of the circuit nor the electromotive force producing the current may be known. The measurement of quantity can be obtained from that of a current by a make-and-break apparatus, or "Wippe," in a well-known manner, or by measuring the swing of a galvanometer-needle when a single instantaneous discharge is allowed to pass through it (Appendix C, § 25). If, therefore, we could measure resistance in absolute measure, the whole system of practical absolute measurement would be complete, since, when the current and resistance are known, equation (1) (Ohm's law) directly gives the electromotive force producing the current. The object of the experiments of the Subcommittee (made at King's College, by the kind permission of the Principal) was therefore to determine, in the absolute system, the resistance of a certain piece of wire, in order from this one careful determination to construct the material representative of the absolute unit with which all other resistances would be compared by well-known methods.

There are several means by which the absolute resistance of a wire can be measured. Starting from equation (3), Professor Thomson, in 1851, determined the absolute resistance of a wire by means of Dr. Joule's experimental measurement of the heat developed in the wire by a current†; and by this method he obtained a result which agrees within about 5 per cent. with our latest experiments. This method is the simplest of all, so far as the mental conception is concerned, and is probably susceptible of very considerable accuracy.

Indirect methods depending on the electromotive force induced in a wire moving across a magnetic field have, however, now been more accurately applied; but, before describing these methods, it will be necessary to point out the connexion between the electromotive force induced in the above manner and the fundamental equations adopted for the absolute system. The exact sense in which the terms are employed is defined in the accompanying footnote, along with some simple corollaries from those definitions‡.

* The resultant electro-magnetic force (f) exerted at the centre of the coil by a current (C) will, by equation (5), be $f=\frac{CL}{k^2}$, and the short magnet hung in the centre will experience a couple acting in a direction perpendicular to the plane of the coil equal to $\frac{CLml}{k^2}$, where ml = the product of the strength of one of the poles into the length of the magnet, or, in other words, its magnetic moment. The strength of the couple acting perpendicularly to the axis of the magnet, when it has deflected to an angle d under the influence of the current, will be $\cos d\,\frac{CLml}{k^2}$; at the same ime the equal and opposite couple exerted on the magnet by the earth's magnetism will be $\sin d\,Hml$; hence

$$C=\frac{Hk^2}{L}\times\frac{\sin d}{\cos d}=\frac{Hk^2}{L}\tan d.$$

† Phil. Mag. vol. ii. ser. 4, 1851, p. 551.

‡ *Definition* 1.—A magnetic field is any space in the neighbourhood of a magnet.

Definition 2.—The unit magnetic pole is that which, at a unit distance from a similar pole, is repelled with unit force.

A current (C) in a straight conductor of length (L) crossing the lines of force of a magnetic field of the intensity (S) at right angles will experience the same force (f) as if all the points of the conductor were at the unit distance from a pole of the strength (S). The force in this case exerted on the magnet is, by equation (5), equal to SLC, and, conversely, an equal force is exerted by the magnet on the current. Hence we have equation (7), expressing the value of the force (f) exerted on a current crossing a magnetic field at right angles,

$$f = SLC. \quad \ldots \ldots \ldots \quad (7)$$

Let us imagine this straight conductor to have its two ends resting on two conducting-rails of large section in connexion with the earth, and let the whole sensible resistance (R) of the circuit thus formed be constant for all positions of the conductor. Let us further imagine the rails so placed that when the conductor slips along them it moves perpendicularly to the magnetic lines of force and to its own length. By experiment we know that when the conductor is moved along the rails cutting these lines of force, a current will be developed in the circuit, and that the action of the magnetic force on this current will cause a resistance (f) to the motion (due to electro-magnetic causes only); and, by equation (7), we find that this resistance $f=SLC$.

Let the motion be uniform, and its velocity be called V; and let the work done in the unit of time in overcoming the resistance to motion due to electro-magnetic causes be called W; then $W=VSLC$. But this force produces no other effect than the current, and the work done by the current must therefore be $=W$, or equivalent to that done in moving the conductor against the force f; but, by equation (3), $W=C^2R$, and hence

$$R = \frac{VSL}{C}. \quad \ldots \ldots \ldots \quad (8)$$

Definition 3.—The intensity of a magnetic field at any point is equal to the force which the unit pole would experience at that point.

Corollary 1.—A pole of given strength (S) will produce a magnetic field which (if uninfluenced by other magnetic forces) will at the unit distance from the pole be of the intensity S, *i. e.* numerically equal to the strength of the pole; for, at that distance, the force exerted on a unit pole would, by def. 2, be equal to S, and hence, by def. 3, the intensity of the magnetic field at that point would be equal to S.

Definition 4.—The direction of the force in the field is the direction in which any pole is urged by the magnetism of the field; this is the direction which a short, balanced, freely suspended magnet would assume.

Remark.—The properties of a magnetic field, as shown by Dr. Faraday, may be conveniently and accurately conceived as represented by *lines* of force (each line representing a force of constant intensity). The direction of the lines will indicate the direction of the force at all points; and the number of lines which pass through the unit area of cross section will represent the magnetic intensity of the field resolved perpendicularly to that area.

Definition 5.—A uniform magnetic field is one in which the intensity is equal throughout, and hence, as demonstrated by Professor W. Thomson, the lines of force parallel.

Example.—The earth is a great magnet. The instrument-room, where experiments are tried, is a magnetic field. The dipping-needle is an instrument by which the direction of the lines of force is found. The intensity of the field is found by a method described in the 'Admiralty Manual,' 3rd edit., article "Terrestrial Magnetism." The number of lines of force passing through the unit of area perpendicularly to the dipping-needle in the room must be conceived as proportional to this intensity, and the direction to correspond with that of the dipping-needle. The magnitude and direction of the earth's force at a point are generally expressed by resolving it into two components, one horizontal and the other vertical. The mean horizontal component in England for 1862 was at Kew = 3·8154 British units, or 1·7592 metrical; *i. e.* a unit pole weighing one gramme, and free to move in a horizontal plane, would, under the action of the earth's horizontal force, acquire, at the end of a second, a velocity equal to 1·7592 metres per second. (*Vide* also Appendix C, §§ 5 to 12.) If the centimetre is taken as the fundamental unit of length, ·17592 will be the mean value of the horizontal force.

It has already been shown that C and S can be obtained in absolute measure; hence the second member of equation (8) contains no unknown quantities, and, by the experiment described, the absolute resistance (R) of a wire might be determined. One curious consequence of these considerations is, that the resistance of a conductor in absolute measure is really expressed by a velocity; for, by equation (8), when $SL=C$ we have $R=V$, that is to say, the resistance of a conductor may be expressed or defined as equal to the velocity with which it must move, if placed in the conditions described, in order to generate a current equal to the product of the length of the conductor into the intensity of the magnetic field; or more simply, the resistance of a circuit is the velocity with which a conductor of unit length must move across a magnetic field of unit intensity in order to generate a unit current in the circuit. Moreover it can be shown that this velocity is independent of the magnitude of the fundamental units on which the expression of the magnetic intensity of the field or strength of the current is based, and hence that electrical resistance really is measured by an absolute velocity in nature, quite independently of the units of time and space in which it is expressed (Appendix C, § 39). By equation (8) we have $C=\frac{VSL}{R}$, but by equation (1) $C=\frac{E}{R}$, hence

$$E=VSL; \quad \ldots \ldots \ldots \quad (9)$$

that is to say, the electromotive force produced between two ends of a straight conductor moved perpendicularly to its own length and to the lines of force of a magnetic field is equal to the product of the intensity of the field into the length of the conductor and the velocity of the motion; or, more simply, the unit length of a conductor moving with unit velocity perpendicularly across the lines of force of a magnetic field will produce a unit electromotive force (or difference of potential) between its two ends. This was by Weber made a fundamental equation, in place of equation (3), first shown by Thomson and Helmholtz to be consistent with Weber's electro-magnetic equation. These simple and beautiful relations between inductive effects and the simple voltaic effects first described are well adapted to show the rational and coherent character of the absolute system.

The experiment last described, as a method of finding the absolute resistance of a conductor by measuring the velocity of motion of a straight wire, would be barely practicable; but it will be easily understood that we can, by calculation, pass from this simple case to the more complex case of a circular coil of known dimensions revolving with known velocity about an axis in a magnetic field of known intensity. Weber, from these elements, determined the absolute resistance of many wires; but this method requires that the intensity of the magnetic field be known; and the determination of this element is laborious, while its value, for the earth at least, is very inconstant. A method due to Professor Thomson, by which a knowledge of this element is rendered unnecessary, has therefore been adopted in the experiments of the Subcommittee at King's College. In this plan a small magnet, screened from the effect of the air, is hung at the centre of a revolving coil, which is divided into two parts to allow the suspending fibre to pass freely.

By calculation it can be shown that when the coil revolves round a vertical axis, the couple exerted on a magnetic needle of the moment ml, when deflected to the angle d, will be $\frac{L^2 VH}{4k^2 R} ml \cos d$.

The equal and opposite couple caused by the earth's magnetism will be $\mathrm{H}ml \sin d$. Hence

$$\tan d = \frac{\mathrm{L}^2\,\mathrm{V}}{4k^2\,\mathrm{R}}$$

or

$$\mathrm{R} = \frac{\mathrm{L}^2\,\mathrm{V}}{4k^2 \tan d}, \qquad (10)$$

an equation from which the earth's magnetic force and the moment of the suspended magnet have been eliminated, and by which the absolute resistance (R) can be calculated in terms of the length (L), the velocity (V), the radius (k), and the deflection (d). The resistance thus calculated is expressed in electro-magnetic absolute units, because equation (10) is a simple consequence of equations (1), (3), and (5)—fundamental equations in the electro-magnetic system. The essence of Professor Thomson's method consists in substituting, by aid of the laws of electro-magnetic induction, the measurements of a velocity and a deflection for the more complex and therefore less accurate measurements of work and force required in the simple fundamental equations. But, however simple in theory the method may be, the practical determination of the absolute resistance of a conductor by its means required great care and very numerous precautions,—some of an obvious character, while the need of others only became apparent during the course of the experiments.

The apparatus consisted of two circular coils of copper wire, about one foot in diameter, placed side by side, and connected in series; these coils revolved on a vertical axis, and were driven by a belt from a hand-winch, fitted with Huyghens's gear to produce a sensibly constant driving-power. A small magnet, with a mirror attached, was hung in the centre of the two coils, and the deflections of this magnet were read by a telescope from the reflection of a scale in the mirror. A frictional governor controlled the speed of the revolving coil. The details and a drawing of the apparatus are given in Appendix D and Plate II.; but a short account may fitly be given here of the points of chief practical importance, the difficulties encountered, and the improvements still desirable.

It is essential that the dimensions of the coil be very accurately known, that the axis on which it revolves should be truly vertical, and that, except in the coil itself, no currents affecting the position of the magnet be induced in any part of the apparatus. To measure the angular deflection the distance of the scale from the mirror is required, and the scale must be truly perpendicular to the line joining its middle point with the suspension-fibre. All these conditions were fulfilled without difficulty; but the scale by the reflection of which the deflections were measured was, towards the end of the experiments, found not to be very accurately divided; and although a correction for this inaccuracy has been applied in the calculations, an improvement can in future experiments be effected by the use of a more perfect scale. The magnet was suspended by a single silk fibre, eight feet long, inside a wooden case, and by suitable adjustments was brought very carefully to the centre of the coils. The whole suspended system was so screened from currents of air, and so well protected from vibration, that when the coil revolved at its full speed of 350 revolutions per minute, the reflection in the mirror was as clear and undisturbed as when the coil was at rest. The torsion of the long fibre was determined by experiment, and the slight necessary corrections applied in the calculations. The Huyghens's gearing for the driving hand-winch was somewhat roughly constructed, and could certainly be improved; nevertheless there was little difficulty in maintaining a sensibly

constant driving-power for twenty minutes at a time. The speed of the coil was controlled by a frictional governor of novel form, designed by Mr. Jenkin for another purpose, and lent for the experiments in question. The action of this governor, combined with that of the driving-gear, was such that in many experiments the oscillations in deflection due to a change of speed were not so great as those due to the passage of steamers in the river when all parts of the apparatus were at rest; so that the deflections during twenty minutes could be quite as accurately observed as the slightly imperfect zero-point from which they were measured. Still better results are expected with a larger governor, made specially for the apparatus, on the joint plans of Professor Thomson and Mr. Jenkin. The oscillations produced by the passage of steamers on the Thames at no great distance from the place of experiments were of very sensible magnitude; and although by carefully observing the limit of every oscillation during every experiment the error due to this cause was in great part eliminated, it is desirable that any future experiments should be conducted in some spot free from all local magnetic disturbance.

The speed of the coil was determined by observing on a chronometer the instant at which a small gong was struck by a detent released once in every hundred revolutions. Mr. Balfour Stewart's skill in this kind of observation enabled him thus to determine the velocity with great accuracy, especially as the observations frequently lasted for twenty minutes without material alteration in the speed.

During the operation of coiling the wire, the circumference of the core and of each successive layer was carefully measured by means of a steel riband applied first to the coil, and then to a standard scale, allowance being made for the half thickness of the steel. From this the mean radius and depth of the coil and the effective length of the wire were determined. It was considered advisable, however, in order to check any error in counting the number of windings of the coil, to measure the length of the wire when uncoiled. This was effected without stretching the wire, in a manner amusing from its simplicity. At the conclusion of the experiments, the wire to be measured was uncoiled in the Museum at King's College and lay in awkward bends on the planked floor. The straight planks formed an obvious contrast to the crooked wire, and a joint between the planks was found where the opening was just sufficient to hold the wire when pushed into this little groove. Held in this way, the wire when measured was quite straight, and yet was never stretched.

No other measurements than those already described are required by the simple theory; but this theory, as hitherto stated, stands in need of various slight corrections. The currents induced by the earth's magnetism are modified by the currents induced by the little suspended magnet, and also by the induction of the coil on itself. The force deflecting the magnet is also modified by the lateral distance of the coils from the vertical axis. An elaborate analysis of the corrections required on these grounds was made by Professor Maxwell (Appendix D), and to allow of these corrections, the moment of the suspended magnet was measured, and the position of every turn of the copper coil carefully observed. An experimental determination of the induction of the coil on itself, by a method due to Professor Maxwell, agreed with the calculated correction within one quarter per cent.

The resistance of the copper coil measured by these laborious experiments varied each day, and during each day, according to the temperature; and, moreover, this temperature could at no time be determined with sufficient accuracy. It was therefore intended that at each experiment a small German-

silver coil, at a known temperature, should have been prepared exactly equal in resistance to the copper coil during that experiment, and these small coils were to have been kept as permanent records of the resistance of the copper coil on each occasion; but this resistance was found to vary so rapidly that the little copies could not be accurately adjusted with sufficient rapidity, and the resistance of the copper coil was therefore simply measured at the beginning and end of each experiment, in terms of an arbitrary unit. This proportional measurement was made with rapidity and precision by a new method, which, it is believed, is superior to the usual plan depending on the division or calibration of a comparatively short wire in the Wheatstone balance. (Appendix D, Part II.)

One unforeseen difficulty was caused by the change of direction of the earth's magnetic force during each experiment. Our method is indeed independent of the intensity of the earth's magnetism, but depends essentially on its direction, since it depends on the value of a deflection from the magnetic meridian. When this source of error was discovered by the continual and gradual change of zero observed, the absolute time of each experiment was noted, and a continuous correction obtained from the contemporaneous records at Kew, which agreed closely with the total changes observed at the beginning and end of each experiment. As the change of zero frequently reached three or four divisions in the course of the day, and as the whole deflection seldom exceeded 300 divisions, the importance of this correction is apparent.

The presence of stationary masses of iron does not affect the experiments injuriously, so long as the uniformity of the magnetic field in which the coil revolves is undisturbed—a point carefully tested before the experiments began; but a change in the position of iron in the neighbourhood during any experiment produces a corresponding error in the result, and the serious effect of moving very small masses of iron at a great distance from the coil was only fully appreciated in the later experiments.

When it is considered that the method described is the simplest known, the discrepancy between the few determinations hitherto made in absolute measurement will cause no surprise. The time, labour, and money required could hardly be expected to be given by any one person, and in researches of this kind the value of the cooperation secured by the committees of the Association is especially evident.

The absolute unit of the Subcommittee is about eight per cent. larger than the unit as derived from a German-silver coil lately measured by Professor Weber. It is about six and a half per cent. larger than the unit as derived from a value published by Professor Weber of Dr. Siemens's mercury units. It is about five per cent. smaller than the unit as derived from coils issued by Professor Thomson in 1858, based on Jacobi's standard and a previous determination by Professor Weber. It is about five per cent smaller than Thomson's determination from Joule's silver wire. It agrees most closely with an old determination of a copper standard made by Weber for Professor Thomson, which it exceeds by only a very small fraction.

The experiments of the Subcommittee agree much better than the above one with another. Owing to the gradual improvement in the method and apparatus, the experiments of the last three days are alone considered satisfactory. On the first day the maximum deviation in six distinct experiments from their mean result was 2·4 per cent. On the second day the maximum deviation in four experiments from their mean was 1·3 per cent. On the

third day the maximum deviation in five experiments from their mean was 1·15 per cent. The maximum deviation in the means of the three days' experiments from the mean of the whole is only four tenths per cent.

These results are not unsatisfactory, and are perhaps more accurate than any measurement yet made of the relative values of heat and work—a measurement corresponding to a great extent in its nature with that undertaken by the Committee. Nevertheless, considering the discrepancy of the various independent results, the Committee are of opinion that it is essential that the results of the Subcommittee should be checked by a fresh series of experiments with a new coil in a distinct place, when every separate measurement will necessarily be repeated. The Subcommittee especially urge the repetition of the experiments, as with the improvements already enumerated, and other minor alterations, they confidently expect a considerably closer approximation to the absolute unit than they have hitherto obtained. It will be well here to remark that, according to the resolution of the Committee of 1861, the coils, when issued, will not be called absolute units, but the units of the British Association; so that any subsequent improvement in experimental absolute measurement will not entail a change in the standard, but only a trifling correction in those calculations which involve the correlation of the physical forces.

It is now time to leave the question of absolute measurement and pass to some of the other points under the consideration of the Committee. Dr. Matthiessen has, by careful experiment, proved the permanence for a year at least of the electrical resistance of certain wires; but he has detected a change in others, due, apparently, to the influence of time. Certain specimens of silver, gold, and copper have varied; but other specimens of the same metals have remained constant. All the specimens of platinum and gold-silver alloy have remained constant, and all the specimens of German silver have changed considerably. It is proposed to continue and extend these experiments, and it is much to be hoped that the defect observed in the German silver tested will not be found common to all the varieties of this alloy, in other respects so well adapted for the construction of resistance-coils. Dr. Matthiessen found no difference in the resistance of wires of any of the above metals before and after the passage of a powerful current transmitted through them continually for a fortnight. The details of these experiments are given in Appendix A. Dr. Matthiessen has also continued his experiments with the object of finding an alloy with a minimum variation of resistance due to change of temperature, but has been unable to produce a wire superior in this respect to the silver-platinum alloy mentioned in Appendix A of the Report of last year, as decreasing in conducting power 3·1 per cent. between 0° and 100° Centigrade. German silver was found to decrease under the same circumstances 4·4 per cent.

The valuable experiments by Mr. Sabine, for Dr. Werner Siemens of Berlin, on the reproduction of standards by means of mercury, although not undertaken for the Committee, yet bear so directly on the subject before them that the results cannot be allowed to pass unmentioned. Dr. Siemens has conclusively proved that he can, in his laboratory, reproduce a standard by means of mercury with an error of less thon 0·05 per cent. This admirable result, while it seriously affects the question of the best material for the construction and reproduction of the standard, leaves, of course, the question of the best magnitude for the standard quite untouched. Dr. Matthiessen thinks that several of the solid metals are equally fitted for the purposes of reproduction, and, if aided by the Association, is disposed to put his conviction to

experimental proof. It is especially desirable that the various methods proposed should be tested by the concordance of the results obtained from a number of independent observers.

With reference to the construction of the material standard, it is proposed that the British-Association unit shall be represented by several equal standards made of the different metals, which, so far as our limited experience goes, show the greatest signs of constancy. Two at least of those standards would be made of mercury, in the manner proposed by Dr. Siemens. The permanent agreement between several of these standards would afford the strongest possible proof of their constancy.

Passing to other electrical measurements, the Committee have to report that Professor W. Thomson has successfully constructed a material standard gauge by which electromotive force or difference of potentials can be directly measured. This instrument is founded on a measurement of the electrical attraction exerted on a small movable portion of a large conducting-plane by another large parallel plane fixed at a constant distance, and electrified to a different potential. The force exerted is ultimately measured by the torsion of a platinum wire; but the difference of potential corresponding to any one gauge is simply indicated by the motion of an index to a sighted position. If the planes are brought sufficiently close, with a given torsion in the platinum wire, the movable piece will be in a condition of unstable equilibrium when its index is in the sighted position, but if moved to a greater distance the equilibrium will be stable; hence, by a correct choice of the distance between the two planes, or initial torsion in the platinum wire, as compared with the difference of potential to be measured, any required delicacy of indication is obtained. The constancy of the gauge, like that of all standards, depends simply on the constancy of the materials of which it is constructed, and there is no reason to apprehend any special difficulty in the present case.

Professor Thomson has also on the same principle constructed an electrometer in which the distance between the parallel planes is made variable, and is adjusted by a micrometer-screw. The plane conductor, of which the small movable index forms part, is in this instrument permanently maintained at a high potential by connexion with the inner coating of a Leyden jar, and the other plane is connected with the body to be tested. Calculation, confirmed by experiment, shows that in these instruments the difference of potentials between any two bodies, successively tested, is directly proportional to the difference of the distances between the parallel planes required in each case to bring the index to its sighted position. This difference of distance is the same whatever be the charge of the Leyden jar, provided only it remains constant during the comparison of the two bodies. With this limitation, the indications of the instrument may be called independent of the charge of the Leyden jar. There can be little doubt that gauges of electromotive force and electrometers, fulfilling the above conditions, will shortly become as necessary to all practical electricians as standards of resistance and sets of resistance-coils.

No progress has been made in the measurement of currents, and much remains to be done in this respect. The method already described, depending on the use of a tangent galvanometer, requires a knowledge of the horizontal force of the earth's magnetism, and is, therefore, in most cases beyond the reach of observers where greater accuracy is required than can be obtained by taking their value from the scientific almanacs. Next year it is hoped that this want may be remedied; and the present Report may fitly conclude by the

enumeration of objects to be pursued by the Committee, if reappointed at the present Meeting:—

1st. The experiments on the determination of the absolute unit of resistance will be continued.

2nd. Immediately on the conclusion of these experiments, equal standards, constructed of such metals as promise the greatest constancy, will be deposited at Kew, where the permanence of their equality will be rigorously tested.

3rd. Unit resistance-coils of the best known construction will be issued to the public.

4th. The experiments already begun on the permanence of the electrical resistance of wires and alloys under various circumstanees will be continued and extended.

5th. The experiments on the reproduction of standards by chemical means will be continued.

6th. Experiments on the best construction of gauges of electromotive force or difference of potential, and on electrometers, will be continued.

7th. A standard galvanometer, for the measurement of currents in absolute measure, will be constructed, and electro-dynamometers for the same purpose compared with the standard instrument, and issued to the public.

8th. Experiments on the ratio between the electrostatic units and the electro-magnetic units will be undertaken.

9th. Experiments will be made on the development of heat in conductors of known absolute resistance with currents of known absolute magnitude. The results of this experiment will give, by equation (3), a new and very accurate determination of the mechanical value of the unit of heat.

The conclusion of the experiments on absolute resistance, and the adoption of the absolute system as the basis of all electrical measurement, will, it is hoped, allow considerable progress to be made in most of these researches.

Appendix A.—*On the Electrical Permanency of Metals and Alloys.* *By* A. Matthiessen, *F.R.S.*

The following are the results obtained with the metals and alloys described in Appendix B of the Report on Standards of Electrical Resistance by your Committee:—

The wires to be experimented on were—

1. Silver: hard-drawn .... 2. Silver: annealed	Cut from the same piece; pure.
3. Silver: hard-drawn .... 4. Silver: annealed	Cut from the same piece, but different from 1 and 2; pure.
5. Copper: hard-drawn ...... 6. Copper: annealed..........	Cut from the same piece; pure.
7. Copper: hard-drawn ...... 8. Copper: annealed..........	Cut from the same piece, but different from 5 and 6; pure.
9. Gold: hard-drawn .... 10. Gold: annealed	Cut from the same piece; pure.
11. Gold: hard-drawn .... 12. Gold: annealed	Cut from the same piece, but different from 9 and 10; pure.

13. Platinum: hard-drawn ....
14. Platinum: hard-drawn .... } Cut from the same piece; commercial.

15. Gold-silver alloy: hard-drawn
16. Gold-silver alloy: hard-drawn } Cut from same piece. Made by Messrs. Johnson and Matthey.

17. German silver: annealed....
18. German silver: annealed....
19. German silver: annealed.... } Cut from the same piece. No. 19 arranged with longer connectors, and used as normal wire with which the rest were compared.

These were first tested on May 9th, 1862, and at intervals between that date and June 14th, 1863, when they were last tested. During the time when not used, they were hung up in a room where in the winter a fire was kept all day, so that the temperature may have varied at times some 10 or 12 degrees in the twenty-four hours.

The following Table contains the results of the first and last comparisons. I have taken the conducting power in the first in all cases equal to 100 as compared with No. 19; in the last I have assumed that the conducting power of No. 15 has remained unaltered:—

	Conducting powers found, as compared with No. 19=100.				Conducting power found, as compared with No. 15=100.	
	1.		2.		3.	
	May 9, 1862.	T.	June 14, 1863.	T.		T.
1. Silver: hard-drawn	100·00	20·2	103·700	20·0	103·915	20·0
2. Silver: annealed	100·00	20·2	99·740	20·1	99·947	20·1
3. Silver: hard-drawn	100·00	20·2	102·590	20·2	102·807	20·2
4. Silver: annealed	100·00	20·2	99·825	20·0	100·031	20·0
5. Copper: hard-drawn	100·00	20·1	100·040	20·2	100·248	20·2
6. Copper: annealed	100·00	20·1	99·807	20·0	100·015	20·0
7. Copper: hard-drawn	100·00	20·0	99·941	19·8	100·149	19·8
8. Copper: annealed	100·00	20·0	95·358	20·4	95·556	20·4
9. Gold: hard-drawn	100·00	20·0	99·838	20·2	100·045	20·2
10. Gold: annealed	100·00	20·0	99·855	20·0	100·062	20·0
11. Gold: hard-drawn	100·00	20·0	99·662	20·2	99·869	20·2
12. Gold: annealed	100·00	20·0	99·670	20·3	99·877	20·3
13. Platinum: hard-drawn	100·00	20·0	99·744	20·2	99·951	20·2
14. Platinum: hard-drawn	100·00	20·0	99·792	20·2	99·999	20·2
15. Gold-silver alloy: hard-drawn	100·00	20·0	99·793	20·2	100·000	20·2
16. Gold-silver alloy: hard-drawn	100·00	19·9	99·756	20·3	99·963	20·3
17. German silver: annealed	100·00	20·3	99·955	20·0	100·162	20·0
18. German silver: annealed	100·00	20·3	99·938	20·0	100·145	20·0
19. German silver: annealed	......	......	......	......	100·217	20·2

From the above it would appear that if the conducting power of No. 19 has remained constant, that of all the others has altered; but supposing such to be the case, it will be found on comparing the values that the conducting powers have all altered in a like extent. Is it probable? Is it not more probable that the conducting power of the German silver has changed, than that that of all the others should have altered in the same degree? If that of the gold-silver alloy (No. 15) be called 100·00 instead of 99·793, then, as will be seen from column 3, very few show any change in

their conducting power. Those which show no sensible change are as follows:—

		Values taken from column 3.
No. 2.	Silver: annealed	99·947
No. 4.	Silver: annealed	100·031
No. 6.	Copper: annealed	100·015
No. 9.	Gold: hard-drawn	100·045
No. 10.	Gold: annealed	100·062
No. 13.	Platinum: hard-drawn	99·951
No. 14.	Platinum: hard-drawn	99·999
No. 15.	Gold-silver alloy: hard-drawn	100·000
No. 16.	Gold-silver alloy: hard-drawn	99·963

The differences in the above are probably due to temperature; for as the wires are in tubes filled with carbonic-acid gas, we can never be absolutely sure that wire has exactly the same temperature as the bath. In properly made resistance-coils this source of error is materially diminished, and in some experiments which are about to be made to further test the electrical permanency of metals and alloys this source of error will be almost entirely obviated. It may be here again mentioned, that the reason of placing the wires in glass tubes filled with carbonic-acid gas was to obviate the oxidation of the metal or alloy by the oxygen of the air, or from the acids produced by the oxidation of the oil or fat with which the wires are covered when drawn, as the holes in the draw-plates are generally oiled or greased, &c.

Those whose conducting-power has changed are as follows:—

		Values taken from column 3.
No. 1.	Silver: hard-drawn	103·915
No. 3.	Silver: hard-drawn	102·807
No. 5.	Copper: hard-drawn	100·248
No. 7.	Copper: hard-drawn	100·149
No. 8.	Copper: annealed	95·556
No. 11.	Gold: hard-drawn	99·869
No. 12.	Gold: annealed	99·877
No. 17.	German silver: annealed	100·162
No. 18.	German silver: annealed	100·145
No. 19.	German silver: annealed	100·217

The cause of the change in the conducting powers of the alloys Nos. 1, 3, 5, 7 is undoubtedly due to their becoming somewhat annealed by age*. With No. 8 the alteration may be attributed to faulty soldering. That the conducting power of the German silver experimented with has altered is not a proof that all German silver will do so; for we find the gold wires Nos. 9 and 10 not altered, but Nos. 11 and 12 (which were cut from the same piece, but a different one from the one from which Nos. 9 and 10 were taken) have altered. Further experiments are, however, required to find whether the metals and alloys given above as constant in their conducting power are so or not.

Schröder van der Kolk states† that the conducting power of copper wire undergoes a change when even weak currents are allowed to pass through it. In order to see whether that of the above wires would suffer any change, the following experiment was arranged:—Nos. 1, 2, 5, 6, 9, 10, 13, 15, 17 were connected together, and a current from two Bunsen's cells was allowed to pass through them day and night for six days. The cells were cleaned

* *Suprà*, p. 16; and Brit. Assoc. Report, 1862, p. 140.

† Pogg. Ann. vol. cx. p. 452.

every morning and evening, and the dilute sulphuric acid renewed. The experiment was carried out soon after June 14, 1863. In the subjoined Table the conducting powers are given as found before and after the trial, compared with No. 19.

	Conducting power observed, as compared with No. 19 = 100.			
	Before.	T.	After.	T.
No. 1........	103·700	20·0	103·775	20·2
No. 2........	99·740	20·1	99·733	20·2
No. 5........	100·040	20·2	100·045	20·2
No. 6........	99·807	20·0	99·865	20·0
No. 9........	99·838	20·2	99·860	20·2
No. 10........	99·855	20·0	99·807	20·2
No. 13........	99·744	20·2	99·766	20·2
No. 15........	99·793	20·2	99·762	20·2
No. 17........	99·955	20·0	99·926	20·2

From the above numbers it will be seen that the conducting power has not changed, the differences in the values being in all probability due, as above stated, to temperature.

If the passage of a current really altered the conducting power of a wire, then of what use would resistance-coils be? The above experiments prove that a much stronger current than is used for testing the resistance of a wire has no effect on it.

Appendix B.—*On the Variation of the Electric Resistance of Alloys due to Change of Temperature.* By A. Matthiessen, *F.R.S.*

In the Appendix to the Report of your Committee read at the Meeting held last year, I gave a Table containing the results of experiments with some alloys, made with a view to find out the alloy whose conducting power decreases least with an increase of temperature. With the same apparatus, &c., I have, in conjunction with Dr. C. Vogt, experimented with the following alloys.

(With each series the formula deduced from the observations for the correction of the conducting power for temperature is given, where λ is equal to the conducting power at the temperature t° C. Silver (hard-drawn) is taken at $0^{\circ}=100$.)

Composition of alloy by weight.		Length 226 mm.; diameter 0·470 mm.	
		T.	Conducting power.
(1) Gold..........	95·3	12·0	2·3573
Iron..........	4·7	56·0	2·3138
Made from pure metals.			
Hard-drawn.		100·0	2·2798

$$\lambda=2{\cdot}3708-0{\cdot}0011555t+0{\cdot}000002454t^2.$$

		Length 284 mm.; diameter 1·217 mm.	
		T.	Conducting power.
(2) Gold..........	95·0	15·0	2·0819
Iron..........	5·0	57·5	2·0424
Hard-drawn.		100·0	2·0067

$$\lambda=2{\cdot}0967-0{\cdot}0010057t+0{\cdot}000001052t^2.$$

This and the two following alloys were made by Messrs. Johnson and Matthey. No. 2 was made to check the results obtained with No. 1; for

those given with Nos. 3 and 4 appeared to show that some mistake had been made with No. 1. That this was not the case is proved by No. 2. It is, however, a very curious fact that the percentage decrement increases in this manner, for in no other series of alloys has this behaviour been noticed. Its cause may be attributed to the existence of chemical combinations in the solid alloys of gold and iron.

Nos. 3 and 4 are very brittle, and therefore difficult to draw.

(3) Gold .... 90·0
Iron .... 10·0
Hard-drawn.

Length 184 mm.; diameter 0·943 mm.

T.	Conducting power.
14·0	1·9822
57·0	1·7951
100·0	1·7010

$$\lambda = 2{\cdot}0632 - 0{\cdot}0061367t + 0{\cdot}00002513t^2.$$

(4) Gold............ 85·0
Iron............ 15·0
Hard-drawn.

Length 145 mm.; diameter 0·758 mm.

T.	Conducting power.
15·0	2·6239
57·5	2·2732
100·0	1·9926

$$\lambda = 2{\cdot}7645 - 0{\cdot}0096586t + 0{\cdot}00001940t^2.$$

(5) Silver .... 75·0
Palladium ...... 25·0
Made by Messrs. Johnson and Matthey.
Hard-drawn.

Length 520 mm.; diameter 0·802 mm.

T.	Conducting power.
11·0	8·4846
55·5	8·3577
100·0	8·2256

$$\lambda = 8{\cdot}5152 - 0{\cdot}0027644t - 0{\cdot}000001313t^2.$$

This alloy was formerly used by dentists on account of its elasticity. It was tested, as it appeared to answer some of the conditions required.

(6) Copper.......... 63·3
Zinc............ 36·7
Made from pure metals.
Hard-drawn.

Length 296·6 mm.; diameter 0·576 mm.

T.	Conducting power.
15·72	21·807
23·75	21·562
39·28	21·116
54·38	20·693
69·31	20·300
84·63	19·897
99·43	19·327

$$\lambda = 22{\cdot}274 - 0{\cdot}030601t + 0{\cdot}00002980t^2.$$

(7) Copper.......... 75·0
Zinc............ 25·0
Made from pure metals.
Hard-drawn.

Length 190 mm.; diameter 0·381 mm.

T.	Conducting power.
13·47	21·704
24·07	21·413
39·21	21·020
53·65	20·647
69·03	20·268
83·71	19·915
98·97	19·565

$$\lambda = 22{\cdot}076 - 0{\cdot}028100t + 0{\cdot}00002945t^2.$$

These alloys are given, as they approach in composition to that of brass.

It seemed very desirable to test the influence of temperature on the alloy, as it was proposed by Jacobi as a unit of electric resistance.

(8) Copper .... 90·3
Tin .... 9·7
Made from pure metals.
Hard-drawn.

Length 322·5 mm.; diameter 0·524 mm.

T.	Conducting power.
15·43	12·058
23·40	11·990
40·35	11·852
54·75	11·737
69·78	11·619
84·66	11·499
98·70	11·391

$$\lambda = 12{\cdot}186 - 0{\cdot}0084168t + 0{\cdot}000003700t^2.$$

(9) Copper .... 89·7
Tin .... 10·3
Made from pure metals.
Hard-drawn.

Length 429 mm.; diameter 0·627 mm.

T.	Conducting power.
11·0	10·1386
55·5	9·8710
100·0	9·6526

$$\lambda = 10{\cdot}212 - 0{\cdot}0068043t + 0{\cdot}00001210t^2.$$

These alloys are given, as they approach in composition to that of ordinary gun-metal.

(10) Gun-metal (Austrian).
Copper.
Zinc.
Iron.
A specimen obtained through the kindness of Mr. F. Abel.
Hard-drawn.

Length 904·5 mm.; diameter 0·650 mm.

T.	Conducting power.
13·0	26·336
56·5	24·056
100·0	22·121

$$\lambda = 27{\cdot}084 - 0{\cdot}058750t + 0{\cdot}00009116t^2.$$

The conducting power of this alloy increased by heating to 100° for one day 5·7 per cent.—a larger increment than has been observed with any alloy. Generally, the conducting power of an alloy either remains constant or only varies 0·1 or 0·2 per cent. under the same conditions.

(11) Proof gold.
Hard-drawn.

Length 1564 mm.; diameter 0·525 mm.

T.	Conducting power.
15·0	68·969
57·5	60·179
100·0	53·387

$$\lambda = 72{\cdot}548 - 0{\cdot}24692t + 0{\cdot}0005531t^2.$$

(12) Standard silver.
Hard-drawn.

Length 2328 mm.; diameter 0·525 mm.

T.	Conducting power.
12·0	78·015
56·0	69·301
100·0	61·949

$$\lambda = 80{\cdot}628 - 0{\cdot}22196t + 0{\cdot}0003518t^2.$$

In the following Table I have given the results here obtained, with those of last year, in such a manner that they may be easily compared:—

	Conducting power at 0°.	Percentage decrement in conducting power between 0° & 100°.
Pure iron *	16·81	39·2
Pure thallium *	9·16	31·4
Other pure metals in a solid state	....	29·3
Gold, with 15 p.c. iron	2·76	27·9
Proof gold	72·55	26·4
Standard silver	80·63	23·2
Gun-metal (Austrian)	27·08	18·3
Gold, with 10 p.c. iron	2·06	17·5
Gold, with 14·3 p.c. silver and 7·4 p.c. copper	44·47	15·5
Copper, with 36·7 p.c. zinc	22·27	12·4
Copper, with 25 p.c. zinc	22·08	11·5
Silver, with 5 p.c. platinum *	31·64	11·3
Silver, with 9·8 p.c. platinum *	18·04	7·1
Copper, with 9·7 p.c. tin	12·19	6·6
The gold-silver alloy *	15·03	6·5
Platinum, with 33·4 p.c. iridium	4·54	5·9
Copper, with 10·3 p.c. tin	10·21	5·2
Gold, with 18·1 p.c. silver and 15·4 p.c. copper *	10·6	5·2
Gold, with 15·2 p.c. silver and 26·5 p.c. copper *	12·02	4·8
German silver *	7·80	4·4
Gold, with 5 p.c. iron	2·10	4·3
Gold, with 4·7 p.c. iron	2·37	3·8
Silver, with 25 p.c. palladium	8·52	3·4
Silver, with 33·4 p.c. platinum †	6·70	3·1

It will be observed that I have not yet been able to find an alloy whose conducting power decreases between 0° and 100° less than that of the alloy of silver with 33·4 p.c. platinum; and from results obtained in this direction in conjunction with Dr. Vogt, I am of opinion there will be great difficulty in doing so. We have already tested upwards of 100 alloys, and it is curious how few we have found whose conducting power varies less than that of German silver between 0° and 100°.

Appendix C.—*On the Elementary Relations between Electrical Measurements. By* Professor J. Clerk Maxwell *and* Mr. Fleeming Jenkin.

Part I.—Introductory.

1. *Objects of Treatise.*—The progress and extension of the electric telegraph has made a practical knowledge of electric and magnetic phenomena necessary to a large number of persons who are more or less occupied in the construction and working of the lines, and interesting to many others who are unwilling to be ignorant of the use of the network of wires which surrounds them. The discoveries of Volta and Galvani, of Oersted, and of Faraday are familiar in the mouths of all who talk of science, while the results of those discoveries are the foundation of branches of industry conducted by many who have perhaps never heard of those illustrious names. Between the student's mere knowledge of the history of discovery and the workman's

* Proc. Roy. Soc. xii. p. 472 (1863). † *Suprà*, p. 12; and Brit. Assoc. Rep. 1862, p. 131.

practical familiarity with particular operations which can only be communicated to others by direct imitation, we are in want of a set of rules, or rather principles, by which the laws remembered in their abstract form can be applied to estimate the forces required to effect any given practical result.

We may be called on to construct electrical apparatus for a particular purpose. In order to know how many cells are required for the battery, and of what size they should be, we require to know the strength of current required, the electromotive force of the cells, and the resistance of the circuit. If we know the results of previous scientific inquiry, and are acquainted with the method of adapting them to the case before us, we may discover the proper arrangement at once. If we are unable to make any estimate of what is required before constructing the apparatus, we may have to encounter numerous failures which might have been avoided if we had known how to make a proper use of existing data.

All exact knowledge is founded on the comparison of one quantity with another. In many experimental researches conducted by single individuals, the absolute values of those quantities are of no importance; but whenever many persons are to act together, it is necessary that they should have a common understanding of the measures to be employed. The object of the present treatise is to assist in attaining this common understanding as to electrical measurements.

2. *Derivation of Units from fundamental Standards.*—Every distinct kind of quantity requires a standard of its own, and these standards might be chosen quite independently of each other, and in many cases have been so chosen; but it is possible to deduce all standards of quantity from the fundamental standards adopted for length, time, and mass; and it is of great scientific and practical importance to deduce them from these standards in a systematic manner. Thus it is easy to understand what a square foot is when we know what a linear foot is, or to find the number of cubic feet in a room from its length, breadth, and height; because the foot, the square foot, and the cubic foot are parts of the same system of units. But the pint, gallon, &c. form another set of measures of volume which has been formed without reference to the system based on length; and in order to reduce the one set of numbers to the other, we have to multiply by a troublesome fraction, difficult to remember, and therefore a fruitful source of error.

The varieties of weights and measures which formerly prevailed in this country, when different measures were adopted for different kinds of goods, may be taken as an example of the principle of unsystematized standards, while the modern French system, in which every thing is derived from the elementary standards, exhibits the simplicity of the systematic arrangement.

In the opinion of the most practical and the most scientific men, a system in which every unit is derived from the primary units with decimal subdivisions is the best whenever it can be introduced. It is easily learnt; it renders calculation of all kinds simpler; it is more readily accepted by the world at large; and it bears the stamp of the authority, not of this or that legislator or man of science, but of nature.

The phenomena by which electricity is known to us are of a mechanical kind, and therefore they must be measured by mechanical units or standards. Our task is to explain how these units may be derived from the elementary ones; in other words, we shall endeavour to show how all electric phenomena may be measured in terms of time, mass, and space only, referring briefly in each case to a practical method of effecting the observation.

3. *Standard Mechanical Units.*—In this country the standard of length is

one yard, but a foot is the unit popularly adopted. In France it is the ten millionth part of the distance from the pole to the equator, measured along the earth's surface, according to the calculations of Delambre; and this measure is called a metre, and is equal to 3·280899 feet, or 39·37079 inches.

In the original Report the metre was taken as the fundamental unit of length; the gramme, or French standard of weight, is not, however, systematically derived from the metre, being the weight not of a cubic metre, but of a cubic centimetre of water. This consideration has led several Members of the Committee in subsequent writings to adopt the centimetre as the fundamental unit of length. To facilitate comparison with these writings, constants based on the centimetre will be given, besides those for the metre.

The standard unit of time in all civilized countries is deduced from the time of rotation of the earth about its axis. The sidereal day, or the true period of rotation of the earth, can be ascertained with great exactness by the ordinary observations of astronomers; and the mean solar day can be deduced from this by our knowledge of the length of the year. The unit of time adopted in all physical researches is one second of mean solar time.

The standard unit of mass is in this country the avoirdupois pound, as we received it from our ancestors. The grain is one 7000th of a pound. In the French system it is the gramme derived from the unit of length, by the use of water at a standard temperature as a standard of density. The weight of one cubic centimetre of water is a gramme $=15{\cdot}43235$ grains $={\cdot}00220462$ lb.

A Table, showing the relative value of the standard and derived units in the British and metrical system is given in § 55.

The unit of force adopted in this treatise is that force which will produce a unit of velocity in a free unit mass, by acting on it during a unit of time. This unit of force is equal to the weight of the unit mass divided by g, where g is the accelerating force of gravity, the value of which, as depending on the place of observation, is given in § 55. In this country it is about 32·2 feet, or 981 centimetres.

A unit of force still very generally used is the weight of the standard mass. This is called the gravitation unit of force, and measurements of force, work, &c. in which it is used are called gravitation measurements. The gravitation unit is equal to the absolute unit multiplied by g.

The unit of work adopted in this treatise is the unit of force, defined as above, acting through the unit of space (*vide* § 55).

4. *Dimensions of Derived Units.*—The name of every quantity consists of two factors or components, and may be written thus, $Q[Q]$.

The first, or numerical factor, Q, is a number, integral or fractional. The second, or denominational factor, $[Q]$, is the name of an individual thing of the same kind as the quantity to be expressed, the magnitude of which is agreed on among men. Thus, in the expression 28 lb., 28 is the numerical part represented by Q, and lb. is the denominational part represented by $[Q]$. When Q is unity, then the quantity expressed is the unit, $1[Q]$, or simply $[Q]$. In the example $[Q]$ is one pound; that is, a piece of platinum preserved in the Exchequer Chambers, and marked "P. S. 1844, 1 lb.," or some copy of the same.

We shall use the symbols $[L]$, $[M]$, and $[T]$ enclosed in square brackets to denote the standards or units of length, mass, and time; and symbols without brackets, such as L, M, T, to denote the number of such units in the quantity to be expressed. Thus if $[L]$ denotes a centimetre and L the number 978, $L[L]$ denotes 978 centimetres. Similarly, if $[l]$ denotes 1 foot and l the number 32·088, $l[l]$ denotes 32·088 feet.

Now these quantities express the same distance measured in two different ways, so that

$$L[L] = l[l];$$

but 1 foot is 30·479 centimetres, or

$$[l] = 30{\cdot}479\,[L].$$

Hence

$$L = 30{\cdot}479\,l;$$

or the numerical factor of the expression of a given quantity varies inversely as the magnitude of the unit employed.

In passing from one system of measurements to another, we first consider the magnitude of the units employed in the two systems and then determine the numerical factors so that the quantity expressed may be the same. Every measurement of which we have to speak involves as factors measurements of time, space, and mass only; but these measurements enter sometimes at one power and sometimes at another. In passing from one set of fundamental units to another, and for other purposes, it is necessary to know at what power each of these fundamental measurements enters into the derived measure.

Thus the value of a force is directly proportional to a length and a mass, but inversely proportional to the square of a time. This is expressed by saying that the *dimensions* of a force are $\left[\frac{LM}{T^2}\right]$; in other words, if we wish to pass from the English to the French system of measurements, the French unit of force will be to the English as $\frac{{\cdot}0328 \times 15{\cdot}43}{1} : 1$, or as ·506 to 1; because there are ·0328 feet in a centimetre, and 15·43 grains in a gramme. If the metre be adopted as the unit of length, the French unit of force will be to the English as 50·6 to 1. If the minute were chosen as the unit of time, the unit of force would, in either system, be $\frac{1}{3600}$ of that founded on the second as unit.

A Table of the dimensions of every unit adopted in the present treatise is given in § 55.

Part II.—The Measurement of Magnetic Phenomena.

5. *Magnets and Magnetic Poles.*—Certain natural bodies, as the iron ore called loadstone, the earth itself, and pieces of steel after being subjected to certain treatment, are found to possess the following properties, and are called magnets.

If one of these bodies be free to turn in any direction, the presence of another will cause it to set itself in a position which is conveniently described or defined by reference to certain imaginary lines occupying a fixed position in the two bodies, and called their magnetic axes. One object of our magnetic measurements will be to determine the force which one magnet exerts upon another. It is found by experiment that the greatest manifestation of force exerted by one long thin magnet on another occurs very near the ends of the two bars, and that the two ends of any one long thin magnet possess opposite qualities. This peculiarity has caused the name of "poles" to be given to

the ends of long magnets; and this conception of a magnet, as having two poles capable of exerting opposite forces joined by a bar exerting no force, is so much the most familiar that we shall not hesitate to employ it, especially as many of the properties of magnets may be correctly expressed in this way; but it must be borne in mind, in speaking of poles, that they do not really exist as points or centres of force at the ends of the bar, except in the case of long, infinitely thin, uniformly magnetized rods.

If we mark the poles of any two magnets which possess similar qualities, we find that the two marked poles repel each other, that two unmarked poles also repel each other; but that a marked and an unmarked pole attract each other. The pole which is repelled from the northern regions of the earth is called the positive pole; the other end the negative pole. The negative pole is generally marked N by British instrument-makers, and is sometimes called the north pole of the magnet, though it is obviously similar to the earth's south pole.

The strength of the pole is necessarily defined as proportional to the force it is capable of exerting on any other pole. Hence the force f exerted between two poles of the strengths m and m_1 must be proportional to the product $m\,m_1$. The force f is also found to be inversely proportional to the square of the distance, D, separating the poles, and to depend on no other quantity; hence we have, unless an absurd and useless coefficient be introduced,

$$f=\frac{m\,m_1}{D^2}. \qquad (1)$$

From which equation it follows that the unit pole will be that which at unit distance repels another similar pole with unit force; f will be an attraction or a repulsion according as the poles are of opposite or the same kinds. The dimensions of the unit magnetic pole are $\left[\frac{L^{\frac{3}{2}}M^{\frac{1}{2}}}{T}\right]$.

6. *Magnetic Field.*—It is clear that the presence of a magnet in some way modifies the surrounding space, since any other magnet brought into that space experiences a peculiar force. The neighbourhood of a magnet is, for convenience, called a magnetic field; and for the same reason the effect produced by a magnet is often spoken of as due to the magnetic field, instead of to the magnet itself. This mode of expression is the more proper, inasmuch as the same or a similar condition of space may be produced by the passage of electrical currents in the neighbourhood, without the presence of a magnet. Since the peculiarity of the magnetic field consists in the presence of a certain force, we may numerically express the properties of the field by measuring the strength and direction of the force, or, as it may be worded, the intensity of the field and the direction of the lines of force.

This direction at any point is the direction in which the force tends to move a free pole; and the intensity, H, of the field is completely defined as proportional to the force, f, with which it acts on a free pole; but this force, f, is also proportional to the strength, m, of the pole introduced into the field, and it depends on no other quantities; hence

$$f=m\mathrm{H}, \qquad (2)$$

and therefore the field of unit intensity will be that which acts with unit force on the unit pole.

The dimensions of [H] are $\left[\frac{M^{\frac{1}{2}}}{L^{\frac{1}{2}}T}\right]$.

The lines of force produced by a long thin bar-magnet near its poles radiate from the poles, and the intensity of the field is equal to the quotient of the strength of the pole divided by the square of the distance from the pole; thus the unit field is produced at the unit distance from the unit pole.

In a uniform magnetic field the lines of force, as may be demonstrated, are parallel; such a field can only be produced by special combinations of magnets, but a small field at a great distance from any one pole is sensibly uniform. Thus, in any room unaffected by the neighbourhood of iron or magnets, the magnetic field due to the earth is sensibly uniform; its direction is that assumed by the dipping-needle.

7. *Magnetic Moment.*—When a bar magnet is placed in a uniform field two equal opposite and parallel forces act on its poles, and tend to set it with the line joining those poles in the direction of the force of the field. When the magnet is so placed that the line joining the poles is at right angles to the lines of force in the field, this tendency to turn or "couple," G, is proportional to the intensity of the field, H, the strength of the poles, m, and the distance between them, l; or

$$G = mlH. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

ml, or the product of the strength of the poles into the length between them, is called the magnetic moment of the magnet; and from equation (3) it follows that, in a field of unit intensity, the couple actually experienced by any magnet in the above position measures its moment. The dimensions of the unit of magnetic moment are evidently $\left[\frac{L^{\frac{5}{2}} M^{\frac{1}{2}}}{T}\right]$.

8. *Intensity of Magnetization.*—The intensity of magnetization of a magnet is measured by its magnetic moment divided by its volume.

The dimensions of the unit of magnetization are therefore $\left[\frac{M^{\frac{1}{2}}}{L^{\frac{1}{2}} T}\right]$, the same as in the case of intensity of field.

9. *Coefficient of Magnetic Induction.*—When certain bodies, such as soft iron, &c., are placed in the magnetic field, they become magnetized by "induction;" so that the intensity of magnetization is (except when great) nearly proportional to the intensity of the field.

In diamagnetic bodies, such as bismuth, the direction of magnetization is opposite to that of the field. In paramagnetic bodies, such as iron, nickel, &c., the direction of magnetization is the same as that of the field.

The coefficient of magnetic induction is the ratio of the intensity of magnetization to the intensity of the magnetic force within the body, and is therefore a *numerical* quantity, positive for paramagnetic bodies, negative for diamagnetic bodies.

10. *Magnetic Potentials and Equipotential Surfaces.*—If we take a very long magnet, and, keeping one pole well out of the way, move the other pole from one point to another of the magnetic field, we shall find that the forces in the field do work on the pole, or that they act as a resistance to its motion, according as the motion is with or contrary to the force acting on the pole. If the pole moves at right angles to the force, no work is done.

The *magnetic potential* at any point in a magnetic field is measured by the work done against the magnetic forces on a unit pole during its motion from an infinite distance from the magnet producing the field to the point in question, supposing the unit pole to exercise no influence on the magnetic field in question. The idea of potential as a mathematical quantity having different

values at different points of space was brought into form by Laplace*. The name of potential, and the application to a great number of electric and magnetic investigations, were introduced by George Green in his Essay on Electricity (Nottingham, 1828).

An equipotential surface in a magnetic field is a surface so drawn that the potential of all its points are equal. By drawing a series of equipotential surfaces corresponding to potentials 1, 2, 3 . . . . . . *n*, we may map out any magnetic field, so as to indicate its properties.

The magnetic force at any point is perpendicular to the equipotential surface at that point, and its intensity is the reciprocal of the distance between one surface and the next at that point. The dimensions of the unit of magnetic potential are $\left[\frac{L^{\frac{1}{2}} M^{\frac{1}{2}}}{T}\right]$.

11. *Lines of Magnetic Force.*—There is another way of exploring the magnetic field, and indicating the direction and magnitude of the force at any point. The conception and application of this method in all its completeness is due to Faraday†. The full importance of this method cannot be recognized till we come to electro-magnetic phenomena (§§ 22, 23, & 24).

A line whose direction at any point always coincides with that of the force acting on the pole of a magnet at that point, is called a line of magnetic force. By drawing a sufficient number of such lines we may indicate the *direction* of the force in every part of the magnetic field; but by drawing them according to rule, we may indicate the intensity of the force at any point as well as its direction. It has been shown ‡ that if, in any part of their course, the number of lines passing through unit of area is proportional to the intensity there, the same proportion between the number of lines in unit of area and the intensity will hold good in every part of the course of the lines.

All that we have to do, therefore, is to space out the lines in any part of their course, so that the number of lines which start from unit of area is *equal* to the number representing the intensity of the field there. The intensity at any other part of the field will then be measured by the number of lines which pass through unit of area there; each line indicates a constant and equal force.

12. *Relation between Lines of Force and Equipotential Surfaces.*—The lines of force are always perpendicular to the equipotential surfaces; and the number of lines passing through unit of area of an equipotential surface is the reciprocal of the distance between that equipotential surface and the next in order—a statement made above in slightly different language.

In a uniform field the lines of force are straight, parallel, and equidistant; and the equipotential surfaces are planes perpendicular to the lines of force, and equidistant from each other.

If one magnetic pole of strength m be alone in the field, its lines of force are straight lines, radiating from the pole equally in all directions; and their number is $4\pi m$. The equipotential surfaces are a series of spheres whose centres are at the pole, and whose radii are m, $\frac{1}{2}m$, $\frac{1}{3}m$, $\frac{1}{4}m$, &c. In other magnetic arrangements these lines and surfaces are more complicated; but in all cases the calculation is simple, and in many cases the lines and surfaces can be graphically constructed without any calculation.

* Mécanique Céleste, liv. iii.

† Experimental Researches, vol. iii. art. 3122 *et passim*.

‡ *Vide* Maxwell on Faraday's Lines of Force, Cambridge Phil. Trans. 1857.

Part III.—Measurement of Electric Phenomena by their Electromagnetic Effects.

13. *Preliminary.*—Before treating of electrical measurements, the exact meaning in which the words "quantity," "current," "electromotive force," and "resistance" are used must be explained. But in giving these explanations, we shall assume the reader to be acquainted with the meaning of such expressions as conductor, insulator, voltaic battery, &c.

14. *Meaning of the words "Electric Quantity."*—When two light conducting bodies are connected with the same pole of a voltaic battery, while the other pole is connected with the earth, they may be observed to repel one another. The two poles produce equal and similar effects. When the two bodies are connected with opposite poles, they attract one another. Bodies, when in a condition to exert this peculiar force one on the other, are said to be electrified or charged with electricity. These words are mere names given to a peculiar condition of matter. If a piece of glass and a piece of resin are rubbed together, the glass will be found to be in the same condition as an insulated body connected with the copper pole of the battery, and the resin in the same condition as the body connected with the zinc pole of the battery. The former is said to be positively and the latter negatively electrified. The propriety of this antithesis will soon appear. The force with which one electrified body acts on another, even at a constant distance, varies with different circumstances. When the force between the two bodies at a constant distance, and separated by air, is observed to increase, it is said to be due to an increase in the quantity of electricity; and the quantity at any spot is defined as proportional to the force with which it acts, through air, on some other constant quantity at a distance. If two bodies, charged each with a given quantity of electricity, are incorporated, the single body thus composed will be charged with the sum of the two quantities. It is this fact which justifies the use of the word "quantity."

Thus the quality in virtue of which a body exerts the peculiar force described is called electricity, and its quantity is measured (*cæteris paribus*) by measuring force.

The quantity, thus defined, produced on two similar balls similarly circumstanced, but connected with opposite poles of a voltaic battery, is equal, but opposite; so that the sum of these two equal and opposite quantities is zero; hence the conception of positive and negative quantities.

In speaking of a quantity of electricity, we need not conceive it as a separate thing, or entity distinct from ponderable matter, any more than in speaking of sound we conceive it as having a distinct existence. Still it is convenient to speak of the intensity or velocity of sound to avoid tedious circumlocution; and quite similarly we may speak of electricity without for a moment imagining that any real electric fluid exists.

The laws according to which this force varies, as the shape of the conductors, their combinations, and their distances are varied, have been established by Coulomb, Poisson, Green, W. Thomson, and others. These will be found accurately described, independently of all hypothesis, in papers by Professor W. Thomson, published in the Cambridge Mathematical Journal, vol. i. p. 75 (1846), and a series of papers in 1848 and 1849.

15. *Meaning of the words "Electric Current."*—When two balls charged by the opposite poles of a battery with opposite and equal quantities of electricity are joined by a conductor, they lose in a very short time their peculiar properties, and assume a neutral condition intermediate between the

positive and negative states, exhibiting no electrical symptoms whatever, and hence described as unelectrified, or containing no electricity. But, during the first moment of their junction, the conductor is found to possess certain new and peculiar properties: any one part of the conductor exerts a force upon any other part of the conductor; it exerts a force on any magnet in the neighbourhood; and if any part of the conductor be formed by one of those compound bodies called electrolytes, a certain portion of this body will be decomposed. These peculiar effects are said to be due to a current of electricity in the conductor. The positive quantity, or excess, is conceived as flowing into the deficiency represented by the negative quantity; so that the whole combination is reduced to the neutral condition. This neutral condition is similar to that of the earth where the experiment is tried. If the balls are continually recharged by the battery, and discharged or neutralized by the wire, a rapid succession of the so-called currents will be sent; and it is found that the force with which a magnet is deflected by this rapid succession of currents is proportional (*cœteris paribus*) to the quantity of electricity passed through the conductor per second; it is also found that the amount of chemical action, measured by the weights of the particular substances decomposed, is proportional to the same quantity. The currents just described are intermittent; but a wire or conductor, used simply to join the two poles of a battery, acquires permanently the same properties as when used to discharge the balls as above with great rapidity; and the greater the rapidity with which the balls are discharged, the more perfect the similarity of the condition of the wire in the two cases. The wire in the latter case is therefore said to convey a permanent current of electricity, the magnitude or strength of which is defined as proportional to the quantity conveyed per second. This definition is expressed by the equation

$$C=\frac{Q}{t}, \qquad (4)$$

where C is the current, Q the quantity, and t the time. A permanent current flowing through a wire may be measured by the force which it exerts on a magnet; the actual quantity it conveys may be obtained by comparing this force with the force exerted, under otherwise similar conditions, when a known quantity is sent through the same wire by discharges. The strength of a permanent current is found at any one time to be equal in all parts of the conductor. Conductors conveying currents exert a peculiar force one upon another; and during their increase or decrease they produce currents in neighbouring conductors. Similar effects are produced as they approach or recede from neighbouring conductors. The laws according to which currents act upon magnets and upon one another will be found in the writings of Ampère and Weber.

16. *Meaning of the words "Electromotive Force."*—Hitherto we have spoken simply of statical effects; but it is found that a current of electricity, as above defined, cannot exist without effecting work or its equivalent. Thus it either heats the conductor, or raises a weight, or magnetizes soft iron, or effects chemical decomposition; in fine, in some shape it effects work, and this work bears a definite relation to the current. Work done presupposes a force in action. The immediate force producing a current, or, in other words, causing the transfer of a certain quantity of electricity, is called an electromotive force. This force is necessarily assumed as ultimately due to that part of a circuit where a "degradation" or consumption of energy takes place: thus we speak of the electromotive force of the voltaic or thermo-

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electric couple; but the term is also used, independently of the source of power, to express the fact that, however caused, a certain force tending to do work by setting electricity in motion does, under certain circumstances, exist between two points of a conductor or between two separate bodies. But equal quantities of electricity transferred in a given time do not necessarily or usually produce equal amounts of work; and the electromotive force between two points, the proximate cause of the work, is defined as proportional to the amount of work done between those points when a given quantity of electricity is transferred from one point to another. Thus if, with equal currents in two distinct conductors, the work done in the one is double that done in the other in the same time, the electromotive force in the first case is said to be double that in the second; but if the work done in two circuits is found strictly proportional to the two currents, the electromotive force acting on the two currents is said to be the same. Defined in this way, the electromotive force of a voltaic battery is found to be constant so long as the materials of which it is formed remain in a similar or constant condition. The above definitions, in mathematical language, give $W=ECt$,

$$\text{or} \quad E=\frac{W}{Ct}, \quad \dots\dots\dots\dots \quad (5)$$

where E is the electromotive force, and W the work done. Thus the electromotive force producing a current in a conductor is equal to the ratio between the work done in the unit of time and the current effecting the work. This conception of the relations of work, electromotive force, current, and quantity will be aided by the following analogy:—A quantity of electricity may be compared to a quantity or given mass of water; currents of water in pipes in which equal quantities pass each spot in equal times then correspond to equal currents of electricity; electromotive force corresponds to the head of water producing the current. Thus if, with two pipes conveying equal currents, the head forcing the water through the first were double that forcing it through the second, the work done by the water in flowing through the first pipe would necessarily be twice that done by the water in the second pipe; but if twice as much water passed through the first pipe as passes through the second, the work done by water in the first pipe would again be doubled. This corresponds exactly with the increase of work done by the electrical current when the electromotive force is doubled and when the quantity is doubled.

Thus, to recapitulate, the quality of a battery or source of electricity, in virtue of which it tends to do work by the transfer of electricity from one point to another, is called its electromotive force, and this force is measured by measuring the work done during the transfer of a given quantity of electricity between these points. The relations between electromotive force and work were first fully explained in a paper by Professor W. Thomson, on the application of the principle of mechanical effect to the measurement of electromotive forces, published in the 'Philosophical Magazine' for December 1851.

17. *Meaning of the words "Electric Resistance."*—It is found by experiment that even when the electromotive force between two points remains constant, so that the work done by the transfer of a given quantity of electricity remains constant, nevertheless, by modifying the material and form of the conductor, this transfer may be made to take place in very different times; or, in other words, currents of very different magnitudes are produced, and very different amounts of work are done, in the unit of time. The quality of

the conductor in virtue of which it prevents the performance of more than a certain amount of work in a given time by a given electromotive force is called its electrical resistance. The resistance of a conductor is therefore inversely proportional to the work done in it when a given electromotive force is maintained between its two ends; and hence, by equation (5), it is inversely proportional to the currents which will then be produced in the respective conductors. But it is found by experiment that the current produced in any case in any one conductor is simply proportional to the electromotive force between its ends; hence the ratio $\frac{E}{C}$ will be a constant quantity, to which the resistance as above defined must be proportional, and may with convenience be made equal; thus

$$R=\frac{E}{C}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

an equation expressing Ohm's law. In order to carry on the parallel with the pipes of water, the resistance overcome by the water must be of such nature that twice the quantity of water will flow through any one pipe when twice the head is applied. This would not be the result of a constant mechanical resistance, but of a resistance which increased in direct proportion to the speed of the current; thus the electrical resistance must not be looked on as analogous to a simple mechanical resistance, but rather to a coefficient by which the speed of the current must be multiplied to obtain the whole mechanical resistance. Thus if the electrical resistance of a conductor be called R, the work, W, is not equal to CRt, but $C \times CR \times t$, or

$$W=C^2Rt\,*, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where C may be looked on as analogous to a quantity moving at a certain speed, and CR as analogous to the mechanical resistance which it meets with in its progress, and which increases in direct proportion to the quantity conveyed in the unit of time.

18. *Measurement of Electric Currents by their Action on a Magnetic Needle.*—In 1820, Oersted discovered the action of an electric current upon a magnet at a distance, and one method of measurement may be based on this action. Let us suppose the current to be in the circumference of a vertical circle, so that in the upper part it runs from left to right. Then a magnet suspended in the centre of the circle will turn with the end which points to the north away from the observer. This may be taken as the simplest case, as every part of the circuit is at the same distance from the magnet, and tends to turn it the same way. The force is proportional to the moment of the magnet, to the strength of the current as defined by § 15, to its length, and inversely to the square of its distance from the magnet.

Let the moment of the magnet be ml, the strength of the current C, the radius of the circle k, the number of times the current passes round the circle n, the angle between the axes of the magnet and the plane of the circle θ, and the moment tending to turn the magnet G, then

$$G=mlC\,.\,2\pi nk\,\frac{1}{k^2}\cos\theta, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

* By equation (5) we have $W=CEt$; but by equation (6) $R=\frac{E}{C}$; hence $W=C^2Rt$.—Q.E.D.

which will be unity if ml, C, k, and the length of the circuit be unity, and if $\theta=0°$.

The unit of current founded on this relation, and called the electro-magnetic unit, is therefore that current of which the unit of length placed along the circumference of a circle of unit radius produces a unit of magnetic force at the centre.

The usual way of measuring C, the strength of a current, is by making it describe a circle about a magnet, the plane of the circle being vertical and magnetic north and south. Thus, if H be the intensity of the horizontal component of terrestrial magnetism, and G the moment of this on the magnet, $G=ml\mathrm{H}\sin\theta$, whence the strength of the current—

$$C=\frac{k}{2\pi n}\,\mathrm{H}\tan\theta, \quad \dots\dots\dots \quad (9)$$

where k is the radius of the circle, n the number of turns, H the intensity of the horizontal part of the earth's magnetic force as determined by the usual method, and θ the angle of deviation of the magnet suspended in the centre of the circle. As the strength of the current is proportional to the tangent of the angle θ, an instrument constructed on this plan is called a tangent galvanometer. The instrument called a sine galvanomter may also be used, provided the coil is circular. The equation is similar to that just given, substituting $\sin\theta$ for $\tan\theta$.

To find the dimensions of [C], the unit electric current, we must consider that what we observe is the force acting between a magnetic pole, m, and a current of given length, L, at a given distance, L_1, and that this force $=\frac{mCL}{L_1^{\,2}}$. Hence the dimensions of [C], the unit electric current, are $\left[\frac{L^{\frac{1}{2}}M^{\frac{1}{2}}}{T}\right]$.

19. *Measurement of Electric Currents by their mutual action on one another.*—Hitherto we have spoken of the measurement of currents as dependent on their action upon magnets; but this measurement in the same units can as simply be founded on their mutual action upon one another. Ampère has investigated the laws of mechanical action between conductors carrying currents. He has shown that the action of a small closed circuit at a distance is the same as that of a small magnet, provided the axis of the magnet be placed normal to the plane of the circuit, and the moment of the magnet be equal to the product of the current into the area of the circuit which it traverses.

Thus, let two small circuits, having areas A and A_1, be placed at a great distance, D, from each other in such a way that their planes are at right angles to each other, and that the line D is in the intersection of the planes. Now let currents, C and C_1, circulate in these conductors; a force will act between them tending to make their planes parallel, and the direction of the currents opposite. The moment of this couple will be

$$G=\frac{AC\times A_1C_1}{D^3}. \quad \dots\dots\dots\dots \quad (10)$$

Hence the unit electric current conducted round two circuits of unit area in vertical planes at right angles to each other, one circuit being at a great distance, D, vertically above the other, will cause a couple to act between the

circuits of a magnitude $\frac{1}{D^3}$. The definition of the unit current (identical with the unit founded on the relations given in § 18) might be founded on this action quite independently of the idea of magnetism.

20. *Weber's Electro-dynamometer.*—The measurement described in the last paragraph is only accurate when D is very great, and therefore the moment to be measured very small. Hence it is better to make the experimental measurements in another form. For this purpose, let a length (l) of wire be made into a circular coil of radius k; let a length (l_1) of wire be made into a coil of very much smaller radius, k_1. Let the second coil be hung in the centre of the first, the planes being vertical and at the angle θ. Then, if a current C traverses both coils, the moment of the force tending to bring them parallel will be

$$G = \tfrac{1}{2} C^2 \frac{l l_1 k_1}{k^2} \sin\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

This force may be measured in mechanical units by the angle through which it turns the suspended coil, the forces called into play by the mechanical arrangements of suspension being known from the construction of the instrument. Weber used a bifilar suspension, by which the weight of the smaller coil was used to resist the moment produced by the action of the currents.

21. *Comparison of the Electro-magnetic and Electro-chemical action of Currents.*—Currents of electricity, when passed through certain compound substances, decompose them; and it is found that, with any given substance, the weight of the body decomposed in a given time is proportional to the strength of the current as already defined with reference to its electro-magnetic effect. The voltameter is an apparatus of this kind, in which water is the substance decomposed. Special precautions have to be taken, in carrying this method of measurement into effect, to prevent variations in the resistance of the circuit, and consequently in the strength of the current. This subject is more fully treated in Part V. §§ 53, 54.

22. *Magnetic Field near a Current.*—Since a current exerts a force on the pole of a magnet in its neighbourhood, it may be said to produce a magnetic field (§ 6), and, by exploring this field with a magnet, we may draw lines of force and equipotential surfaces of the same nature as those already described for magnetic fields caused by the presence of magnets.

When the current is a straight line of indefinite length, like a telegraph-wire, a magnetic pole in its neighbourhood is urged by a force tending to turn it round the wire, so that this force is at any point perpendicular to the plane passing through this point and the axis of the current.

The equipotential surfaces are therefore a series of planes passing through the axis of the current, and inclined at equal angles to each other. The number of these planes is $4\pi C$, where C is the strength of the current.

The lines of magnetic force are circles having their centres in the axis of the current, and their planes perpendicular to it. The intensity of the magnetic force at a distance, k, from the current is the reciprocal of the distance between two equipotential surfaces, which shows the force to be $\frac{2C}{k}$.

The work done on a unit magnetic pole in going completely round the current is $4\pi C$, whatever the path which the pole describes.

23. *Mechanical Action of a Magnetic Field on a closed Conductor conveying a Current.*—When there is mechanical action between a conductor carrying a

current and a magnet, the force acting on the conductor must be equal and opposite to that acting on the magnet. Every part of the conductor is therefore acted on by a force perpendicular to the plane passing through its own direction and the lines of magnetic force due to the magnet, and equal to the product of the length of the conductor into the strength of the current, the intensity of the magnetic field, and the sine of the angle between the lines of force and the direction of the current. This may be more concisely expressed by saying, that if a conductor carrying a current is moved in a magnetic field, the work done on the conductor by the electro-magnetic forces is equal to the product of the strength of the current into the *number* of lines of force which it cuts during its motion.

Hence we arrive at the following general law, for determining the mechanical action on a closed conductor carrying a current and placed in a magnetic field :—

Draw the lines of magnetic force. Count the number which pass through the area enclosed by the circuit of the conductor, then any motion which increases this number will be aided by the electro-magnetic forces; so that the work done during the motion will be the product of the strength of the current and the number of additional lines of force.

For instance, let the lines of force be due to a single magnetic pole of strength m. These are $4\pi m$ in number, and are in this case straight lines radiating equally in all directions from the pole. Describe a sphere about the pole, and project the circuit on its surface by lines drawn to the pole. The surface of the area so described on the sphere will measure the solid angle subtended by the circuit at the pole. Let this solid angle $=\omega$, then the number of lines passing through the closed surface will be $m\omega$; and if C be the strength of the current, the amount of work done by bringing the magnet and circuit from an infinite distance apart to their present position will be $Cm\omega$. This shows that the magnetic potential of a closed circuit carrying a unit current with respect to a unit magnetic pole placed at any point is equal to the solid angle which the circuit subtends at that point.

By considering at what points the circuit subtends equal solid angles, we may form an idea of the surfaces of equal potential. They form a series of sheets, all intersecting each other in the circuit itself, which forms the boundary of every sheet. The number of sheets is $4\pi C$, where C is the strength of the current. The lines of magnetic force intersect these surfaces at right angles, and therefore form a system of rings encircling every point of the circuit. When we have studied the general form of the lines of force, we can form some idea of the electro-magnetic action of that current, after which the difficulties of numerical calculation arise entirely from the imperfection of our mathematical skill.

24. *General Law of the Mechanical Action between Electric Currents and other Electric Currents or Magnets.*—Draw the lines of magnetic force due to all the currents, magnets, &c. in the field, supposing the strength of each current or magnet to be reduced from its actual value to unity. Call the number of lines of force due to a circuit or magnet, which pass through another circuit, the potential coefficient between the one and the other. This number is to be reckoned positive when the lines of force pass through the circuit in the same direction as those due to a current in that circuit, and negative when they pass in the opposite direction.

If we now ascertain the change of the potential coefficient due to any displacement, this increment multiplied by the product of the strengths of the currents or magnets will be the amount of work done by the mutual action of

these two bodies during the displacement. The determination of the actual value of the potential coefficient of two things, in various cases, is an important part of mathematics as applied to electricity. (See the mathematical discussion of the experiments, Appendix D.)

25. *Electro-magnetic Measurement of Electric Quantity.*—A conducting body insulated at all points from the neighbouring conductors may in various ways be electrified, or made to hold a quantity of electricity. This quantity (§ 14) is perfectly definite in any given circumstances; it cannot be augmented or diminished so long as the conductor is insulated, and is called the charge of the conductor. Its magnitude depends on the dimensions and shape and position of the insulated and the neighbouring conductors, on the insulating material, and finally on the electromotive force between the insulated and the neighbouring conductors at the time when the charge was produced. The well-known Leyden jar is an arrangement by which a considerable charge can be obtained on a small conductor with moderate electromotive force between the inner and outer coatings which constitute respectively the "insulated" and "neighbouring" conductors referred to in general. We need not enter into the general laws determining the charge, since our object is only to show how it may be measured when already existing; but it may be well to state that the quantity on the charged insulated conductor necessarily implies an equal and opposite quantity on the surrounding or neighbouring conductors.

We have already defined the magnitude of a current of electricity as simply proportional to the quantity of electricity conveyed in a given time, and we have shown a method of measuring currents consonant with this definition. The unit quantity will therefore be that conveyed by the unit current as above defined in the unit of time. Thus if a unit current is allowed to flow for a unit of time in a wire connecting the two coatings of a Leyden phial, the quantity which one coating loses or which the other gains is the electromagnetic unit quantity*. The measurement thus defined of the quantity in a given statical charge can be made by observing the swing of a galvanometer-needle produced by allowing the charge to pass through the coil of the galvanometer in a time extremely short compared with that occupied by an oscillation of the needle.

Let Q be the whole quantity of electricity in an instantaneous current, then

$$Q=2\,\frac{C_1 t}{\pi}\sin\tfrac{1}{2}i, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where C_1 = the strength of a current giving a unit deflection (45° on a tangent or 90° on a sine galvanometer), t = half the period or time of a complete oscillation of the needle of the galvanometer under the influence of terrestrial magnetism alone, and i = the angle to which the needle is observed to swing from a position of rest when the discharge takes place; C_1 is a constant which need only be determined once for each instrument, provided the horizontal force of the earth's magnetism remain unchanged. In the case of the tangent galvanometer, the formula for obtaining it has already been given. From equations (9) and (12) we have for a tangent galvanometer

$$Q=\frac{k}{\pi^2 n}\,Ht\sin\tfrac{1}{2}i, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

* Weber calls this quantity two units—a fact which must not be lost sight of in comparing his results with those of the Committee.

where, as before, k=the radius of the coil, and n=the number of turns made by the wire round the coil.

The quantity in a given charge which can be continually reproduced under fixed conditions may be measured by allowing a succession of discharges to pass at regular and very short intervals through a galvanometer, so as to produce a permanent deflection. The value of a current producing this deflection can be ascertained; and the quotient of this value by the number of discharges taking place in a 'second' gives the value of each charge in electro-magnetic measure.

To find the dimensions of [Q], we simply observe that the unit of electricity is that which is transferred by the unit current in the unit of time. Multiplying the dimensions of [C] by [T], we find the dimensions of [Q] are $[L^{\frac{1}{2}} M^{\frac{1}{2}}]$.

26. *Electric Capacity of a Conductor.*—It is found by experiment that, other circumstances remaining the same, the charge on an insulated conductor is simply proportional to the electromotive force between it and the surrounding conductors, or, in other words, to the difference of potentials (47). The charge that would be produced by the unit electromotive force is said to measure the electric capacity of a conductor. Thus, generally, the capacity of a conductor $S=\frac{Q}{E}$, where Q is the whole quantity in the charge produced by the electromotive force E. When the electromotive force producing the charge is capable of maintaining a current, the capacity of the conductor may be obtained without a knowledge of the value either of Q or E, provided we have the means of measuring the resistance of a circuit in electro-magnetic measure. For let R_1 be the resistance of a circuit, in which the given electromotive force E will produce the unit deflection on a tangent galvanometer, then, from equations (6) and (12), we have

$$S=2\frac{t\sin\frac{1}{2}i}{\pi R_1}, \qquad (14)$$

where t and i retain the same signification as in equation (13) (§ 25).

27. *Direct Measurement of Electromotive Force.*—The meaning of the words "electromotive force" has already been explained (§ 16); this force tends to do work by means of a current or transfer of electricity, and may therefore be said to produce and maintain the current. In any given combination in which electric currents flow, the immediate source of the power by which the work is done is said to produce the electromotive force. The sources of power producing electromotive force are various. Of these, chemical action in the voltaic battery, unequal distribution of temperature in circuits of different conductors, the friction of different substances, magneto-electric induction, and simple electric induction are the most familiar. An electromotive force may exist between two points of a conductor, or between two points of an insulator, or between an insulator and a conductor,—in fine, between any points whatever. This electromotive force may be capable of maintaining a current for a long time, as in a voltaic battery, or may instantly cease after producing a current of no sensible duration, as when two points of the atmosphere at different potentials (§ 47) are joined by a conductor; but in every case in which a constant electromotive force E is maintained between any two points, however situated, the work spent or gained in transferring a quantity, Q, of electricity from one of those points to the other will be constant; nor will this work be affected by the manner or method of the transfer. If

the electricity be slowly conveyed as a static charge on an insulated ball, the work will be spent or gained in accelerating or retarding the ball; if the electricity be conveyed rapidly through a conductor of small resistance, or more slowly through a conductor of great resistance, the work may be spent in heating the conductor, or it may electrolyze a solution, or be thermo-electrically or mechanically used; but in all cases the change effected, measured as equivalent to work done, will be the same, and equal to EQ. Hence the electromotive force from the point A to the point B is unity, if a unit of mechanical work is gained in the transfer of a unit of electricity from A to B. This general definition is due to Professor W. Thomson.

The direct measurement of electromotive force may be made by the measurement in any given case of the work done by the transfer of a given quantity of electricity. The ratio between the numbers measuring the work done and the quantity transferred would measure the electromotive force. This measurement has been made by Dr. Joule and Professor Thomson, by determining the heat developed in a wire by a given current measured as in § 18*.

28. *Indirect Measurements of Electromotive Force.*—The direct method of measurement is in most cases inconvenient, and in many impossible; but the indirect methods are numerous and easily applied. The relation between the current, C, the resistance, R, and the electromotive force, E, expressed by Ohm's law (equation 6), will determine the electromotive force of a battery whenever R and C are known. A second indirect method depends on the measurement of the statical force with which two bodies attract one another when the given electromotive force is maintained between them. This method is fully treated in Part IV. (43). The phenomenon on which it is based admits of an easy comparison between various electromotive forces by electrometers. This method is applicable even to those cases in which the electromotive force to be measured is incapable of maintaining a current. The laws of chemical electrolysis and electro-magnetic induction afford two other indirect methods of estimating electromotive force in special cases (54 and 31).

29. *Measurement of Electric Resistance.*—We have already stated that the resistance of a conductor is that property in virtue of which it limits the amount of work performed by a given electromotive force in a given time, and we have shown that it may be measured by the ratio $\frac{E}{C}$ of the electromotive force between two ends of a conductor to the current maintained by it. The unit resistance is therefore that in which the unit electromotive force produces the unit current, and therefore performs the unit of work in the unit of time. If in any circuit we can measure the current and electromotive force, or even the ratio of these magnitudes, we should, *ipso facto*, have measured the resistance of the circuit. The methods by which this ratio has been measured, founded on the laws of electro-magnetic induction, are fully described in Appendix D. Other methods may be founded on the measurement of currents and electromotive forces described in 18, 19, 20, 27, and 28. Lastly, a method founded on the gradual loss of charge through very great resistances will be found in Part IV. (45). The equation (25) there given for electrostatic measure is applicable to electro-magnetic measure when the capacity and difference of potentials are expressed in electro-magnetic units.

* Phil. Mag. vol. ii. 4th ser. 1851, p. 551.

30. *Electric Resistance in Electro-magnetic Units is measured by an Absolute Velocity.*—The dimensions of [R] are found, by comparing those of [E] and [C], to be $\left[\frac{L}{T}\right]$, or those of a simple velocity. This velocity, as was pointed out by Weber, is an absolute velocity in nature quite independent of the magnitude of the fundamental units in which it is expressed. The following illustration, due to Professor Thomson, will show how a velocity may express a resistance, and also how that expression may be independent of the magnitude of the units of time and space.

Let a wire of any material be bent into an arc of $57\frac{1}{4}°$ with any radius, k. Let this arc be placed in the magnetic meridian of any magnetic field, with a magnet of any strength freely suspended in the centre of the arc. Let two vertical wires or rails, separated by a distance equal to k, be attached by a wire to the ends of the arc; and let a cross piece slide along these rails inducing a current in the arc. Then it may be shown that the speed required to produce a deflection of 45° on the magnet will measure the resistance of the circuit, which is assumed to be constant. This speed will be the same whatever be the value of k, or the intensity of the magnetic field, or the moment of the magnet. In this form the experiment could not be easily carried out; but if a length, l, of wire be taken and rolled into a circular coil at the radius k, and the distance between the vertical rails be taken equal to $\frac{k^2}{l}$, then if the resistance of the circuit be the same as in the previous case, the deflection of 45° will be produced by the same velocity in the cross piece, measuring that resistance; or, generally, if the distance between the rails be $p\frac{k^2}{l}$, then p times the velocity required to produce the unit deflection (45°) will measure the resistance. The truth of this proposition can easily be established when the laws of magneto-electric induction have been understood (31).

31. *Magneto-electric Induction.*—Let a conducting circuit be placed in a magnetic field. Let C be the intensity of any current in that circuit; E the magnitude of the electromotive force acting in the circuit. Let the circuit be so moved that the number of lines of magnetic force (11) passing through the area which it encloses is increased by N in the time t, then (23) the electro-magnetic forces will contribute towards the motion an amount of work measured by CN. Now Q, the quantity of electricity which passes, is equal to Ct; so that the work done on the current is EQ or CEt. By the principle of conservation of energy, the work done by the electro-magnetic forces must be at the expense of that done by the electromotive forces, or

$$\mathrm{CN}+\mathrm{CE}t=0\,;$$

or dividing by Ct, we find that

$$\mathrm{E}=-\frac{\mathrm{N}}{t}\,; \quad \ldots\ldots\ldots \quad (15)$$

or, in other words, if the number of lines of force passing through the area enclosed by a circuit be increased, an electromotive force in the negative direction will act in the circuit measured by the number of lines of force added per second.

If R be the resistance of the circuit, we have, by Ohm's law (equation 6), E=CR; and therefore

$$\mathrm{N}=-\mathrm{E}t=-\mathrm{RC}t=-\mathrm{RQ}\,; \quad \ldots\ldots \quad (16)$$

or, in other words, if the number of lines of magnetic force passing through the area enclosed by the circuit is altered, a current will be produced in the circuit in the direction opposite to that of a current which would have produced lines of force in the direction of those added, and the quantity of electricity which passes multiplied by the resistance of the circuit measures the number of additional lines passing through the area enclosed by the circuit.

The facts of magneto-electric induction were discovered by Faraday, and described by him in the First Series of his "Experimental Researches in Electricity," read to the Royal Society, 24th November, 1831.

He has shown* the relation between the induced current and the lines of force cut by the circuit treated as a surface or area, and he has also described the state of a conductor in a field of force as a state the change of which is a cause of currents. He calls it the electrotonic state; and, as we have just seen, the electrotonic state may be *measured* by the number of lines of force which pass through the circuit at any time.

The measure of electromotive force used by W. Weber, and derived by him (independently of the principle of conservation of energy) from the motion of a conductor in a magnetic field, is the same as that at which we have arrived; for, from equation (15), we find that the unit electromotive force will be produced by motion in a magnetic field when one line of force is added (or subtracted) per unit of time, and this will occur when in a field of unit intensity a straight bar of unit length, forming part of a circuit otherwise at rest, is moved with unit velocity perpendicularly to the lines of force and to its own direction.

To W. Weber, whose numerical determinations of electrical magnitudes are the starting-point of exact science in electricity, we owe this, the first definition of the unit of electromotive force; but to Professor Helmholtz† and to Professor W. Thomson‡, working independently of each other, we owe the proof of the necessary existence of magneto-electric induction and the determination of electromotive force on strictly mechanical principles.

32. *On Material Standards for the Measurement of Electric Magnitudes.*—The comparison between two different electrical magnitudes of the same nature, *e. g.* between two currents or between two resistances, is in all cases much simpler than the direct measurements of these magnitudes in terms of time, mass, and space, as described in the foregoing pages. Much labour is, therefore, saved by the use of standards of each magnitude; and the construction and diffusion of those standards form part of the duties of the Committee.

Electric currents are most simply compared by "electro-dynamometers" (20)—instruments which, unlike galvanometers, are practically independent of the intensity of the earth's magnetism. When an instrument of this kind has been constructed, with which the values of the currents corresponding to each deflection has been measured (19, 20), other instruments may easily be so compared with this standard, that the relative value of the deflections produced by equal currents on the standard and the copies shall be known. Hence the absolute value of the current indicated by each deflection of each copy will be known in absolute measure. In other words, in order to obtain the electro-magnetic measure of a current in the system described, each observer in possession of an electro-dynamometer which has been compared with

* Experimental Researches, 3082, &c.

† Paper read before the Physical Society of Berlin, 1847 (*vide* Taylor's Scientific Memoirs, part ii. Feb. 1853, p. 114).

‡ Transactions of the British Association, 1848; Phil. Mag., Dec. 1851.

the standard instrument will simply multiply by a constant number the deflection produced by the current on his instrument (or the tangent or sine of the deflection, according to the particular construction of the instrument).

Electric quantities may be compared by the swing of the needle of a galvanometer of any kind. They may be measured by any one in possession of a standard electro-dynamometer, or resistance-coil, since the observer will then be in a position directly to determine C, in equation (12), or R, in equation (14).

Capacities may be compared by the methods described (26); and a Leyden jar or condenser (41) of unit capacity, and copies derived from it, may be prepared and distributed. The owner of such a condenser, if he can measure electromotive force, can determine the quantity in his condenser.

The material standard for *electromotive force* derived from electro-magnetic phenomena would naturally be a conductor of known shape and dimensions, moving in a known manner in a known magnetic field. Such a standard as this would be far too complex to be practically useful: fortunately a very simple and practical standard or gauge of electromotive force can be based on its statical effects, and will be described in treating of those effects (Part IV. 43). A practical standard for approximate measurements might be formed by a voltaic couple, the constituent parts of which were in a standard condition. It is probable that the Daniell's cell may form a practical standard of reference in this way, when its value in electro-magnetic measure is known. This value (centimetre-gramme second) lies between 9×10^7 and 11×10^7 (or 9×10^4 and 11×10^4 metre-gramme second). [*Note*, 1872.—Mr. Latimer Clark's cell equal to $1{\cdot}457 \times 10^8$ centimetre-gramme second series, or $1{\cdot}457 \times 10^5$ metre-gramme second series, is a better standard of E.M.F. This cell is composed of pure mercury as the negative element, the mercury being covered by a paste made by boiling mercurous sulphate in a thoroughly saturated solution of zinc sulphate, the positive element consisting of pure zinc resting on the paste. This element must not be used to produce a current, but forms an excellent standard of E.M.F., when compared with other cells, by any method which does not involve the passage of a current through the cell (*vide* 'Proc. Roy. Soc.' no. 136, 1872).]

Resistances are compared by comparing currents produced in the several conductors by one and the same electromotive force. The unit resistance, determined as in Appendix D, will be represented by a material conductor; simple coils of insulated wire compared with this standard, and issued by the Committee, will allow any observer to measure any resistance in electromagnetic measure.

Part IV.—Measurement of Electric Phenomena by Statical Effects.

33. *Electrostatic Measure of Electric Quantity.*—By the application of a sufficient electromotive force between two parts of a conductor which does not form a circuit, it is possible to communicate to either part a *charge* of electricity which may be maintained in both parts, if properly insulated (14). With the ordinary electromotive forces due to induction or chemical action, and the ordinary size of insulated conductors, the charge of electricity in electro-magnetic measure is exceedingly small; but when the capacity of the conductor is great, as in the case of long submarine cables, the charge may be considerable. By making use of the electromotive force produced by the friction of unlike substances, the charge or electrification even of small bodies may be made to produce visible effects. The electricity in a charge is

not essentially in motion, as is the case with the electricity in a current. In other words, a charge may be permanently maintained without the performance of work. Electricity in this condition is therefore frequently spoken of as statical electricity, and its effects, to distinguish them from those produced by currents, may be called statical effects. The peculiar properties of electrically charged bodies are these:—

1. When one body is charged positively (14), some other body or bodies must be charged negatively to the same extent.

2. Two bodies repel one another when both are charged positively, or both negatively, and attract when oppositely charged.

3. These forces are inversely proportional to the square of the distance of the attracting or repelling charges of electricity.

4. If a body electrified in any given invariable manner be placed in the neighbourhood of any number of electrified bodies, it will experience a force which is the resultant of the forces that would be separately exerted upon it by the different bodies if they were placed in succession in the positions which they actually occupy, without any alteration in their electrical conditions.

From these propositions it follows that, at a given distance, the force, f, with which two small electrified bodies repel one another is proportional to the product of the charges, q and q_1, upon them. But when the distance varies, this force, f, is inversely proportional to the square of the distance, d, between them; hence

$$f=\frac{qq_1}{d^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

When q and q_1 are of dissimilar signs, f becomes negative, *i. e.* there is an attraction, and not a repulsion. This equation is incompatible with the electro-magnetic definitions given in Part III., and, if it be allowed to be fundamental, gives a new definition of the unit quantity of electricity, as that quantity which, if placed at unit distance from another equal quantity of the same kind, repels it with unit force.

34. *Electrostatic System of Units.*—This new measurement of quantity forms the foundation of a distinct system or series of units, which may be called the electrostatic units, and measurements in these units will in these pages be designated by the use of small letters; thus, as Q, C, &c. signify the number of electrostatic units in the same quantities, currents, &c. in *electro-magnetic* measure, so q, c, e, and r, &c. will represent the *electrostatic* measure of quantity, current, electromotive force, resistance, &c.

The relations between current and quantity, between work, current, and electromotive force, and between electromotive force, current, and resistance, remain unchanged by the change from the electro-magnetic to the electrostatic system.

35. *Ratio between Electrostatic and Electro-magnetic Measures of Quantity.*—Since the expression forming the second member of equation (17) represents a force the dimensions of which are $\left[\frac{LM}{T^2}\right]$, the dimensions of $[q]$ are $\left[\frac{L^{\frac{3}{2}}M^{\frac{1}{2}}}{T}\right]$. The dimensions of the unit of electricity, [Q], in the electro-magnetic system are $[L^{\frac{1}{2}}M^{\frac{1}{2}}]$ (25). Hence, since in passing from the one system to the other we must employ the ratio $\frac{q}{Q}$, this ratio will be of the dimension $\left[\frac{L}{T}\right]$; that is to

say, it is of the nature of a velocity. In the present treatise this velocity will be designated by the letter v.

The first estimate of the relation between quantity of electricity measured statically and the quantity transferred by a current in a given time was made by Faraday*. A careful experimental investigation by MM. Weber and Kohlrausch† not only confirms the conclusion that the two kinds of measurements are consistent, but shows that the velocity v is 310,740,000 metres per second—a velocity not differing from the estimated velocity of light more than the different determinations of the latter quantity differ from each other. v must always be a constant real velocity in nature, and should be measured in terms of the system of fundamental units adopted in electrical measurements (3 and 55). A redetermination of v (46) will form part of the present Committee's business in 1863–64. It will be seen that, by definition, the quantity transmitted by an electro-magnetic unit current in the unit time is equal to v electrostatic units of quantity. In the centimetre-gramme second series the value of v will clearly be 100 times as great as that given above.

36. *Electrostatic Measure of Currents.*—In any coherent system, a current is measured by the quantity of electricity which passes in the unit of time (15); if both current and quantity are measured in electrostatic units, then

$$c=\frac{q}{t}. \qquad \qquad (18)$$

The dimensions of [c] are therefore $\left[\frac{L^{\frac{3}{2}}M^{\frac{1}{2}}}{T^2}\right]$; and in order to reduce a current from electro-magnetic to electrostatic measure, we must multiply C by v, or

$$c=v\mathrm{C}. \qquad \qquad (19)$$

37. *Electrostatic Measure of Electromotive Force.*—The statical measure of an electromotive force is the work which would be done by electrical forces during the passage of a unit of electricity from one point to another. The only difference between this definition and the electro-magnetic definition (16 and 27) consists in the change of the unit of electricity from the electro-magnetic to the electrostatic.

Hence if q units of electricity are transferred from one place to another, the electromotive force between those places being e, the work done during the transfer will be qe; but we found (27) that if E and Q be the electro-magnetic measures of the same quantities, the work done would be expressed by QE; hence

$$qe=\mathrm{QE};$$

but (35) $$q=v\mathrm{Q},$$

therefore $$e=\frac{\mathrm{E}}{v}. \qquad \qquad (20)$$

Thus, to reduce electromotive force from electro-magnetic to electrostatic measure, we must divide by v.

The dimensions of e are $\frac{L^{\frac{1}{2}}M^{\frac{1}{2}}}{T}$.

38. *Electrostatic Measure of Resistance.*—If an electromotive force, e, act

* Experimental Researches, series iii. § 361, &c.

† Abhandlungen der König. Sächsischen Ges. Bd. iii. (1857) p. 260; or, Poggendorff's Annalen, Bd. xcix. p. 10 (Aug. 1856).

on a conductor whose resistance in electrostatic measure is r, and produce a current, c, then by Ohm's law

$$r=\frac{e}{c}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Substituting for e and c their equivalents in electro-magnetic measure (equations 19 and 20), we have

$$r=\frac{1}{v^2}\frac{E}{C};$$

but (eq. 7) . . . . $R=\frac{E}{C}$,

and therefore $r=\frac{1}{v^2}R. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$

Hence, to reduce a resistance measured in electro-magnetic units to its electrostatic value, we must divide by v^2.

The dimensions of $[r]$ are $\left[\frac{T}{L}\right]$, or the reciprocal of a velocity.

39. *Electric Resistance in Electrostatic Units is measured by the Reciprocal of an Absolute Velocity.*—We have seen from the last paragraph that the dimensions of r establish this proposition; but the following independent definition, due to Professor W. Thomson, assists the mind in receiving this conception as a necessary natural truth. Conceive a sphere of radius k, charged with a given quantity of electricity, Q. The potential of the sphere, when at a distance from all other bodies, will be $\frac{Q}{k}$ (40, 41, and 47). Let it now be discharged through a certain resistance, r. Then if the sphere could collapse with such a velocity that its potential should remain constant, or, in other words, that the ratio of the quantity on the sphere to its radius should remain constant, during the discharge, then the time occupied by its radius in shrinking the unit of length would measure the resistance of the discharging conductor in electrostatic measure, or the velocity with which its radius diminished would measure the conducting power (50) of the discharging conductor. Thus the conducting power of a few yards of silk in dry weather might be an inch per second, in damp weather a yard per second. The resistance of 1000 miles of pure copper wire, $\frac{1}{16}$ inch in diameter, would be about 0·00000141 of a second per metre, or its conducting power one metre per 0·00000141 of a second, or 709220 metres per second.

40. *Electrostatic Measure of the Capacity of a Conductor.*—The electrostatic capacity of a conductor is equal to the quantity of electricity with which it can be charged by the unit electromotive force. This definition is identical with that given of capacity measured in electro-magnetic units (26). Let s be the capacity of a conductor, q the electricity in it, and e the electromotive force charging it; then

$$q=se. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

From this equation we can see that the dimension of the quantity s is a length only. It will also be seen that

$$s=v^2S, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where S is the electro-magnetic measure of the capacity of the conductor with the electrostatic capacity, s.

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The capacity of a spherical conductor in an open space is, in electrostatic measure, equal to the radius of the sphere—a fact demonstrable from the fundamental equation (17).

Experimentally to determine s, the capacity of the conductor in electrostatic measure, charge it with a quantity, q, of electricity, and measure in any unit its potential (47), e. Then bring it into electrical connexion with another conductor whose capacity, s_1, is known. Measure the potential, e_1, of s and s_1 after the charge is divided between them; then

$$q = se = (s + s_1)e_1,$$

and hence

$$s = \frac{e_1}{e - e_1} s_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

In this measurement we do not require to know e and e_1 in absolute measure, since the ratio of these two quantities only is required. We must, however, know the value of s_1; and hence we must begin either with a spherical conductor in a large open space, whose capacity is measured by its radius, or with some other form of absolute condenser alluded to in the following paragraph.

41. *Absolute Condenser. Practical Measurement of Quantity.*—As soon as the electromotive force of a source of electricity is known in electrostatic measure, the quantity which it will produce in the form of charge on simple figures is known by the laws of electrical distribution experimentally proved by Coulomb. Such simple figures may be termed *absolute condensers.* A sphere in an open space is such a condenser, and its capacity is numerically equal to its radius. A more convenient form is a sphere of radius x, suspended in the centre of a hollow sphere, radius y, the latter being in communication with the earth. The capacity, s, of the internal sphere is then, by calculation,

$$s = \frac{xy}{y - x}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

By a series of condensers of increasing capacity, we may measure the capacity of any condenser, however large. The comparison is made by the method described above (40). Thus, the practical method of measuring quantity in electrostatic measure is first to determine the capacity of the conductor containing the charge, and then to multiply that capacity by the electromotive force producing the charge (43).

42. *Practical Measurement of Currents.*—The electrostatic value of currents can be obtained from equation (21), when e and r are known, or from equation (19), when v and C are known, or by comparison with a succession of discharges of known quantities from an absolute condenser.

43. *Practical Measurement of Electromotive Force.*—The relations expressed by eq. (17) and (23) show that in any given circumstances the force exerted between two bodies due to the effects of statical electricity will be proportional to the electromotive force or difference of potential (47) between them. This fact allows us to construct gauges of electromotive force, or instruments so arranged that a given electromotive force between two parts of the apparatus brings an index into a sighted position. In order that the gauge should serve to *measure* the electromotive force absolutely, it is necessary that two things should be known: first, the distribution of the electricity over the two attracting or repelling masses (or, in other words, the capacity of each part); secondly, the absolute force exerted between them. For simple forms, the distribution or capacity of each part can be calculated from the fundamental principles

(33); the force actually exerted can be weighed by a balance. By these means Professor W. Thomson * determined the electromotive force of a Daniell's cell to be 0·0021 in British electrostatic units, or 0·0002951 in metrical units, or 0·002951 in centimetrical units. This proposition is equivalent to saying that two balls of a metre radius, at a distance d apart, measured in metres, in a large open space, and in connexion with the opposite poles of a Daniell's cell, would attract one another with a force equal

$$\text{to } \frac{0{\cdot}0002951 \times 0{\cdot}0002951}{d^2} \text{ absolute metrical units,}$$

$$\text{or } \frac{0{\cdot}000000008876}{d^2} \text{ gramme weight.}$$

An apparatus by which such a measurement as the foregoing can be carried out is called an absolute electrometer. It will be observed that, although the definition of electromotive force is founded on the idea of work, its practical measurement is effected by observing a force, inasmuch as when this force exerted between two conductors of simple shape is known, the work which the passage of a unit of electricity between them would perform may be calculated by known laws.

44. *Comparison of Electromotive Forces by their Statical Effects.*—This comparison is simpler than the absolute measurement, inasmuch as it is not necessary, in comparing two forces, to know the absolute values of either. Instruments by which the comparison can be made are called electrometers. Their arrangement is of necessity such that the force exerted between two given parts of the instrument shall be proportional to the difference of potential between them †. This force may be variable and measured by the torsion of a wire, as in Thomson's reflecting electrometer, or it may be constant, and the electromotive forces producing it may be compared by measuring the distance between the two electrified bodies at which these attract each other with that constant force. The latter arrangement is adopted in Professor Thomson's portable electrometer, first exhibited at the present meeting of the Association. The indications of a gauge or electrometer not in itself absolute may be reduced to absolute measurement by multiplication into a constant coefficient.

45. *Practical Measurement of Electric Resistance.*—The electrostatic resistance of a conductor of great resistance (such as gutta percha or india rubber) might be directly obtained in the following manner:—Let a body of known capacity, s (40), be charged to a given potential, P (47), and let it be gradually discharged through the conductor of great resistance, r. Let the time, t, be noted at the end of which the potential of the body has fallen to p. The rate of loss of electricity will then be $\frac{p}{sr}$. Hence $p = \mathrm{P}\epsilon^{-\frac{t}{sr}}$ and $\frac{t}{sr} = \log_\epsilon \frac{\mathrm{P}}{p}$. Hence

$$r = \frac{t}{s \log_\epsilon \frac{\mathrm{P}}{p}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

from which equation r can be deduced, if s, t, and the ratio $\frac{\mathrm{P}}{p}$ be known, t can be directly observed, s can be measured (40), and the ratio $\frac{\mathrm{P}}{p}$ can be measured

* Paper read before the Royal Society, February 1860. *Vide* Proceedings of the Royal Society, vol. x. p. 319, and Phil. Mag. vol. xx. 4th ser. (1860) p. 233.

† A bifilar suspension is now used (1872).

by an electrometer (44) in constant connexion with the charged body. This ratio can also be measured by the relative discharges through a galvanometer, first, immediately after the body has been charged to the potential P, and again when, after having been recharged to the potential P, it has, after a time t, fallen to potential p. (This latter plan has long been practically used by Messrs. Siemens, although the results have not been expressed in absolute measure.)

Unfortunately, in those bodies, such as gutta percha and india rubber, the resistance of which is sufficiently great to make t a measurable time, the phenomenon of absorption due to continued electrification * so complicates the experiment as to render it practically unavailable for any exact determination. The apparent effect of absorption is to cause r, the resistance of the material, to be a quantity variable with the time t; and the laws of the variation are very imperfectly known.

46. *Experimental Determination of the Ratio, v, between Electro-magnetic and Electrostatic Measures of Quantity.*—In order to obtain the value of v, it is necessary and sufficient that we should obtain a common electrostatic and electro-magnetic measure of some one quantity, current, resistance, electromotive force, or capacity. There are thus five known methods by which the value can be obtained:—

1°. By a common measure of quantity. Let a condenser of known capacity, s, be prepared (40). Let it be charged to a given potential P (47). Then the quantity in the condenser will be sP in electrostatic measure. The charge can next be measured by discharge through a galvanometer (25) in electro-magnetic measure. The ratio between the two numbers will give the value of v. The only difficulty in this method consists in the measurement of the potential P, entailing the measurement of an absolute force between two electrified bodies. This method was proposed and adopted by Weber †.

2°. By a comparison of the measure of electromotive force. The electromotive force produced by a battery, in electrostatic measure, can be directly weighed (43). Its electromotive force, in electro-magnetic measure, can be obtained from the current it produces in a given resistance (28). The ratio of the two numbers will give the value of v. This method has been carried out by Professor W. Thomson, who was not, however, at the time in possession of the means of determining accurately either the absolute resistance of his circuit or the absolute value of the current ‡.

3°. By a common measure of resistance. We know (29 and 45) how to measure resistances in electro-magnetic and electrostatic measure. The ratio between these measures is equal to v^2. The measure of resistance in electrostatic measure is not as yet susceptible of great accuracy.

4°. By a comparison of currents. The electro-magnetic value of a current produced by a rapid succession of discharges from a condenser of capacity s can be measured (18, 19). The electrostatic value of the current will be known if the potential to which the condenser is charged be known. The ratio of the two numbers is equal to v.

5°. By a common measure of capacity. The two measurements can be effected by the methods given (26 and 40). The ratio between the two

* *Vide* Transactions of British Association, 1859, p. 248, and Report of the Committee of Board of Trade on Submarine Cables, pp. 136 & 464.

† Pogg. Ann., Aug. 1856, Bd. xcix. p. 10. Abhandlungen der Kön. Sächsischen Gesellschaft, vol. iii. (1857) p. 266.

‡ Paper read before the Royal Society, February 1860. *Vide* Proceedings of the Royal Society, vol. x. p. 319.

measurements will give v^2. This method would probably yield very accurate results.

Part V.—Electrical Measurements derived from the Five Elementary Measurements; and Conclusion.

47. *Electric Potential.*—The word "potential," as applied by G. Green to the condition of an electrified body and the space surrounding it, is now coming into extensive use, but is perhaps less generally understood than any other electrical term. Electric potential is defined by Prof. W. Thomson as follows*:

"The potential, at any point in the neighbourhood of or within an electrified body, is the quantity of work that would be required to bring a unit of positive electricity from an infinite distance to that point, if the given distribution of electricity remained unaltered."

It will be observed that this definition is exactly analogous to that given of magnetic potential (10), with the substitution of the unit quantity of electricity for the unit magnetic pole. (Analogous definitions might be given of gravitation potential, heat potential; and every one of these potentials coexist at every point of space quite independently one of the other.) In another paper† Professor Thomson describes electric potential as follows:—

"The amount of work required to move a unit of electricity against electric repulsion from any one position to any other position is equal to the excess of the electric potential of the first position above the electric potential of the second position."

The two definitions given are virtually identical, since the potential at every point of infinity is zero; and it will be seen that the difference of potential defined in the second passage quoted is identical with what we have called the electromotive force between the two points (16 and 27).

When, instead of a difference of potentials, *the potential* simply of a point is spoken of, the difference of potential between the point and the earth is referred to, or, as we might say, the electromotive force between the point and the earth.

The potential at all points close to the surface and in the interior of any simple metallic body is constant; that is to say, no electromotive force can be produced in a simple metallic body by mere electrical distribution; the potential *at* the body may therefore be called the potential *of* the body. The potential of a metallic body varies according to the distribution, dimensions, position, and electrification of all surrounding bodies. It also depends on the substance forming the dielectric.

In any given circumstances, the potential of the body will be simply proportional to the quantity of electricity with which it is charged; but if the circumstances are altered, the potential will vary although the total amount of the charge may remain constant.

In a closed circuit in which a current circulates, the potential of all parts of the circuit is different; the difference depends on the resistance of each part and on the electromotive force of the source of electricity, *i. e.* on the difference of potentials which it is capable of causing when its two electrodes are separated by an insulator or dielectric. The different parts of a conductor moving in a magnetic field are maintained at different potentials, inasmuch as we have shown that an electromotive force is produced in this case. The

* Paper read before the British Association, 1852. *Vide* Phil. Mag. 1853, p. 288.

† Paper read before the Royal Society, February 1860. *Vide* Proceedings of the Royal Society, vol. x. p. 334.

potential of a body moving in an electric field (*i. e.* in the neighbourhood of electrified bodies) is constantly changing, but at any given moment the potential of all the parts is equal. The use of the word "potential" has the following advantages: it enables us to be more concise than if we were continually obliged to use the circumlocution, "electromotive force between the point and the earth;" and it avoids the conception of a force capable of generating a current, which almost necessarily, although falsely, is attached to "electromotive force."

Equipotential surfaces and lines of force in an electric field may be constructed for statically electrified bodies; these surfaces and lines may be drawn on similar principles and possess analogous properties to those described in a magnetic field (10). It is hardly necessary to observe that the magnetic and the electric fields are totally distinct, and coexist without producing any mutual influence or interference.

The rate of variation of electric potential per unit of length along a line of force is at any point equal to the electrostatic force at that point, *i. e.* to the force which a unit of electricity placed there would experience. The unit difference of potential is identical with the unit electromotive force; and the electrometer spoken of as measuring electromotive force measures potentials or differences of potential.

48. *Density, Resultant Electric Force, Electric Pressure.*—The three following definitions are taken almost literally from a paper by Professor W. Thomson*. Our treatise would be incomplete without reference to these terms, and Professor Thomson's definitions can hardly be improved.

"*Electric Density.*—This term was introduced by Coulomb to designate the quantity of electricity per unit of area in any part of the surface of a conductor. He showed how to measure it, though not in absolute measure, by his proof-plane.

"*Resultant Electric Force.*—The resultant force in air or other insulating fluid in the neighbourhood of an electrified body is the force which a unit of electricity concentrated at that point would experience if it exercised no influence on the electric distributions in its neighbourhood. The resultant force at any point in the air close to the surface of a conductor is perpendicular to the surface, and equal to $4\pi\rho$, if ρ designates the electric density of the surface in the neighbourhood.

"*Electric Pressure from the Surface of a Conductor balanced by Air.*—A thin metallic shell or liquid film (as, for instance, a soap-bubble), if electrified, experiences a real mechanical force in a direction perpendicular to the surface outwards, equal in amount per unit of area to $2\pi\rho^2$, ρ denoting, as before, the electric density at the part of the surface considered. In the case of a soap-bubble its effect will be to cause a slight enlargement of the bubble on electrification with either vitreous or resinous electricity, and a corresponding collapse on being perfectly discharged. In every case we may consider it as constituting a deduction from the amount of air-pressure which the body experiences when unelectrified. The amount of deduction being different in different parts according to the square of the electric density, its resultant action on the whole body disturbs its equilibrium, and constitutes in fact the resultant electric force experienced by the body."

49. *Tension.*—The use of this word has been intentionally avoided by us in this treatise, because the term has been somewhat loosely used by various writers, sometimes apparently expressing what we have called the density,

* Paper read before the Royal Society, Feb. 1860. *Vide* Proc. R. S. vol. x. p. 319 (1860), and Phil. Mag. vol. xx. ser. 4 (1860), p. 322.

and at others diminution of air-pressure. By some writers it has been used in the sense of a magnitude proportional to potential or difference of potentials, but without the conception of absolute measurement, or without reference to the idea of work essential in the conception of potential.

50. *Conducting Power, Specific Resistance, and Specific Conducting Power.*

Conducting Power, or Conductivity.—These expressions are employed to signify the reciprocal of the resistance of any conductor. Thus, if the resistance of a wire be expressed by the number 2, its conducting power will be 0·5.

Specific Resistance referred to unit of Mass.—The specific resistance of a material at a given temperature may be defined as the resistance of the unit mass formed into a conductor of unit length and of uniform section. Thus the specific resistance of a metal in the metrical system is the resistance of a wire of that metal one metre long and weighing one gramme. If the centimetre is used as the fundamental unit, the specific resistance of a metal is the resistance of a wire of that metal one centimetre long and weighing one gramme.

The Specific Conducting Power of a material is the reciprocal of its specific resistance.

Specific resistance, referred to unit of volume, is the resistance opposed by the unit cube of the material to the passage of electricity between two opposed faces. It may easily be deduced from the specific resistance referred to unit of mass, when the specific gravity of the material is known.

Specific Conducting Power may also be referred to unit of volume. It is of course the reciprocal of the specific resistance referred to the same unit.

It is somewhat more convenient to refer the resistance to the unit of mass in the case of long uniform conductors, such as metal wires, of which the size is frequently and easily measured by the weight per foot or metre or centimetre; and it is, on the other hand, more convenient to refer to the unit of volume bodies, such as gutta percha, glass, &c., which do not generally occur as conducting-rods of uniform section, while their dimensions can always be measured with at least as much accuracy as their weights.

51. *Specific Inductive Capacity**.—Faraday discovered that the capacity of a conductor does not depend simply on its dimensions or on its position relatively to other conductors, but is influenced in amount by the nature of the insulator or dielectric separating it from them. The laws of induction are assumed to be the same in all insulating materials, although the amount be different. The name "inductive capacity" is given to that quality of an insulator in virtue of which it affects the capacity of the conductor it surrounds; and this quality is measured by reference to air, which is assumed to possess the unit inductive capacity. The specific inductive capacity of a material is therefore equal to the quotient of the capacity of any conductor insulated by that material from the surrounded conductors, divided by the capacity of the same conductor in the same position separated from them by air only. It is not improbable that this view of induction may be hereafter modified.

52. *Heat produced in a Conductor by a Current.*—The work done in driving a current, C, for a unit of time through a conductor whose resistance is R, by an electromotive force E, is $EC = RC^2$ (§ 17). This work is lost as electrical energy, and is transformed into heat. As Dr. Joule has ascertained the quantity of mechanical work equivalent to one unit of heat, we can calculate

* Experimental Researches, series xi.

the quantity of heat produced in a conductor in a given time, if we know C and R in absolute measure. In the series of units founded on the centimetre, gramme, and second, if we call the total heat Θ, taking as unit the quantity required to raise one gramme of water one degree Centigrade, we have

$$\Theta = \frac{RC^2 t}{4157 \times 10^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (28)$$

If the metre is used instead of the centimetre the divisor is 4157; and in the British system, founded on feet, grains, and seconds, with a unit of heat equal to the quantity required to raise one grain one degree Fahrenheit, the divisor is 24·861.

53. *Electrochemical Equivalents.*—Dr. Faraday has shown * that when an electric current passes through certain substances and decomposes them, the quantity of each substance decomposed is proportional to the quantity of electricity which passes. Hence we may call that quantity of a substance which is decomposed by unit current in unit time the electrochemical equivalent of that substance.

This equivalent is a certain number of grammes of the substance. The equivalents of different substances are in the proportion of their combining numbers; and if all chemical compounds were electrolytes, we should be able to construct experimentally a table of equivalents in which the weight of each substance decomposed by a unit of electricity would be given. The electrochemical equivalent of water, in electro-magnetic measure, is about 0·02 in the British, 0·00092† in the centimetrical system, and ·0092 in the metrical system. The electrochemical equivalents of all other electrolytes can be deduced from this measurement with the aid of their combining numbers.

54. *Electromotive Force of Chemical Affinity.*—When two substances having a tendency to combine are brought together and enter into combination, they enter into a new state, in which the intrinsic energy of the system is generally less than it was before, that is, the substances are less able to effect chemical changes, or to produce heat or mechanical action, than before.

The energy thus lost appears during the combination as heat or electrical or mechanical action, and can be measured in many cases ‡.

The energy given out during the combination of two substances may, like all other forms of energy, be considered as the product of two factors §—the tendency to combine, and the amount of combination effected. Now the amount of combination may be measured by the number of electrochemical equivalents which enter into combination; so that the tendency to combine may also be ascertained by dividing the energy given out by the number of electrochemical equivalents which enter into combination.

If the whole energy appears in the form of electric currents, the energy of the current is measured by the product of the electromotive force and the quantity of electricity which passes. Now the quantity of electricity which passes is equal to the number of electrochemical equivalents which enter on either side into combination. Hence the total energy given out, divided by this number, will give the electromotive force of combination. Thus, if N

* Experimental Researches, series vii.

† ·0009375 by Weber and Kohlrausch.

‡ Report British Association, 1850, p. 63, and Phil. Mag. vol. xxxii. ser. 3. See papers by Prof. Andrews, and Favre and Silbermann, "On the Heat given out in Chemical Action," Comptes Rendus, vols. xxxvi. & xxxvii.

§ *See* Rankine "On the General Law of Transformation of Energy," Phil. Mag. 1853.

electrochemical equivalents enter into combination under a chemical affinity I, and in doing so give out energy equal to W, either as heat or as electrical action, then

$$NI = W.$$

But if W be given out as electrical action, and causes a quantity of electricity Q to traverse a conductor under an electromotive force E, we shall have

$$W = EQ.$$

By the definition of electrochemical equivalents, $Q = N$, therefore

$$I = E;$$

or the force of chemical affinity may in these cases be measured as electromotive force.

This method of ascertaining the electromotive force due to chemical combination, which gives us a clear insight into the meaning and the measurement of "chemical affinity," is due to Professor W. Thomson*.

The field of investigation presented to us by these considerations is very wide. We have to measure the intrinsic energy of substances as dependent on volume, temperature, and state of combination. When this is done, the energy due to any combination will be found by subtracting the energy of the compound from that of the components before combination.

As the tendency to increase in volume is measured as pressure, and as the tendency to part with heat is measured by the temperature, so in chemical dynamics the tendency to combine will be properly measured by the electromotive force of combination.

55. *Tables of Dimensions and other Constants*† :—

Fundamental Units.

Length $= L$. Time $= T$. Mass $= M$.

Derived Mechanical Units.

$$\text{Work} = W = \frac{L^2 M}{T^2}. \qquad \text{Force} = F = \frac{LM}{T^2}. \qquad \text{Velocity} = V = \frac{L}{T}.$$

Derived Magnetical Units.

Strength of the pole of a magnet .... $m = L^{\frac{3}{2}}\ T^{-1}\ M^{\frac{1}{2}}$

Moment of a magnet .... $ml = L^{\frac{5}{2}}\ T^{-1}\ M^{\frac{1}{2}}$

Intensity of magnetic field.......... $H = L^{-\frac{1}{2}}\ T^{-1}\ M^{\frac{1}{2}}$

* "On the Mechanical Theory of Electrolysis," Phil. Mag. Dec. 1851.

† The first Table of Dimensions was given by Fourier, 'Théorie de la Chaleur,' p. 157.

Table of Dimensions.

Name of Quantity.	Electrostatic system.		Electro-magnetic system.		Number of electrostatic units in one electro-magnetic unit.
	Symbol.	Dimensions of unit.	Symbol.	Dimensions of unit.	
Electrostatic Pair.					
Quantity of electricity	q	$[L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-1}]$	Q	$[L^{\frac{1}{2}} M^{\frac{1}{2}}]$	v
Electromotive force	e	$[L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-1}]$	E	$[L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-2}]$	v^{-1}
Their product: electrostatic energy	..	$[L^2 M T^{-2}]$	..	$[L^2 M T^{-2}]$	identical
Ratio of the first to the second: capacity of an accumulator.	s	$[L]$	S	$L^{-1} T^2$	v^2
Electro-magnetic Pair.					
Electro-magnetic momentum of a circuit, also strength of magnetic pole	m	$[L^{\frac{1}{2}} M^{\frac{1}{2}}]$	..	$[L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-1}]$	v^{-1}
Strength of electric current, also magnetic potential....	c	$[L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-2}]$	C	$[L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-1}]$	v
Their product, electrokinetic energy	..	$[L^2 M T^{-2}]$	..	$[L^2 M T^{-2}]$	identical
Ratio of the first to the second: coefficient of electromagnetic induction of two circuits ..	..	$[L^{-1} T^2]$	..	$[L]$	v^{-2}
Pair for Conduction and Resistance.					
Electromotive force	e	$[L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-1}]$	E	$[L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-2}]$	v^{-1}
Strength of electric current ..	c	$[L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-2}]$	C	$[L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-1}]$	v
Their product: rate at which energy is transformed into heat	..	$[L^2 M T^{-3}]$	..	$[L^2 M T^{-3}]$	identical
Ratio of the first to the second: resistance of a conductor ..	r	$[L^{-1} T]$	R	$[L T^{-1}]$	v^{-2}

All men of science are agreed to use the second of mean solar time as the unit of time. In all the primary quantities the dimensions of M are the same, namely $\frac{1}{2}$. The principal differences, therefore, are in the dimensions of L.

We therefore arrange the different quantities in groups, first, with respect to the dimensions of L, and then with respect to M and T, thus :—

Electrostatic system.	Electro-magnetic system.	Dimensions.	Group.
——	Moment of a magnet	$L^{\frac{5}{2}} M^{\frac{1}{2}} T^{-1}$	I.
Quantity of electricity ...	Strength of magnetic pole.	$L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-1}$	II.
Strength of current......	Electromotive force......	$L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-2}$	
Strength of magnetic pole.	Quantity of electricity ..	$L^{\frac{1}{2}} M^{\frac{1}{2}}$	III.
Electromotive force	Strength of current......	$L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-1}$	
Magnetic intensity	Electric force at a point..	$L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-2}$	
Electric force and electric induction	Magnetic force and magnetic induction........	$L^{-\frac{1}{2}} M^{\frac{1}{2}} T^{-1}$	IV.
Density of electric current	——	$L^{-\frac{1}{2}} M^{\frac{1}{2}} T^{-2}$	
Magnetic induction......	Electric induction	$L^{-\frac{3}{2}} M^{\frac{1}{2}}$	V.
——	Density of electric current	$L^{-\frac{3}{2}} M^{\frac{1}{2}} T^{-1}$	

The Electrostatic and Electro-magnetic System of Units.

The electrostatic system begins with the definition of the unit of electricity, as determined by the mechanical force between two electrified bodies.

The electro-magnetic system begins with the definition of the strength of a unit magnetic pole, as determined by the mechanical force between two poles.

The form of the definition is precisely the same in both cases. Hence the electrostatic unit of electricity is of the same dimensions as the electro-magnetic unit magnetic pole, and the series of derived units of the one system form a series having respectively the same dimensions as another series belonging to the other system.

The most instructive method of exhibiting the relations of these quantities is to arrange them in pairs, the product of each pair being either a quantity of mechanical energy, or the work done in unit of time, or energy existing in unit of volume, or work done in unit of volume in unit of time. The ratio of the two quantities is in several cases a quantity of importance in electrical science.

Let v be the ratio of the electrostatic to the electro-magnetic unit of quantity (35 and 46); then v=310,740,000 metres per second approximately, and we have

$$q=vQ \quad \Big| \quad c=vC \quad \Big| \quad e=\frac{1}{v}E \quad \Big| \quad r=\frac{1}{v^2}R \quad \Big| \quad s=v^2S$$

Table for the Conversion of British (foot-grain-second) System to Centimetrical (centimetre-gramme-second) System.

	Number of centimetrical units contained in a British unit.	Number of British units contained in a centimetrical unit.
1° for M	0·0647989	15·43235
2° for L, $\frac{v}{1}$ R, $\frac{1}{r}$ and V	30·47945	·03280899
3° for F (also for foot-grains and centimetre-grammes).	1·97504	·506320
4° for W	60·198	·01661185
5° for H and electrochemical equivalents.	·0461085	21·6880
6° for Q, C, and e	1·40536	·711561
7° for E, m, q, and c	42·8346	·0233456
8° for heat	0·0359994	27·7782

1 volt .... $=10^8$ absolute units of electromotive force.
1 ohm ...... $=10^9$ centimetres per second.
 ,, ... $=3{\cdot}2809\times10^7$ feet per second.
 ,, ... $=1$ quadrant of the meridian through Paris per second.
v............ $=31{\cdot}074$ ohms by Weber and Kohlrausch.
,, ... $=28{\cdot}2$ ohms by Thomson.
,, ... $=28{\cdot}8$ ohms by Maxwell.
Velocity of light $=29{\cdot}8$ ohms by Foucault.

The intensity of gravity at many different stations has been determined by experiment. Where it has not been so determined, it may be calculated by the formula

$$g=\mathrm{G}\,(1-0{\cdot}0025659\cos 2\lambda)\left\{1-\left(2-\frac{3}{2}\frac{\rho'}{\rho}\right)\frac{z}{r}\right\},$$

where g denotes the intensity of gravity at the station.

G the intensity of gravity at latitude 45° at the level of the sea.

G $=980{\cdot}533$ centimetres, or $32{\cdot}1703$ feet.

λ is the latitude of the station.

The last factor is a correction for the height of the station.

z is the height of the station in centimetres or feet.

r is the mean radius of the earth.

$r=636{,}619{,}800$ centimetres, or 20,886,852 feet; ρ is the mean density of the earth, about 5·5 times that of water; ρ' is the mean density of the hill on which the station is placed. If we suppose this about half the density of the earth as a whole, the factor for correction due to height becomes

$$1-1{\cdot}32\frac{z}{r}, \text{ nearly.}$$

British System.—Relation between Absolute and other Units.

One absolute unit of $\left\{\begin{matrix}\text{force}\\ \text{work}\end{matrix}\right. = 0{\cdot}0310666 \begin{matrix}\text{weight of a grain}\\ \text{foot-grains}\end{matrix}\left.\right\}$ in London.

In London $\left\{\begin{matrix}\text{weight of a grain}\\ \text{one foot-grain}\end{matrix}\right. = 32{\cdot}1889$ absolute units of $\left\{\begin{matrix}\text{force.}\\ \text{work.}\end{matrix}\right.$

One absolute unit of $\left\{\begin{matrix}\text{force}\\ \text{work}\end{matrix}\right. = \frac{1}{g}\left\{\begin{matrix}\text{unit weight}\\ \text{unit weight} \times \text{unit length}\end{matrix}\right\}$ everywhere.

g in British system $= 32{\cdot}088\ (1 + 0{\cdot}005133 \sin^2 \lambda)$, where $\lambda =$ the latitude of the place at which the observation is made.

Heat.—The unit of heat is the quantity required to raise the temperature of one grain of water at its maximum density 1° Fahrenheit.

Absolute mechanical equivalent of unit of heat $= 24861 = 772$ foot-grains at *Manchester.*

Thermal equivalent of an absolute unit of work $= 0{\cdot}000040224$.

Thermal equivalent of a foot-grain at Manchester $= 0{\cdot}0012953$.

Electrochemical equivalent of water $= 0{\cdot}02$, nearly.

Centimetrical System.—Relation between Absolute and other Units.

One absolute unit of $\left\{\begin{matrix}\text{force}\\ \text{work}\end{matrix}\right. = 0{\cdot}0010195 \begin{matrix}\text{weight of a gramme}\\ \text{centimetre-gramme}\end{matrix}\left.\right\}$ at Paris.

At Paris $\left\{\begin{matrix}\text{the weight of a gramme}\\ \text{or centimetre-gramme}\end{matrix}\right. = 980{\cdot}868$ absolute units of $\left\{\begin{matrix}\text{force.}\\ \text{work.}\end{matrix}\right.$

One absolute unit of $\left\{\begin{matrix}\text{force}\\ \text{work}\end{matrix}\right. = \frac{1}{g}\begin{matrix}\text{unit weight}\\ \text{unit weight} \times \text{unit length}\end{matrix}\left.\right\}$ everywhere.

g in metrical system $= 978{\cdot}024\ (1 + 0{\cdot}005133 \sin^2 \lambda)$, where $\lambda =$ the latitude of the place where the experiment is made.

Heat.—The unit of heat is the quantity required to raise one gramme of water at its maximum density 1° Centigrade.

Absolute mechanical equivalent of the unit of heat $= 4157{\cdot}25 \times 10^4 =$ $42354{\cdot}2$ centimetre-grammes at *Manchester.*

Thermal equivalent of an absolute unit of work $= 0{\cdot}00024054 \times 10^{-4}$.

Thermal equivalent of a centimetre-gramme at Manchester $= 0{\cdot}0000236154$.

Electrochemical equivalent of water $= 0{\cdot}00092$, nearly.

1 horse-power = 33,000 foot-pounds per minute.
„ = 14·732 foot-tons per minute.
„ = 456,233,300 centimetre-gramme weight per minute.
„ = 7,603,388·8 centimetre-gramme weight per second.
„ = 7,462,455,683 absolute units of work per second.
„ = 746×10^7 absolute units of work per second approximately.

Electromotive force of one Daniell's cell, as estimated by Thomson in 1851,
$= 107 \times 10^6$ absolute units.
$= 1{\cdot}07$ volt.

And 1 volt through 1 ohm decomposes $\cdot 00092 \times \frac{10^8}{10^9} = \cdot 000092$ gramme of water per second, and hence decomposes $\cdot 000092 \times \frac{65}{18} = \cdot 000332$ gramme of zinc per second $= \frac{1}{3000}$ gramme per second, very nearly $= 28{\cdot}8$ grammes per day approximately.

Activity = rate of doing work $= \frac{E^2}{R}$ for a galvanic element.

$= 10^7$ for 1 volt through 1 ohm.

Or 1 volt-ohm uses $\frac{1}{3000}$ gramme of zinc, and does 10^7 absolute units of work per second.

1 horse-power = 746 volt-ohms, and is equivalent to the consumption of $\frac{746}{3000}$ grammes of zinc per second in a Daniell's battery, or 895·2 grammes per hour, or $21\frac{1}{2}$ kilogrammes per day, very nearly.

Table for the Conversion of British (foot-grain-second) System to Metrical (metre-gramme-second) System.

	Number of metrical units contained in a British unit.	Log.	Log.	Number of British units contained in a metrical unit.
1° or M................	0·0647989	$\bar{2}$·8115678	1·1884321	15·43235
2° for L, $\frac{v}{I}$, R, $\frac{1}{r}$ and V....	0·3047945	$\bar{1}$·4840071	0·5159929	3·280899
3° for F (also for foot-grains and metre-grammes).	0·0197504	$\bar{2}$·2955749	1·7044250	50·6320
4° for W	0·0060198	$\bar{3}$·7795820	2·2204179	166·1185
5° for H and electrochemical equivalents.	0·461085	$\bar{1}$·6637804	0·3362196	2·16880
6° for Q, C, and *e*........	0·140536	$\bar{1}$·1477874	0·8522125	7·11561
7° for E, *m*, *q*, and *c*......	0·0428346	$\bar{2}$·6317949	1·3682051	23·3456
8° for heat..............	0·0359994	$\bar{2}$·5562953	1·4437046	27·7782

Metrical System.—Relation between Absolute and other Units.

$$\text{One absolute unit of} \begin{cases} \text{force} \\ \text{work} \end{cases} = 0{\cdot}10195 \begin{rcases} \text{weight of a gramme} \\ \text{metre-gramme} \end{rcases} \text{at Paris.}$$

$$\text{At Paris} \begin{cases} \text{the weight of a gramme} \\ \text{or metre-gramme} \end{cases} = 9{\cdot}80868 \text{ absolute units of} \begin{cases} \text{force.} \\ \text{work.} \end{cases}$$

$$\text{One absolute unit of} \begin{cases} \text{force} \\ \text{work} \end{cases} = \frac{1}{g} \begin{rcases} \text{unit weight} \\ \text{unit weight} \times \text{unit length} \end{rcases} \text{everywhere.}$$

g in metrical system $= 9{\cdot}78024\ (1 + 0{\cdot}005133 \sin^2 \lambda)$, where $\lambda =$ the latitude of the place where the experiment is made.

Heat.—The unit of heat is the quantity required to raise one gramme of water at its maximum density 1° Centigrade.

Absolute mechanical equivalent of the unit of heat = 4157·25 = 423·542 metre-grammes at *Manchester*.

Thermal equivalent of an absolute unit of work = 0·00024054.

Thermal equivalent of a metre-gramme at *Manchester* = 0·00236154.

Electrochemical equivalent of water = 0·0092 nearly.

56. *Magnitude of Units and Nomenclature.*—In connexion with the system of measurement explained in this treatise, two points hitherto unmentioned deserve attention—first, the absolute magnitude of the units, and, secondly, the nomenclature.

The absolute magnitude is in most cases an inconvenient one, leading to the use either of exceedingly small or exceedingly large numbers. Thus the units of electro-magnetic resistance and electromotive force and quantity, and of electrostatic currents, are inconveniently small; the unit of electrostatic resistance is inconveniently large. Decimal multiples and submultiples of these units will therefore probably have to be adopted in practice. The choice of these multiples and submultiples forms part of the business of the Committee.

The nomenclature hitherto adopted is extremely defective. In referring to each measurement, we have to say that the number expresses the value in electrostatic or electro-magnetic absolute units: if a multiple is to be used, this multiple will also have to be named; and further, the standard units of length, mass, and time have to be referred to, inasmuch as some writers use the pound and some the grain, some the metre and some the millimetre, as fundamental units. This cumbrous diction, and the risk of error imported by it, would be avoided if each unit received a short distinctive name in the manner proposed by Sir Charles Bright and Mr. Latimer Clark, in a paper read before the British Association at Manchester, 1861.

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Appendix D.—*Description of an Experimental Measurement of Electrical Resistance, made at King's College. By* Professor J. Clerk Maxwell *and* Messrs. Balfour Stewart *and* Fleeming Jenkin. (Parts I., III., and IV., by Professor Maxwell. Part II., by Mr. Fleeming Jenkin.)

Part I.—General Description of the Method Employed.

In the general Report of the Committee, and in Appendix C, it has already been shown that the most important aid to the exact science of electricity would be the determination of the resistance of a wire in absolute measure, and the publication of standards of resistance derived from this wire. This has already been done by Weber*; but it is desirable that the determination of a quantity so important should not be left in the hands of a single person. Weber has employed two methods.

1st. By suddenly turning a coil of wire about an axis so as to alter its position relatively to the terrestrial magnetic lines of force, he produced an electromotive force acting for a short time in the coil. This coil was connected with another fixed coil having a magnet suspended in its centre. The current generated by the electromotive force passed through both coils and

* Pogg. Ann. Bd. lxxxii. p. 337 (March 1851); Electrische Maasbestimmungen, Leipzig, Wiedemann; Memoirs of the Royal Society of Sciences of Saxony, vol. i. p. 197; and Phil. Mag. 1861.

gave the magnet a sudden impulse, the amount of which was measured by its extreme deflection.

Thus an electromotive force of short duration produced a current of short duration. The total amount of electromotive force depended on the size of the movable coil and on the intensity of terrestrial magnetism. The total amount of the current is measured by the impulse given to the magnet, and the mechanical value of the impulse is measured by the angle through which it swings. The resistance of the whole circuit, consisting of both coils, is then ascertained by dividing the electromotive force by the current.

2nd. Weber's second method consisted in causing a powerful magnet to oscillate within a coil of wire. By the motion of the magnet currents are produced in the coil, and these, reacting on the magnet, retard its motion. The rate of diminution of the amplitude of the oscillations, when compared with the rate of diminution when the circuit is broken, affords the means of determining the resistance of the circuit.

Professor W. Thomson has designed an apparatus by which the resistance of a coil can be determined in electro-magnetic measure by the observation of the constant deflection of a magnet, and his arrangement has been adopted for the experiments made by the Committee.

The coil of wire is made to revolve about a vertical diameter with constant velocity. The motion of the coil among the lines of force due to the earth's magnetism produces induced currents in the coil which are alternately in opposite directions with respect to the coil itself, the direction changing as the plane of the coil passes through the east and west direction. If we consider the direction of the current with respect to a fixed line in the east and west direction, we shall find that the changes in the current are accompanied with changes in the face of the coil presented to the east, so that the absolute direction of the current, as seen from the east, remains always the same. If a magnet be suspended in the centre of the coil, it will be deflected from the north and south line by the action of these currents, and will be turned in the same direction as the coil revolves. The force producing this deflection is continually varying in magnitude and direction; but as the periodic time is small, the oscillations of the magnet may be rendered insensible by increasing the mass of the apparatus along with which it is suspended. The resistance of the coil may be found when we know the dimensions of the coil, the velocity of rotation, and the deflection of the magnet. The intensity of terrestrial magnetism enters into the measurement of the electromotive force, and also into the measurement of the current; but the measure of the resistance, which is the ratio of these two quantities, is quite independent of the value of the magnetic intensity.

Part II.—Description of the Apparatus.

For convenience of description, the apparatus with which the experiments were made may be divided into five parts:—1°, the driving gear; 2°, the revolving coil; 3°, the governor; 4°, the scale, with its telescope, by which the deflections of the magnet were observed; 5°, the electric balance, by which the resistance of the copper coil was compared with a German-silver arbitrary standard.

The general arrangement of the first four parts is shown in the diagram, fig. 4, Plate II.

The *driving gear* consisted of a leaden flywheel X, on a shaft A, turned by hand, and communicating its motion by a band, $b\, b_1\, b_2 \ldots$, arranged in a way equivalent to Huyghens's gearing, to a shaft B, a pulley on which drove

the revolving coil by a simple band $a\, a_1\, a_2 \ldots .$ The arrangement of the band $b\, b_1\, b_2 \ldots .$ communicating the motion of shaft A to shaft B may be easily understood from the diagram. C C are two guide-pulleys running loose on pins attached to the main framing. D D are two loose pulleys maintained at a constant distance by the strut E, to which the weight W is hung.

When the rotation of shaft B is opposed by a sufficient resistance, the effect of turning the flywheel in the direction shown by the arrow is to lift the weight W from the ground, tending to turn the shaft B with a definite force, which will be sensibly constant so long as the weight is kept off the ground and the band $b\, b_1\, b_2 \ldots .$ remains unaltered in length. Wherever, as in the present experiments, the resistance increases with the speed of rotation, the speed of the driving-wheel can easily be regulated by hand, so as to keep the weight from falling so low as to touch the ground, or rising so high as to foul the gear; and thus, with a little care, a constant driving force can be applied to the shaft B and to the machinery connected with it.

The *revolving coil* formed the most important part of the apparatus. It is shown one-fifth full size in figs. 1 and 2, Pl. II.

A strong brass frame, H H, was bolted down by three brass bolts, F F F, dowelled into a heavy stone. It could be accurately levelled by three stout screws, G G G. The brass rings, I I_1, on which the insulated copper wire was coiled, were supported on the frame by a pivot, J, working in lignum vitæ, and by a hollow bearing, K, working in brass: this bearing worked in a kind of stuffing-box, *k* (fig. 3), which, by three screws and a flat spring washer between it and the frame at *j*, could be adjusted to fit the collar *e* with great nicety, preventing all tendency to bind or shake. Supported in this way the coil revolved with the utmost freedom and steadiness.

The coil of copper wire was necessarily divided into two parts on the two rings I I_1, to permit the suspension of the magnet S. The two brass rings were each formed of two distinct halves, insulated from one another by vulcanite at the flanges $f\, f_1$. This insulation was necessary to prevent the induction of currents in the brass rings. These rings, after being bolted together, were turned with great accuracy by Messrs. Elliott Brothers. The insulated copper wire was wound in one direction on both rings; the inner end of the second was soldered to the outer end of the first; the two extreme ends of the conductor thus formed were soldered to two copper terminals, $h\, h'$, insulated by the vulcanite piece, *x*, bolted to the brass rings. Each terminal was provided with a strong copper binding-screw, and had a mercury-cup drilled into its upper surface. The two coils could be joined, so as to form a closed circuit, by a short copper bar between the binding-screws. The bars, binding-screws, and nuts were amalgamated to ensure perfect contact. When the copper coils were to be connected with the electric balance, the short copper bar was removed and the required connexions were made by short copper rods, one quarter of an inch in diameter, dipping at one end into the mercury-cup on the terminals $h\, h'$, and at the other end into the mercury-cups of the electric balance. The absence of all induced currents influencing the suspended magnet when the circuit was broken at $h\, h'$ was repeatedly proved by experiment.

Rotation was communicated to the coils by a catgut band simply making half a turn round the small V-pulley *l*. The band could be tightened as required by the jockey pulley *z* and weight *w* (fig. 4).

A short screw of large diameter, *n*, gearing into a spur-wheel of one hundred teeth, *o*, formed the counter from which the speed of rotation was obtained, as follows:—A pin, *p*, on the wheel *o* lifted the spring *q* as it

passed; this spring in its rebound struck the gong M. The blow was of course repeated at every hundred revolutions, and the time of each blow was observed on a chronometer. The arrangement was equally adapted for rotation in either direction.

A second V-pulley, r, served for the band $c\,c$, communicating motion to the governor by which the speed was controlled.

The manner in which the suspended magnet was introduced to the centre of the coil is best seen in fig. 3. A brass tripod, N, bolted to the main frame, supported the long brass tube O, which passed freely through the hollow bearing at K. A cylindrical wooden box, P, slipped on to the end of the tube O. The magnet hung inside this box, the lower part of which could be removed to allow the exact position of the magnet to be verified. The support N also carried a short brass tube R, on which the glass case T could be secured by a little sliding tube. The mirror t, attached to the magnet S by a rigid brass wire, hung inside this glass case by a single cocoon-fibre about eight feet long. This fibre was protected against currents of air by a wooden case (not shown in the Plate), extending from the point of support down to the glass case. A little sliding paper prolongation of the wooden case made it nearly wind-proof by fitting at the bottom against the main brass frame. An opening in the case allowed the mirror to be seen. The fibre at the top was suspended from a torsion-head, by which it could be turned; it could also be raised and lowered by a small barrel, and was adjustable in a horizontal plane by three set screws. The care taken in suspending the magnet and in protecting it both against currents of air and vibration was repaid by success, for the image of the scale reflected in the magnet was as clear and steady when the coil was making 400 revolutions per minute as when it was at rest.

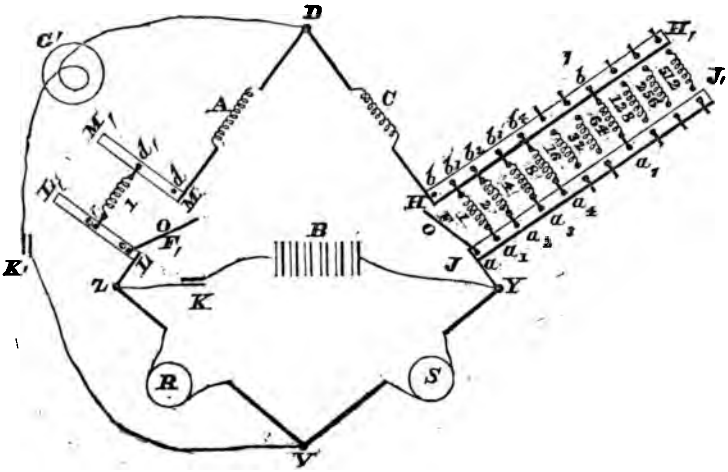
The *governor* used was lent by one of the Committee and will not be described in detail, as an improved governor on the same principle will be adopted in future experiments, in describing which an account of its construction will be given. It may be said, however, that the little instrument actually employed generally controlled the speed to such uniformity as allowed the deflections to be observed with as much accuracy as the zero-point.

The *scale* and *telescope* hardly require special description; they were arranged in the usual manner for this kind of experiment, at about three metres from the mirror. The scale was an engine-divided paper scale nailed to a wooden bar. This plan will in future experiments be abandoned, as variations in the weather had a very perceptible influence on the scale.

The annexed (p. 100) diagram shows the *electric balance* by which the copper coil C was compared with an arbitrary German-silver standard S before and after each induction experiment. The arrangement is that of the ordinary Wheatstone's balance, as described in Appendix H of the Report of your Committee for 1862. A and C represent the arms of the balance as there described, S the German-silver standard, and R the copper coil to be measured. $J\,J_1$, $H\,H_1$, $M\,M_1$, and $L\,L_1$ are four stout copper bars with mercury-cups at $a\,a_1\,a_2\,..$, $b\,b_1\,b_2\,..$, $c\,c_1$, and $d\,d_1$. Two short copper rods, F and F_1, can be used to connect a with b and c with d. When this is done the arrangement is exactly that of the simple Wheatstone balance with the keys at K and K_1, and described in Appendix H of the last Report. A and C were coils formed of about 300 inches of No. 31* German-silver wire, and were adjusted to equality with extreme nicety, and each assumed equal to 100 arbitrary units. If R on any occasion had been exactly equal to S, the galvanometer G′

* Diameter = 0·01 inch.

would have been unaffected on depressing the keys $K K_1$, when a was joined to b and c to d by F and F_1, rods of no sensible resistance. This exact equality between R and S could never be obtained, owing to slight changes in temperature, which affected the two coils very differently. The object of the modifica-



tions introduced was to allow the ratio between S and R, differing by a small amount only, to be measured with great accuracy.

For this purpose a number of German-silver coils were adjusted, representing 1, 2, 4, 8 .... 512 in the arbitrary units, equal to the hundredth part of A or C. These coils were so arranged that any one or more of them could be introduced between the bars $H H_1$ and $J J_1$. A single coil, equal to 1 in the same arbitrary unit, could be introduced between the bars $L L_1$ and $M M_1$. In the diagram this coil is shown in its position and the rod F_1 withdrawn. Similarly F is withdrawn from between H and G′, and the coil 1 joins a_1 and b_1 in the bars $H H_1$ and $J J_1$. If no other coils were placed between $H H_1$ and $J J_1$, the arms of the balance would now be 101 and 101 respectively, instead of 100 and 100; but the ratio would still be that of equality. Let us now suppose that, when the circuit with the battery is completed, the galvanometer by its deflection shows that R is bigger than S, we can reduce the resistance of the arm between D and Y by various small graduated and definite amounts by introducing the coils 2, 4, 8, &c. between $H H_1$ and $J J_1$. Let us first suppose the coil 2 introduced. The resistance between H and J will be the reciprocal of 1·5 or 0·6667; for where various resistances are added in multiple arc, the resistance of the compound arc is the reciprocal of the sum of their conducting powers, and the conducting power of a wire is the reciprocal of its resistance. The ratio between the two arms will now be 101 : 100·6667. Let us suppose that on completing the circuit the galvanometer still deflects in the same direction as before, the arm between D and Y must be still further reduced by including fresh coils between $H H_1$ and $J J_1$. It is very easy by trial to find the combination which maintains the galvanometer at zero when the circuit is completed. Let us suppose that, as in the diagram, the coils included were 1, 2, 4, 8, and 64. The reciprocals of these numbers are 1, 0·5, 0·25, 0·125, and 0·015625. The conducting power

between H and J is therefore 1·890625, the sum of these numbers. The resistance between H and J is 0·52893, the reciprocal of the last number, and the ratio between the arms will be 101 : 100·52893. A little consideration will show that with the coils named any ratio between 101 to 100·5 and 101 to 101 can be obtained by steps not exceeding 0·00195, the reciprocal of 512, the largest coil or smallest conducting power which can be included between the copper bars $H H_1$ and $J J_1$. By substituting the rod F for the coil 1 between $L L_1$ and $M M_1$, the observer can obtain a fresh series of ratios with the same steps between 101 to 100 and 100·5 to 100. In this way it will be seen that unless the coils R and S differ by more than one per cent., their ratio can be measured in the manner described within 0·002 per cent.

It should further be observed that extreme accuracy in the coils 1, 2, 4, &c. is not necessary, since an error of one per cent. in the sum of these, as compared with their true relative value to the coil C, would only affect the final result 0·01 per cent.

The position of R and S in the balance relatively to A and C, &c. is of course interchangeable.

The diagram is not intended at all to represent the practical arrangement, but simply to show the connexions. The electric balance described in Appendix H of last year's Report (Plate I. figs. 1 to 6, Report 1862) was used with a stout copper rod between the cups $e\,e_1$, and two additional boards with the copper bars $H H_1$, $J J_1$, $L L_1$, and $M M_1$ fitted as in the above diagram. The coils 1, 2, 4, &c. had amalgamated copper terminals, which simply dropped into mercury-cups on the copper bars. The observations could be made very rapidly and accurately, as the galvanometer was sensitive enough with four Daniell's cells to indicate the addition or subtraction of the 512 coil with perfect distinctness. The reduction of the observations to find the ratio seems somewhat complicated at first, but with the aid of a table of reciprocals it takes but little time. No improvement seems necessary in this part of the apparatus. The idea of using large coils combined with small ones in multiple arc to obtain extremely minute differences of resistance was suggested to the writer by Professor W. Thomson, and will be found useful in very many ways.

Part III.—Mathematical Theory of the Experiment.

A circular coil of copper wire is made to revolve with uniform velocity about a vertical diameter. A small magnet is suspended by a silken fibre in the middle of the coil. Its position is observed when the coil is at rest, and when the coil revolves with velocity ω the magnet is deflected through an angle ϕ. Currents are induced in the coil by the action of the earth's magnetism, and these act on the magnet and deflect it from the magnetic meridian. By observing the deflection and the velocity of rotation, we can determine the resistance of the coil in electro-magnetic units.

In determining the strength of the current we may neglect the motion of the suspended magnet, as it is found, both by theory and by experiment, to be insensible. We have therefore, in the first place, to determine the electromagnetic potential of the coil with respect to the earth's magnetism, with respect to the suspended magnet, and with respect to itself.

1st. Let H be the horizontal component of the earth's magnetism.
γ the strength of the current in the coil.
G the total area enclosed by all the windings of the wire.
θ the angle between the plane of the coil and the magnetic meridian.

Then the potential of the coil with respect to the earth is

$$-\mathrm{H}\gamma\mathrm{G}\sin\theta.$$

2nd. Let M be the magnetic moment of the suspended magnet.

ϕ the angle between the axis of the magnet and the magnetic meridian.

K the magnetic force at the centre of the coil due to unit current in the wire.

Then the potential of the coil with respect to the magnet is

$$-\mathrm{M}\gamma\mathrm{K}\sin(\theta-\phi).$$

3rd. Let $\frac{1}{2}$L be the potential of the coil on itself for unit current. Then the potential due to a current γ is

$$\tfrac{1}{2}\mathrm{L}\gamma^2.$$

Let P be the electromotive force, and R the resistance, then the work spent in keeping up the current is Pγ in unit of time; or, since $\mathrm{P}=\mathrm{R}\gamma$, the work spent in keeping up the current for a time δt is

$$\mathrm{R}\gamma^2\,\delta t.$$

If the current is at the same time increased from γ to $\gamma+\delta\gamma$, the work spent in increasing the current will be

$$\mathrm{L}\gamma\,\delta\gamma.$$

If the angular motion of the coil be $\delta\theta$, the work spent in keeping up the rotation against the electro-magnetic force is

$$\mathrm{H}\gamma\mathrm{G}\cos\theta d\theta+\mathrm{M}\gamma\mathrm{K}\cos(\theta-\phi)d\theta.$$

Since this work is exactly consumed in keeping up or increasing the current, we must have

$$\mathrm{H}\gamma\mathrm{G}\cos\theta d\theta+\mathrm{M}\gamma\mathrm{K}\cos(\theta-\phi)d\theta=\mathrm{R}\gamma^2dt+\mathrm{L}\gamma d\gamma.$$

Since $\theta=\omega t$ and $\frac{d\theta}{dt}=\omega$, the solution of this equation is

$$\gamma=\frac{\omega}{\mathrm{R}^2+\mathrm{L}^2\omega^2}\{\mathrm{GH}(\mathrm{R}\cos\theta+\mathrm{L}\omega\sin\theta)+\mathrm{KM}\big(\mathrm{R}\cos(\theta-\phi)+\mathrm{L}\omega\sin(\theta-\phi)\big)\}$$
$$+\mathrm{C}e^{-\frac{\mathrm{R}}{\mathrm{L}}t},$$

the last term becoming insensible soon after the beginning of the experiment.

We can now find the equation of motion of the magnet.

Let A be its moment of inertia, MHτ the torsion of the fibre per unit of angular rotation, then

$$\mathrm{A}\frac{d^2\phi}{dt^2}=\mathrm{MK}\gamma\cos(\phi-\theta)-\mathrm{MH}(\sin\phi+\tau\phi).$$

Substituting the value of γ and separating terms in θ, we find

$$\mathrm{A}\frac{d^2\phi}{dt^2}=\frac{1}{2}\frac{\mathrm{MK}\omega}{\mathrm{R}^2+\mathrm{L}^2\omega^2}\Big\{\mathrm{GH}(\mathrm{R}\cos\phi+\mathrm{L}\omega\sin\phi)+\mathrm{KMR}\Big\}-\mathrm{MH}(\sin\phi-\mathrm{T}\phi)$$
$$+\frac{1}{2}\frac{\mathrm{MK}\omega}{\mathrm{R}^2+\mathrm{L}^2\omega^2}\Big\{\mathrm{GH}\big(\mathrm{R}\cos(2\theta-\phi)+\mathrm{L}\omega\sin(2\theta-\phi)\big)$$
$$+\mathrm{KM}\big(\mathrm{R}\cos 2(\theta-\phi)+\mathrm{L}\omega\sin 2(\theta-\phi)\big)\Big\}.$$

In order that ϕ may continue as it does nearly constant, the part independent of θ must vanish, or

$$\frac{1}{2}\frac{\mathrm{MK}\omega}{\mathrm{R}^2+\mathrm{L}^2\omega^2}\left\{\mathrm{GH}(\mathrm{R}\cos\phi+\mathrm{L}\omega\sin\phi)+\mathrm{KMR}\right\}-\mathrm{MH}(\sin\phi+\mathrm{T}\phi)=0.$$

This gives the following quadratic equation for R:—

$$\mathrm{R}^2-\frac{1}{2}\mathrm{R}\frac{\mathrm{GK}\omega}{\sin\phi+\tau\phi}\left(\cos\phi+\frac{\mathrm{KM}}{\mathrm{GH}}\right)=\frac{1}{2}\frac{\mathrm{GKL}\omega^2}{1+\tau\frac{\phi}{\sin\phi}}-\mathrm{L}\omega^2.$$

The solution of this equation may be expressed to a sufficient degree of accuracy as follows:—

$$\mathrm{R}=\frac{\mathrm{GK}\omega}{2\tan\phi(1+\tau)}\left\{1+\frac{\mathrm{KM}}{\mathrm{GH}}\sec\phi-\frac{2\mathrm{L}}{\mathrm{GK}}\left(\frac{2\mathrm{L}}{\mathrm{GK}}-1\right)\tan^2\phi\right\}.$$

To determine the quantities occurring in this equation, we must measure the dimensions of the coil, the strength of the magnet, and the force of torsion of the fibre.

1st. Dimensions of the coil.

Let a = mean radius of the coil .... = 0·1566 metre.
n = number of windings of wire .... = 307
l = effective length of wire = $2\pi na$ .... = 302·063 metres.
b = breadth of section of coil perpendicular to the plane of the coil .... = ·0185 metre.
c = depth of section in the plane of the coil ...... = ·0132 „
b' = distance of mean plane of coil from axis of motion .... = ·01915 „
α = angle subtended at axis by radius of coil = 83° 1′.

$$\cos\alpha=\frac{b'}{a}=\cdot12245.$$

Then $\mathrm{G}=\pi na^2\left(1+\frac{1}{12}\frac{c^2}{a^2}\right)$,

$$\mathrm{K}=\frac{2\pi n}{a}\sin^3\alpha\left\{1+\frac{1}{24}\frac{c^2}{a^2}(2-15\sin^2\alpha\cos^2\alpha)\right.$$
$$\left.+\frac{1}{24}\frac{b^2}{a^2}(15\sin^2\alpha\cos^2\alpha-3\sin^2\alpha)\right\},$$

$$\mathrm{GK}=\pi nl\sin^3\alpha\left\{1+\frac{1}{6}\frac{c^2}{a^2}+\frac{5}{8}\frac{b^2-c^2}{a^2}\sin^2\alpha\cos^2\alpha-\frac{1}{8}\frac{b^2}{a^2}\sin^2\alpha\right\}.$$

If the dimensions of the coil are measured in metres, GK will be expressed in metres.

Let T be the time of 100 revolutions of the coil, expressed in seconds, then

$$\mathrm{T}\omega=200\pi,$$

or

$$\omega=\frac{200\pi}{\mathrm{T}}.$$

Let D be the distance of the scale from the mirror, δ the scale-reading measured from the point of the scale which is nearest to the mirror, then

$$\tan 2\phi = \frac{\delta}{D};$$

$$\therefore \frac{1}{2\tan\phi} = \frac{D}{\delta}\left\{1+\frac{1}{4}\frac{\delta^2}{D^2}\right\}.$$

To determine MHr, the coefficient of torsion, let the magnet be turned round so as to twist the fibre nearly 360°. Let the difference of reading due to the torsion be δ', then

$$r = \frac{\delta'}{4\pi D}\frac{1}{1-\frac{\delta'}{4\pi D}}.$$

To determine $\frac{KM}{GH}$, let the suspended magnet A be removed, and let another magnet, which we shall call B, be put in its place. Let the magnet A be now placed east or west of B, at a distance equal to the mean distance of the coil, or $\sqrt{a^2+b'^2}$. Let the deflection of B when the north or south end of A is directed to it be μ, then

$$\frac{KM}{GH} = \tan\mu.$$

The determination of the quantity L, the electro-magnetic capacity of the coil, requires a more complex calculation, which must be explained separately. In the actual experiment the deviation ϕ was always small, and therefore $\tan^2\phi$ was very small, so that the term depending on L was never important. We may now write the value of R,

$$R = \frac{200\pi^2 Dnl\sin^3\alpha}{T\delta}\{1+\text{corrections}\}.$$

In this expression the quantities $Dnl\alpha$ are determined before the experiment is made. The only quantities to be observed are T, the number of seconds in 100 revolutions, and δ, the deviation in millimetres of the scale.

Part IV.—Details of the Experiments.

In the experiments at King's College, June 1863,

n, the number of windings, was 307.

l, the effective length of wire, 302·063 metres.

$\sin^3\alpha = 1 - \cdot021756$.

D, the distance from the mirror to the scale, 2·9853 metres.

Determination of Velocity.

A wheel of 100 teeth turned by an endless screw caused a bell to be struck every 100 revolutions of the coil. The times of the bells, as observed with a chronometer, serve to determine T.

Determination of Deviation.

δ is the difference between the reading of the scale when the magnet is acted on by the earth only, and when it is acted on also by the induced currents in the coil. To determine δ, the reading of the scale is made when the coil is at rest, or when the circuit is broken. Another reading is taken with the connexion complete and the coil in motion. If the earth's magnetism

remains the same, the difference of these readings is the true value of δ; but since the direction of the earth's magnetic action is continually varying, we must find the difference of *declination* between the times of the two readings, and calculate what would have been the undisturbed reading of the scale at the time when the deviation was observed.

In our experiments this correction was made by comparison with the photographic registers of magnetic declination made at Kew at the same time that our experiments were going on.

Corrections.

The corrections being small may be taken separately. Each has to be multiplied by the factor already considered,

$$R=\frac{200\pi^2 Dnl\sin^2\alpha}{T\delta}\{1+A+B+C+D+E+F+G+H+\&c.\}.$$

A. Correction for the dimensions of the section of the coil.

$$A=\frac{1}{6}\frac{c^2}{a^2}+\frac{5}{8}\frac{b^2-c^2}{a^2}\sin^2\alpha\cos^2\alpha-\frac{1}{8}\frac{b^2}{a^2}\sin^2\alpha$$
$$=+\cdot 000075.$$

B. Correction for level. Let the axis of rotation be inclined to the vertical at an angle β measured towards the north, and let the angle of the dipping-needle with the horizontal be I, then there will be a correction,

$$B=-\tan I\sin\beta.$$

In the actual experiment the level was taken with a spirit-level reading to 12″, and found correct to at least that degree of accuracy.

C. Correction for the induction of the suspended magnet on the coil. The strength of the magnet, as compared with that of the magnetic field, was measured by means of a magnetometer from Kew by the ordinary method. The correction found was

$$C=+\tan\mu$$
$$=\cdot 00780.$$

The small magnet generates induction-currents in the coil which react on the magnet, and tend to turn it in the direction in which the coil revolves. If there were no horizontal magnetic force due to the earth, the coil would drag the magnet round after it. In the actual case it makes the deviation greater than it should be by ·0078.

D. Correction for torsion of the fibre.

$$D=-T=-\frac{\delta'}{4\pi D}$$
$$=-\cdot 00132.$$

This correction depends on the relation between the stiffness of the fibre and the directive force of the suspended magnet. The fibre was a single fibre of silk 7 feet long; the magnet was a steel sphere $\frac{5}{16}$ inch diameter, and not magnetized to saturation. The correction for torsion was therefore much larger than if a stronger magnet had been used.

E. Correction for position of suspended magnet.

Let the centre of the magnet be at a distance ζ above or below the centre of the coil, η north or south of the axis of motion, and ξ east or west of the axis, then there will be a correction,

$$E = +\frac{3}{16}(1-4\cot^2 \alpha)\sin^4 \alpha\left\{4\frac{\zeta^2}{a^2}-\frac{\eta^2}{a^2}-3\frac{\xi^2}{a^2}\right\}.$$

Here $a=156{\cdot}6$ millimetres, and the place of the magnet was so adjusted that it could not vary one millimetre in any direction without the error being observed. Hence this correction is negligible.

F. Correction for irregularity in the magnetic field due to iron or magnets near the instrument.

Let t be the time of oscillation of a magnet at the centre of the coil, t_1 and t_2 at distances z above and below that point, then

$$F = +\frac{3a^2}{16z^2}\left\{\frac{2t-(t_1+t_2)}{t}\right\}.$$

This correction may also be neglected.

G. Correction of scale-reading. The quantity observed is $\tan 2\phi$, the quantity to be found is $\tan \phi$. The correction to the value of R is

$$+\frac{1}{4}\frac{\delta^2}{D^2}.$$

H. Correction for electro-magnetic capacity of the coil.

Let L be the value of the electro-magnetic capacity, the correction is

$$-\frac{1}{4}\frac{\delta^2}{D^2}\frac{2L}{GK}\left(\frac{2L}{GK}-1\right).$$

In the actual coil L was found by calculation $=397750$ metres, and by a rough experiment $=398500$ metres.

Now $GK=560245$ metres.

The correction is therefore $-\frac{1}{4}\frac{\delta^2}{D^2}(0{\cdot}596234)=H$.

This correction is of the same form with G, and may be taken along with it.

The complete expression for R is therefore

$$R=\frac{1}{T\delta}538145581730+\frac{\delta}{T}\,3055{\cdot}5.$$

The nature of the electrical action in the experiment may be stated as follows:—

Suppose the plane of the coil to coincide with magnetic north and south, and that the coil is revolving in the direction of the hands of a watch. Then the north side of the coil is moving from west to east, and therefore experiences an electromotive force tending to produce an *upward* current. The south side of the coil is moving from east to west, and therefore there is a tendency to produce a downward current in it. If the circuit is closed there will be a current upwards on the north side, and downward on the south side round the coil.

Now this current will tend to turn the north end of the suspended magnet towards the east; but the earth's magnetic force tends to turn it towards the north; so that the actual position assumed by the magnet must depend on the relation between the strength of the current and the strength of the earth's magnetism. But the strength of the current depends only on the velocity of rotation, the resistance of the coil, and the strength of the earth's magnetism. Hence the position of the magnet will not depend on the strength of the earth's magnetism, but only on the velocity and the resistance of the coil.

We must remember that the coil in its revolution comes into other positions than that which we have mentioned. As the north side moves towards the east, the current continually diminishes till it ceases when it is due east. The current then commences in the opposite direction with respect to the coil; but since the coil itself is now in a reversed position, the effect of the current on the suspended magnet is still to turn the north end to the east. The action of the current on the magnet is therefore of an intermittent nature, and the position of the magnet is not fixed, but continually oscillating. The extent of these oscillations, however, is exceedingly small. In fact if T be the time of vibration of the magnet from rest to rest under the action of the earth, and if t be one quarter of the time of revolution of the coil, and if δ be the deviation as read on the scale, then the same amplitude of these oscillations will be

$$c=\frac{t^2}{T^2}\delta.$$

In the actual experiment $\frac{t}{T}=$ about $\frac{1}{200}$ and δ less than 400 millimetres, so that the whole extent of vibration would be less than $\frac{1}{100}$ of a millimetre on the scale. This vibration was never observed and did not interfere with the distinctness of vision.

The only oscillations observed were the free oscillations of the magnet. They arose from accidental causes at the beginning of the experiment, and were subject to slight alterations in magnitude due to changes of speed of rotation, the passage of iron steamers in the Thames, &c. The time of one vibration was about 9·6 seconds, and by reading the scale at the extremities of every vibration a series of readings was obtained, the intervals between which were proximately equal.

Now since the deviation is proportional to the velocity

$$\delta=Cv=C\frac{dx}{dt};$$

and if we take values of δ at small intervals dt and sum them, we shall get

$$\int\delta.dt=C\int v\,dt=Cx,$$

where x is the whole distance travelled in the time.

Hence all we have to do is to observe the deviation at every oscillation, and to ascertain the whole number of revolutions during the time of observation, and the exact beginning and ending of that time. This was done in the following way.

The coil was made to revolve by means of the driving machine, and its velocity was regulated by the governor. While the required velocity was being attained, the oscillations of the magnet were reduced within convenient limits by means of a quieting bar at a distance. The quieting bar was then put in its proper place and the observation commenced.

One observer, A, took the readings of the scale as seen in the telescope, writing down the deviation at the extremity of every oscillation and thus obtaining a reading every 9·6 seconds.

Another observer, B, with a chronometer, wrote down the times of every third stroke of the bell. The times thus found were at intervals of 300 revolutions. When the observer B noted the time, the observer A made a mark on his paper, so that after the experiment the readings of deviation

could be compared with the readings of the chronometer taken at the same time.

The mean time of revolution between any two times of observation could thus be found and compared with the mean deviation between the same limits of time, and any portion of an experiment accidentally vitiated could be rejected by itself.

The experiments of each day commenced with a comparison, by means of an electric balance*, between the resistance of the experimental coil and that of a German-silver coil (called "June 4").

Then a series of readings of the scale was taken to determine the undisturbed position of the magnet. The times of beginning and ending this series were noted, and called Times of 1st Zero.

Then the coil was made to revolve, and readings of deviation and of time were taken as already described, and called 1st Spin+.

Then the direction of rotation was reversed and a second set of readings obtained, and called 2nd Spin−.

Then the undisturbed position was again observed with a note of the time. This was called 2nd Zero.

Lastly, the resistance was compared again with the standard coil. This series of experiments was then repeated if there was time.

From the values of 1st zero and 2nd zero, together with the information obtained from the photographic registers at Kew, the true value of the undisturbed reading during the 1st spin and 2nd spin was obtained. The difference between this and the actual reading is the deviation δ due to the electric currents. T was got by the chronometer readings. Now let r be the resistance of the standard coil at standard temperature, R the resistance of the experimental coil during the experiment, then by the comparison of resistances we find

$$R = xv,$$

where x is the ratio observed by means of the electric balance. But we also know that $R = \frac{N}{T\delta} +$ correction, where N is a known number given at p. 76. Hence r, the resistance of the standard coil, may be found in absolute measure by the formula

$$r = \frac{N}{xT\delta} + \text{a small correction};$$

the value of $xT\delta$ should therefore be nearly constant.

Thus, on June 23rd, 1863, the experiments were made as follows:—

At $12^h\ 15^m$ the resistance of the copper experimental coil was compared with that of standard coil "June 4" taken at 101, and found to be 101·26.

From $12^h\ 36^m$ to $12^h\ 45^m$ the undisturbed position of the suspended magnet was observed, and found to be 590·28 scale-divisions as the mean of all the readings.

The position of the declinometer at Kew at the same time was 7·689 of its own scale-divisions.

From $12^h\ 47^m\ 51^s{\cdot}5$ to $1^h\ 3^m\ 13^s$ the position of the magnet was again observed while the coil was revolving; 104 readings of the scale were taken, of which the mean was 930·59. This, when corrected for scale-error, gives 931·48 as the true reading. The position of the declinometer at Kew during the same time was 7·679. The resistance, measured after the experiment, was 101·28.

* *Vide* Report, 1862, p. 159, and present Appendix, p. 99.

The number of revolutions was 6300 during the time of observation, so that the time of 100 revolutions was $14^{s}\cdot464$.

By comparing the Kew apparatus with that at King's College, it appears that 1·0 of the Kew scale =19·137 of the King's College scale. The undisturbed readings at King's College were found actually to vary very nearly in this proportion to those at Kew.

Hence it is easy to find the undisturbed reading during any given experiment by comparison with the Kew numbers.

Thus, for the first experiment on June 23rd we get

Corrected undisturbed reading		591·54
Deflected reading		931·48
Deflection δ	=+	339·94
Time of 100 revolutions=T	=	14·464
Product Tδ	=	4916·90
Resistance at time of experiment x	=	101·28
Tδx	=	4979·75

Three other experiments were made on June 23rd. The result of the four experiments was as follows:—

1st experiment.	Positive rotation	$T.\delta.x$=4979·75		
2nd „	Negative	$T.\delta.x$	....	=5071·18
3rd „	Positive	$T.\delta.x$=5093·35		
4th „	Negative	$T.\delta.x$	....	=5007·66
Mean Positive result		5036·55		
Mean Negative result....................			....	=5039·42

Mean result of June 23rd..........	5037·98
Mean result of June 19th..........	5075·77
Mean result of June 16th..........	5046·18
Mean of three days	5053·32

It will be observed that the mean results of each day are more concordant than the individual experiments made on the same day. The errors, therefore, which we have hitherto been unable to get rid of are not of a kind which would have the effect of making the result depend on the arrangements adopted on the day of experiment, but are rather such as would destroy one another in any long series of experiments.

Dividing N by the number just found, we get for the resistance called 100 provisionally,

$$106493470+61100=10655470,$$

the second term being the correction for self-induction and for scale-reading.

Since the coil of German silver, marked June 4th, was called provisionally 101, we find as the result of the experiments for the resistance of "June 4" in absolute measure

107620116 metres per second.

Knowing the absolute resistance of "June 4," we may construct coils of given resistance by known methods.

THIRD REPORT—BATH, SEPTEMBER 14, 1864.

The Committee consists of—Professor Williamson, Professor Wheatstone, Professor W. Thomson, Professor Miller, Dr. A. Matthiessen, Mr. Fleeming Jenkin, Sir Charles Bright, Professor Maxwell, Mr. C. W. Siemens, Mr. Balfour Stewart, Dr. Joule, and Mr. C. F. Varley.

In the present Report it is thought unnecessary again to refer to the objects with which the Committee was appointed, or to recapitulate the arguments for and against the various systems of standards which have been from time to time proposed. The Committee have seen no reason to alter the conclusions previously adopted, and now propose briefly to state the progress made in the practical development of those conclusions, which may be found expressed at length in the Report for 1863.

That Report announced the adoption by the Committee of the absolute electro-magnetic system of measurement, based on the metre, gramme, and second, with certain modifications to facilitate the practical construction or use of the standards; and it further stated that in 1863 the absolute resistance of a certain German-silver coil had been measured with considerable accuracy.

No standards based on the 1863 determination were officially issued, inasmuch as it was felt that a second determination was absolutely required before complete dependence could be placed either on the method employed or on the results obtained. Some coils representing 10 of the British Association units, *i. e.* 10^7 absolute units according to the 1863 determination, were made by Messrs. Elliott Brothers, and a set from 1 to 10000 was made from the 1863 determination by Messrs. Siemens and Halske of Berlin. This last set is intended for Col. Douglas, the Superintendent of the Government telegraph lines in India; and a few of Messrs. Elliotts' coils have been bought by persons who were unwilling to wait for the final experiments by the Committee. None of these coils have been in any way certified as correct by the Committee.

In order thoroughly to test the value of the experiments made in 1863, it was determined that not only every measurement should be made afresh, but that every element in the experiment should be varied. The experiment consisted essentially in causing a coil, or rather two coils, of copper wire to revolve or spin at a certain definite rate, and in observing the deflection of a magnet, suspended within the coil, by the reflection of a scale in a mirror attached to the magnet.

The measurements required in the calculation are the following:—

a. The mean radius of the coils.
n. The number of turns made by the copper conductor forming the coils.
l. The effective length of the wire.
b. The breadth of the section of the coil.
c. The depth of the section of the coil.
b'. The distance of the mean plane of the coil from the axis of rotation.
T. The time of 100 revolutions of the coil.
D. The distance of the scale from the mirror.
δ. The scale-reading during each experiment.

The above measurements are required for what may be called the simple theory, that is to say, the theory omitting all the necessary corrections arising

from self-induction, torsion of fibre, &c. For these corrections it is further necessary to measure—

1st. The coefficient of torsion of the fibre.

2nd. The magnetic moment of the suspended magnet.

3rd. The horizontal component of the earth's magnetism.

4th. The variation of the electrical resistance of the coil during each experiment and between each experiment.

5th. The variation in the direction of the earth's magnetic force.

6th. The irregularities resulting from the unavoidable departures from that relative position of the telescope, mirror, scale, and magnet which would be theoretically most desirable.

In the experiments made at King's College in 1864, every part of the apparatus, except the distance of the mean planes of the two coils from the axis of rotation, was altered; so that every measurement was not only made afresh, but, where susceptible of change, was considerably different in magnitude.

Few of the measurements could be made by the means employed with greater accuracy than one part in 10000, and some of them were not determined even with this degree of accuracy. No very perfect agreement between two entirely distinct series of experiments was therefore to be expected; but the Subcommittee, consisting of Professor Maxwell and Mr. Jenkin, who this year have undertaken the experiments, are fortunately able to report a concordance between the determinations of 1863 and 1864 which is most satisfactory.

The difference between a standard constructed from the mean result of the 1863 experiments and a standard constructed from the mean result of the 1864 experiments would be only 0·16 per cent. The probable error of the 1863 experiments is 0·24 per cent. if the mean of each day's experiments be counted as one only; the probable error of the 1864 experiments is 0·1 per cent. if the mean of each pair of experiments with the coil revolving in two opposite directions be taken as one experiment.

Taking into account the agreement between the means of the two years, we may say that the determination of the Subcommittee does not probably differ from true absolute measurement by 0·08 per cent.

The Committee are of opinion that, in the present state of electrical science, the result now obtained is satisfactory, and will justify the immediate construction of final standards of electrical resistance.

It can hardly be doubted that, with the lapse of time and the inevitable progress of knowledge, still better determinations will some day be made; and that even now, with still greater care and by still further multiplying the number of experiments, a somewhat more perfect agreement between the standards and the theoretical absolute measurement could be ensured.

The Committee had then to consider whether this possibly still more perfect agreement would be worth the very great time, the labour, and the money which would have to be bestowed upon it. It has never been proposed that the British-Association standard should be considered as representing exactly an absolute measurement; whatever may be the state of science, any such pretension could not be well founded, for all that can be done at any time, by the very greatest care, is to reduce the possible error to less than a certain amount. The amount of probable error in the present determination is so small as to be insignificant for any of the present purposes of science, and will always remain insignificant for any practical applications. For these applications it is chiefly important that every copy of the standard,

whatever that may be, should be accurately made—a condition which is quite unaffected by the greater or less discrepancy between the standard and true absolute measurement.

The reproduction of the standard can perhaps be more easily effected, if ever it be necessary, by a given weight of metal or alloy than by a fresh absolute determination.

Meanwhile practical standards of resistance are urgently required, and the Committee are pressed to come to a decision. Defective systems are daily taking firmer root, and the measurement of currents, quantity, capacity, and electromotive force call urgently for the attention of your Committee.

Under these circumstances they have decided to rest content with the results of the experiments now completed, and to commence at once the construction of standard coils.

The details of the experiments on absolute resistance are given in Appendix A.

It may be useful here to mention that the new unit will be roughly equal to 0·0736 times Dr. Matthiessen's mile of copper wire, and more exactly 1·0456 times Siemens's unit, according to standards which have kindly been sent by Dr. Siemens to several members of the Committee and others*.

The questions of chief importance, after the magnitude of the standard has been chosen and determined, concern the choice of a suitable form and material for the actual construction of the standard; and in this choice the permanence of the standard is above all essential.

Dr. Matthiessen has for two years been endeavouring, at the request of the Committee, to discover whether the electrical resistance of various metals, under various conditions, can be considered as constant, or can be proved to alter. His Report for the present year is given in Appendix B, and will be found to confirm, in a great measure, the conclusions arrived at in his Report for 1863.

No variation has been observed by him in the electrical resistance of annealed wires of silver, copper, gold, platinum, nor in the hard-drawn wires of gold, platinum, or of the gold-silver alloy. But a change *has* been observed in the hard-drawn wires of silver and copper—a change most rapid in the first year, but very sensible in the second year; a somewhat capricious change has also been observed in certain annealed German-silver wires, while others have been proved constant. This result has been independently observed by other members of the Committee. In the hard-drawn wires of silver and copper the direction of the change has been such as to bring the resistance of hard-drawn wires more nearly to resemble that of annealed wires, diminishing the resistance; in other words, it is such a change as would be produced by partial annealing.

From these experiments it is clearly undesirable that silver or copper should be used for standards even in their annealed state; and the change in these metals further indicates that for standards of other metals the partially annealed is preferable to the hard-drawn condition.

The experiments on these points must be continued for many years before much reliance can be placed on the results; and meanwhile equal standards must be constructed of various materials, and protected in various ways, for reference and comparison.

The precautions taken to prevent chemical action and mechanical injury are given in Appendix B of the Report for 1863. Coils of wire covered with

* Twenty-five units are within one per cent. equal to the mile of No. 16 copper wire in use by the Electric and International Company. Mr. Varley has promised that for the *future exact* equality shall be aimed at.

silk, baked and imbedded in solid paraffin, appear, at present, to be the most promising form for the unit standards. Authentic copies of the standard coils made of platinum-silver alloy, which appears likely to be permanent, might be issued at about £2 10*s*. each, and coils prepared from these by electrical instrument-makers could be verified at a moderate rate at Kew, where the original standards will be deposited. No officially authentic coil can be issued until the standards themselves have been made.

The reproduction of the standard forms the next point for consideration. Notwithstanding the good results obtained by Professor Thomson's method of making an absolute measurement, the Subcommittee do not recommend the adoption of this process for the reproduction of the standard, which may some day become necessary, owing to the accidental destruction of, or change in, the Kew standards. Dr. Matthiessen, on the other hand, states, with confidence, that a standard may be reproduced by means of metal wires of given weight and length, or by means of mercury, within about 0·01 per cent.; the report of his investigation on this subject, made conjointly with Mr. C. Hockin, is contained in Appendix C, and may be summed up as follows. He first draws a distinction between ordinary care, great care, and absolute care. He considers that with ordinary care the gold-silver alloy is the most suitable material (see Report, 1862) for the reproduction, but when great care is used lead is recommended as the most suitable material; but any reproduction by one material should be checked by others, such as mercury. With absolute care it appears that almost any material might be used. It must be remembered that Dr. Matthiessen considers that he himself has not taken absolute but only great care.

The following Table shows the number of wires of each material tested, their maximum discrepancy, and the probable error in a standard reproduced by similar experiments:—

Metal.	No. of wires.	Maximum discrepancy expressed as a fraction of the whole conducting power.	Probable error.
Silver	3	0·0014	0·00052
Copper	3	0·0011	0·00021
Gold	3	0·0005	0·00011
Lead	4	0·00054	0·00006
Gold-silver alloy	5	0·00073	0·00001
Mercury	3	0·00151	0·00009

Commercially pure lead differed from the chemically pure lead by only about 0·04 per cent.

For an account of the care taken by Dr. Matthiessen in the chemical preparations of the metals he used, and in their subsequent treatment and electrical comparison, we must refer to Dr. Matthiessen's own Report, Appendix C.

With reference to mercury, great difficulty exists in making the experiments; and it is much to be regretted that Dr. Matthiessen's experiments, very accordant in themselves, do not give results agreeing with Dr. Siemens's experiments. The discrepancy will be best explained by the following Table, giving the value of a column of mercury at 0° C. one metre long, and having a cross section equal to one square millimetre, according to various experi-

ments, and with the specific gravity used respectively by Dr. Siemens and Dr. Matthiessen.

Definition.	Value in B. A. units.
1. Mercury unit according to Siemens's standard issued in 1864. Sp. gr. mercury assumed at 13·557	0·9564
2. Mercury unit according to Siemens's experiments made for 1864 standard, but assuming sp. gr. mercury at 13·595*.	0·9534
3. Mercury unit according to Dr. Matthiessen's experiments. Sp. gr. mercury assumed at 13·557	0·9646
4. Mercury unit according to Dr. Matthiessen's experiments. Sp. gr. mercury assumed at 13·595	0·9619
5. Mercury unit according to one set of coils exhibited in 1862 by Dr. Siemens (Berlin)	0·9625
6. Mercury unit according to a second set of coils exhibited in 1862 by Dr. Siemens (London)	0·9742

Dr. Matthiessen considers No. 4 the true value, while Dr. Siemens supports No 1. The Committee do not desire to express any opinion on this subject, but only to draw attention to the great discrepancies which follow the apparently simple definition of the mercury unit (first proposed by Marié Davy). Even now it cannot be said that a trustworthy standard, answering to the definition, exists.

The Committee have little to report concerning the standard instruments for the measurement of currents, quantity, capacity, or electromotive force. The drawings for a standard galvanometer and electro-dynamometer have been begun. An electro-dynamometer, suitable for general use, has been constructed by Professor W. Thomson, and experiments are being made with it.

Professor Thomson has also had some fine apparatus made for the measurement of electrostatic phenomena and their comparison with electromagnetic measurements; but it will be best to describe the instruments when the experiments have been completed.

Dr. Joule has made some preliminary experiments with the view to redetermine the mechanical equivalent of the unit of heat by electrical means.

Thus, although the Committee have not accomplished all that they hoped, they feel that such progress is being made as will justify their reappointment.

They have received assurances that the British-Association system of units will be readily adopted in this kingdom, in India, Australia, and Germany. They believe that it will be accepted in America and in many other parts of the world.

From France no response has yet been obtained.

The Committee wish to express their sincere regret at the death of one of their members, Dr. Esselbach. He had made valuable experiments on the electromotive force of various chemical combinations, and had promised to communicate them to the Committee; but their record is now probably lost.

Before concluding, the Committee have to thank Mr. Charles Hockin for the efficient assistance he has afforded, both in the determination of the resistance unit and in Dr. Matthiessen's researches.

* This is the mean of the values given by Kopp, Regnault, and Balfour Stewart. The discrepancy between the two values is far greater than could be due to any confusion as to the reference of the specific gravity to water at 0° and at maximum density.

	n.	Varley.	German Miles.	Observations.
$\times 10^7$.		0·01190	0·005307	Calculated from the B. A. unit.
....		0·01251	0·005574	From an old determination by Weber.
....		0·02486	9·01108	No measurement made; ratio between Siemens (Berlin) and Jacobi taken from Weber's 'Galvanometrie.'
e		0·03591	0·01655	Measurement taken from a determination in 1862 of a standard sent by Prof. Thomson; does not agree with Weber's own measurement of Siemens's units; by Weber 1 Siemens's unit $= 1{\cdot}025 \times 10^7$ metres-second.
sue ...		0·03737	0·01666	Measurement taken from three coils issued by Messrs. Siemens.
)		0·03757	0·01675	Measurement taken from coils exhibited in 1862 by Messrs. Siemens, Halske & Co. (well (adjusted).
n)		0·03802	0·01695	Measurement taken from coils exhibited in 1862 by Messrs. Siemens, Halske & Co. (well adjusted).
hmad...		**0·03905**	**0·01741**	Equal to 10,000,000 $\frac{\text{metres}}{\text{second}}$, according to experiments of Standard Committee.
....		0·3620	0·1613	From coils exhibited in 1862 (pretty well adjusted).
....		0·3814	0·1700	From coils exhibited in 1862 (indifferently adjusted).
....		0·4072	0·1815	From coils exhibited in 1862 (badly adjusted).
....		0·5306	0·2365	From a coil lent by Dr. Matthiessen (of German-silver wire).
....		1·000	0·4457	From coils lent by Mr. Varley (well adjusted).
....		2·243	1·000	From coils exhibited in 1862 by Messrs. Siemens, Halske & Co.*

Appendix A.—*Description of a further Experimental measurement of Electrical Resistance made at King's College.* By Prof. T. C. Maxwell and Mr. Fleeming Jenkin, *with the assistance of* Mr. Charles Hockin.

The method employed in these experiments has been fully described in Appendix D to the Report of 1863. In the new experiments the elements of the calculation were varied as much as possible; fresh wire was wound on the experimental coils; observations were made with velocities differing widely from one another. Fresh measurements were made of all the corrections required, and greater precautions were taken to avoid local disturbances.

n, the number of windings, was....	313.
l, the effective length of the wire ..	311·118 metres.
the mean circumference	0·993987 metre.
a, the mean radius	0·158194 „
b, the breadth of each coil	0·1841 „
$2b$, the distance from centre to centre of each coil	0·03851 „
c, the depth of the layers	0·01608 „
The weight of the wire and silk....	110 oz. 8 dwt.

$\log \sin^3 a = \bar{1}\cdot9624955$.

D, the distance from the mirror to the scale; 2212 millims. in some experiments, 2116 millims. in others.

The following Table gives the result of the experiments, and the comparison with those of 1863.

Time of 100 revolutions, in seconds.	Values found for coil in terms of 10^7 for each experiment.	Value of B.A. unit in terms of $10^7 \frac{\text{metre}}{\text{seconds}}$, as calculated from each experiment.	Value from mean of each pair of experiments.	Percentage error from mean value.
17·54	4·7201	1·0121	0·9978	−0·22
17·58	4·5914	0·9836		
77·62	4·8848	1·0468	1·0040	+0·40
76·17	4·4871	0·9613		
53·97	4·6607	0·9985	0·9992	−0·08
54·53	4·6666	0·9998		
41·76	4·6279	0·9915	0·9925	−0·75
41·79	4·6275	0·9936		
54·07	4·6496	0·9961	0·9924	−0·76
53·78	4·6146	0·9886		
17·697	4·6108	0·9878	1·0007	+0·07
17·783	4·7318	1·0136		
17·81	4·6452	0·9952	1·0063	+0·63
17·78	4·7489	1·0174		
17·01	4·7567	1·0191	1·0043	+0·43
16·89	4·6187	1·9895		
21·35	4·6834	1·0034	1·0022	+0·22
21·38	4·6727	1·0011		
21·362	4·6526	0·9968	1·0040	+0·40
21·643	4·7134	1·0096		
11·247	4·8658	1·0424	0·9981	−0·19
16·737	4·5305	0·9707		

Probable error of R (1864) = 0·1 per cent.
Probable error of R (1863) = 0·24 per cent.
Difference in two values 1864 and 1863 = 0·16 per cent.
Probable error of two experiments = 0·08 per cent.

In constructing the standard coil, in consideration of the much greater range of velocities used in 1864, the 1864 mean value was allowed to have five times the weight of the mean value obtained in 1863.

Appendix B.—*On the Electrical Permanency of Metals and Alloys.* *By* A. Matthiessen, *F.R.S.*

In Appendix A of the Report of your Committee of last year, I gave the results of some experiments made to test the electrical permanency of some metals and alloys. On August 5 of this year I re-tested them, and give the results in the following Table, taking the conducting power of No. 15=100·00, as was done in last year's Report.

	May 9, 1862.	T.	June 14, 1863.	T.	Aug. 5, 1864.	T.
1. Silver: hard-drawn	100·00	20·2	103·915	20·0	104·397	20·2
2. Silver: annealed........	100·00	20·2	99·947	20·1	100·013	20·1
3. Silver: hard-drawn	100·00	20·2	102·807	20·2	103·655	20·1
4. Silver: annealed........	100·00	20·2	100·031	20·0	100·048	20·0
5. Copper: hard-drawn....	100·00	20·1	100·248	20·2	100·276	20·0
6. Copper: annealed	100·00	20·1	100·015	20·0	100·010	20·1
7. Copper: hard-drawn.....	100·00	20·0	100·149	19·8	100·200	20·2
8. Copper: annealed	100·00	20·0				
9. Gold: hard-drawn......	100·00	20·0	100·045	20·2	100·000	20·2
10. Gold: annealed	100·00	20·0	100·062	20·0	99·960	20·2
11. Gold: hard-drawn......	100·00	20·0	99·869	20·2	99·937	20·0
12. Gold: annealed	100·00	20·0	99·877	20·3	99·960	20·0
13. Platinum: hard-drawn..	100·00	20·0	99·951	20·2	99·989	20·2
14. Platinum: hard-drawn..	100·00	20·0	99·999	20·2	100·008	20·1
15. Gold-silver alloy: hard-drawn	100·00	20·0	100·000	20·2	100·000	20·2
16. Gold-silver alloy: hard-drawn	100·00	19·9	99·963	20·3	99·996	20·0
17. German silver: annealed.	100·00	20·3	100·162	20·0	100·135	20·0
18. German silver: annealed.	100·00	20·3	100·145	20·0	100·152	20·0
19. German silver: annealed.	100·00	..	100·217	20·2	100·193	20·2

From the above it will be seen that the following wires have not sensibly altered in their conducting power during the space of two years:—

No.	May 9, 1862.	June 14, 1863.	August 5, 1864.	Maximum difference corresponds to.
2.	100·00	99·911	99·977	0·25
4.	100·00	99·959	99·976	0·10
6.	100·00	99·979	100·010	0·05
9.	100·00	100·117	100·072	0·30
10.	100·00	100·062	100·032	0·20
*11.	100·00	99·941	99·937	0·15
*12.	100·00	99·985	99·960	0·10
13.	100·00	100·023	100·061	0·15
14.	100·00	100·071	100·044	0·20
15.	100·00	100·000	100·000	
16.	100·00	99·963	99·996	

* Without taking into consideration the corrections due to temperature, I placed in last year's Report these two wires with those whose conducting powers had changed.

All the values have been reduced to the first observed temperature, assuming that all pure metals vary in conducting power alike with temperature. The correction made was the addition or subtraction of 0·036 for each 0°·1, which number corresponds to the correction of conducting power for temperature at 20°. No correction has been made in the cases of No. 15 and 16, for it is so small that it may be neglected, being about 0·006 for each 0°·1.

As stated in last year's Report, the differences may be considered due to temperature; for, as there explained, a difference in the temperature of the wire and the bath might well exist, and we find in most cases a difference in the conducting power corresponding to 0°·1 to 0°·2.

It is interesting to find that hard-drawn silver and copper wires become partially annealed by age, at least the increment in the conducting power would indicate such to be the case. In the case of silver, a decided increment will be observed.

No. 8, copper, annealed, has altered so much, that there can be no doubt that it was badly soldered.

With regard to the alteration observed with the German-silver wires, it may here again be stated that it is not to be assumed that all wires of this alloy will alter in like manner. An example of this has lately come to my notice. About two years ago I made a coil of the gold-silver alloy, which was compared with one of Prof. Thomson's German-silver coils; and having them still in my possession, they have now been re-compared, with the following results:—

July 8th, 1862. Resistance of Thomson's coil being 1 at 18°·4, that of the gold-silver coil was ·88445 at 18°·4.

August 6, 1864. Resistance of Thomson's coil being 1 at 18°·4, that of the gold-silver coil was ·88447 at 18°·4.

It is worthy of remark that the first comparison was made by Dr. C. Vogt, the last by Mr. C. Hockin, and with entirely different apparatus, showing that different observers with different apparatus obtain absolutely the same results when they take great care in making the observations.

The above proves that the conducting power of all specimens of German-silver wire does not alter by age. Further experiments are being made on this subject, and in the course of a year or so we shall be able to say how far German silver may be trusted for making resistance coils.

Appendix C.—*On the Reproduction of Electrical Standards by Chemical Means. By* A. Matthiessen, *F.R.S., and* C. Hockin, *Fellow of St. John's College, Cambridge.*

Having been requested by your Committee to make some experiments with the view of discovering the best method of reproducing a unit of electrical resistance by chemical means, we have carried out the research of which we now propose to give the results.

The experiments have been made with unusual care. It is important to point out the degree of precaution that has been taken to insure trustworthy results. The care taken in these experiments may be called great care as opposed to ordinary care on the one hand and thorough care on the other. By ordinary care is meant the care usually taken in scientific research, where no extraordinary precautions are had recourse to. The sort of accuracy obtained when a unit is reproduced with ordinary care may be seen by reference to former results. For instance, in the determination of the conducting power of mercury, described in 'Phil. Trans.,' results were obtained

differing in some cases by 1·6 per cent. The same degree of accuracy was obtained in the determination of the mercury unit by Dr. W. Siemens, described in 'Phil. Mag.'

On the other hand, in the experiments to be described, and in those made by Mr. Sabine, the results differ by only a few hundredths per cent.

The results of the determinations of the conducting power of the gold-silver alloy, described in the 'Phil. Mag.' Feb. 1861, differ from each other by 1·5 per cent.; the values now found for the same quantity differ by only seven-hundredths per cent. No doubt if greater care had been taken and more perfect instruments used, still better results would have been obtained.

Perhaps the great difference between what is above called great care and ordinary care lies in the time employed. The experimenter using great care has to neglect almost all consideration of time, and repeat his experiments at reasonable intervals, in all cases in which it is possible that by lapse of time such error as at first there is no means of detecting may increase and so become apparent. The meaning of absolute care is clear. When absolute care is taken no precautions are omitted, the best instruments obtained, and every care taken in the manipulation.

The apparatus used in the following research will first be described, the results obtained will be then given, and finally some remarks made on them.

Description of Apparatus.

Battery.—The battery employed was a single Bunsen's cell. The wires connecting it with the bridge ran parallel to each other the whole of their length, so that no attraction was exercised on the magnet of the galvanometer by the current traversing them*.

Balance.—For measuring the resistance of the wires a Wheatstone's balance, as modified by Kirchhoff, was employed. A plan of it is given in Plate III. (fig. 1).

L and R are two resistance coils acting as the arms of the balance. They are joined by the wire $A\,A_1$, along which the block B connected with one end of the galvanometer coil can be moved.

The wire $A\,A'$ of the instrument was made of an alloy containing 85 per cent. of platinum and 15 per cent. of iridium. The advantages of employing this alloy are that it does not readily oxidize, that it does not change much in conducting power with an alteration of temperature, and that it does not alloy with mercury.

S is a standard coil immersed in an oil-bath.

O P is the wire to be measured or compared with the standard S, and is immersed in a large trough of water.

G is an ordinary galvanometer by which approximate results are first observed.

G_1 is a very sensitive Thomson's reflecting galvanometer, by which the final observations are made.

M_1, M_2 &c., m_1, m_2 &c. are mercury cups used to connect the several parts of the circuit by thick copper rods and bars, plainly shown on the drawing. The arrangement shown was found convenient, as it admitted of adjustment to various positions and dimensions of conductors to be compared. The position of B on the wire $A\,A'$ could be observed by a boxwood scale divided into millimetres and a pointer on the block.

K is a key used to complete the battery circuit, and worked by a treadle

* The battery circuit was generally broken, and was closed by pressing down a treadle, placed under the table, with the foot. The terminals were of platinum.

from below. An enlarged section of the block B is given in fig. 3. *a* is a wooden handle by which the rod *b*, with the platinum point *d*, can be depressed so as to come in contact with the wire of the bridge. When the pressure of the hand is removed the spring *e* lifts the handle and breaks the contact. The galvanometer wire is screwed in between the metal plates *f* and *g*. A pad of gutta percha between the knob *h* and the handle prevented any sensible thermal current. To the top of the block was fixed a piece of brass with a slit in it to serve as a pointer. A lens also was fastened to the handle to read fractions of a millimetre on the scale. The body of the block was of lead, with a slab of ebonite at the top and bottom. The block ran on a tramway parallel to the scale and wire of the balance.

A section of one of the mercury cups is given at fig. 2. At the bottom of the cylindrical cup *l m n o* is placed an amalgamated copper plate, and mercury is poured into the cup; the plate is held down by the wooden cylinder *p*, and this is kept in its place by the pin *r s*. This plug fits the cup closely, and is pierced with two or more holes for the terminals to pass through. The cups were propped up with wedges, when placed under the fixed terminals of the balance, that these might press firmly against the metal bottoms of the cups.

Each of the coils R and L had a resistance of about 20 metres of the wire of the instrument. Careful measures were made of the resistance of the wire of the bridge at different points in order to find if there were any very faulty points in it; this was done by putting the coils R and L in their places, and increasing the resistance of one of them by means of a short piece of wire. The effect of this wire was to shift the zero-point. Two coils, differing about one tenth per cent., were then placed in the centre of the instrument and the reading taken; these coils were then reversed and the reading again taken.

Suppose $2\,l$ the resistance of the circuit from the point B to B′ when the short wire is removed, z the change in the zero-point caused by the insertion of the short wire above mentioned, and x the difference of a pair of readings, resistances being expressed in millimetres of the wire A A′, and lengths expressed in millimetres of the scale; then the resistance of a millimetre of the wire of the instrument about the zero-point is

$$=(l+z)\,.\,\frac{a-b}{a+b}\,.\,\frac{2}{x}\,;$$

$\frac{a}{b}$ is the ratio of the two centre coils.

The value of this expression was found for different points from one end to the other of the wire, and did not vary more than two or three tenths of a millimetre, an error not considerable enough to affect the results obtained with the instrument.

The value of the coil R was thus found. It was placed in the mercury cups m'_1, m'_2, and the cups m_1, m_2 were joined by a stout copper bar. Two coils, the ratio of the resistance of which was known, were placed in the two centre cups and the reading taken.

Let $\frac{a}{b}$ be the ratio of two centre coils, x the reading of scale, which was divided from A′ to A, $R+r$ the resistance of the circuit from B′ to the point of wire opposite that end of the scale nearest to R, viz. A′, l the corresponding quantity for the other side of the instrument.

Then clearly

$$\frac{R+r+x}{l+1000-x}=\frac{a}{b},$$

or

$$R+r=\frac{a}{b}(l+1000-x)-x.$$

The readings are given in the following Table:—

Ratio of $\frac{a}{b}$.	Reading.	Value of R+r.	Value of R+r.
24 : 1	120·5	20987+24l	21215
26 : 1	186·5	20964+26l	21210
29 : 1	269	20930+29l	21205
34 : 1	375	20875+34l	21197
36 : 1	409	20867+36l	21208
37 : 1	425·25	20841+37l	21192
39 : 1	454·25	20830+39l	21201
42 : 1	493·25	20790+42l	21188
47 : 1	547·25	20732+47l	21177
55 : 1	613	20672+55l	21193
60 : 1	645·25	20625+60l	21194
68 : 1	688	20528+68l	21173
76 : 1	720·75	20483+76l	21203

Zero-point was at 516.

Resistance of half length of circuit is 21712 millimetres of wire.

All these values are within necessary errors of observation. The first few values are most to be relied on, as the values of r+R depend nearly directly on 1000$-r$.

So many measurements were made in order to find whether the wire tapered towards either end. The similarity of the values found for R+r shows this better, perhaps, than the direct method before described.

A set of similar measurements were made with the coil L in the left-hand mercury cups, and equally good results obtained.

The galvanometer employed was one of Thomson's reflecting galvanometers, made by Messrs. Elliott Brothers. A short coil was employed. The instrument was placed in a deal box, blackened inside, with large apertures to observe through. The spot of light could thus be clearly seen, and the divisions of the scale were sufficiently illuminated to enable the observer to see immediately in which direction the spot of light moved. The instrument was sufficiently delicate to show 0·001 per cent. difference in the ratio of any two nearly equal conductors compared, corresponding to $\frac{1}{10}$ millim. on scale of bridge.

An ordinary galvanometer was also at hand to find about the place of reading on the scale.

The balance employed for weighing was by Liebrich of Giessen, and would weigh to $\frac{1}{10}$ of a milligramme with accuracy. The weights were adjusted by Oertling, and again tested by weighing them against the largest weight (50 grms.). Mr. Balfour Stewart was kind enough to test this weight, and found its value to be exactly 50·000 grms. All weighings made in this research were double weighings.

The measurements of lengths of wires tested were made with a beam-compass. It was furnished with a vernier carrying a telescope. The instrument was fixed horizontally before a window, the ends being clamped to shelves in the wall on either side of the window.

The telescope pointed downwards, and the wires to be measured were laid on a board fixed below the instrument.

With this apparatus measurements could be made with the greatest certainty to $\frac{1}{40}$ of a millimetre, the telescope being sufficiently powerful to show much smaller lengths than this.

We are indebted to Mr. B. Stewart for measuring the values of the divisions of the instrument.

Thermometers.—Two thermometers were employed. They were made by Messrs. Negretti and Zambra. One was divided to $\frac{1}{5}$ of a degree Centigrade, the other to single degrees. The large thermometer was found to be correct by the Kew standard. The zero-points of the thermometers were carefully taken.

Trough.—The wires the resistances of which were to be determined were placed in a glass tube immersed in a trough of water.

The trough was 1·5 m. long by 0·15 m. square section. A stream of water flowed through it, coming in by the tube V (fig. 1) and escaping by the waste-pipe W. This arrangement was adopted because it was found that naphtha or oil soon acted on the wires and altered their resistance, so that they could not be immediately exposed to the action of a liquid. The details of the arrangement will be understood by reference to fig. 4.

The wire to be tested, *a b*, was soldered at its ends to copper bars, as *a c*. On to each of these bars was slipped a piece of glass tubing, as *e f*. These tubes were fastened to the copper bars by india-rubber tubing. The wire, with its connexions, was then placed in the large glass tube A B. The piece of tubing *e f* was then fastened to the bent tube C E D F by india-rubber tubing.

The ends of the terminals *a c* were beaten out flat and amalgamated. The bent tubes were nearly filled with mercury, and the terminal *c* was connected with the mercury cups m'_1, m'_2 of the instrument by copper rods amalgamated at each end.

The resistances of the wires were compared with those of coils of German silver, well varnished, immersed in a cup of oil. The temperature of the oil was determined by the small thermometer before described.

Method of observing.—The wires were placed in the trough and the connexions made. The water was then turned on and allowed to flow for about fifteen minutes. The large thermometer was placed in the trough, and the temperature was read off by means of a lens placed so as to avoid all error of parallax. The small galvanometer was then connected with the electric balance, and the approximate reading found.

The large galvanometer was next connected, and the block handle pressed down until any thermal current that existed had ceased to cause the needle of the galvanometer to oscillate. The battery contact was then made for an instant with the foot. The slight kick given by the spot of light at once showed which way the block had to be moved, without its being necessary to keep the battery on long enough to heat the conductors sensibly.

The observing-room was kept at a very equable temperature by a screen before the window, also the wire of the balance was protected by a piece of boarding from the heat radiating from the observer's body.

After every observation the temperature of the coil and the water in the trough was read off, and if any difference was observed between these read-

ings and those first taken, the observation was rejected and another one taken.

Four observations were made on each wire at intervals of from twenty to forty minutes.

Before noting down the scale-reading all the connectors were moved, and if no change in resistance was observed the connexions were presumed to be good.

All results are given in terms of weight and length, as it is impossible to measure the diameter of a small wire with the accuracy with which the weight can be found; moreover, the cross section of a wire is not generally a circle, and the mean diameter varies slightly from point to point however carefully it may be drawn.

A great oversight was made in not observing the specific gravity of each wire, so that the results of the experiments now made could be compared with former ones. This omission was first made because it was thought that the results of former experiments could be used; but after several measurements had been made it was found that the values of the specific gravity of wires of the same metal, given by different observers, varied so much that it was impossible to find the resistance of a wire of a metal of which the length and sectional area are known, from the resistance of a wire of which the length and weight are known, without taking the specific gravity of the wire actually experimented on.

Silver.

Three silver wires were compared.

No. I. from commercially pure nitrate of silver.
No. II. from French coin.
No. III. from English coin.

The silver was first dissolved in nitric acid and then diluted with water and precipitated by hydrochloric acid. The chloride was then well washed, and afterwards fused with pure carbonate of sodium. The resulting button of silver was fused a second time with borax and a little nitrate of potassium; lastly, before casting, it was fused with a piece of charcoal floating on the top. The mould was about 35 millimetres long by 4½ millimetres diameter. The drawing of the wire was conducted with the utmost care. The wire was annealed only twice during the process.

In drawing all wires the end first entering the hole was reversed at each successive drawing, after it had been drawn down to about one half its required diameter. The wires were twice drawn through each of the smallest holes, the ends being reversed as before.

To measure the harder wires they were straightened by rolling them between two smooth boards, and then passed through a thermometer tube of such a length that the ends just projected from the tube, the long ones being cut into two or three lengths for the purpose. It was found that the wire could be pulled out of the tube and reinserted many times without altering the length by half one tenth of a millimetre. Some care was necessary in soldering the wires to their connexions. A small lump of hot solder was placed in the terminal, and the end of the wire steadily and slowly pushed into it until it set. Thus the boundary between the wire and solder was well defined, and the wire could be cut off at exactly the required point. The wires were weighed and measured after the resistance had been taken.

The care taken in drawing the silver wires accounts for the close agree-

ment of the results. Another wire was drawn as rapidly as possible through the latter holes to harden it, and a difference of 3½ per cent. was found in its conducting power.

The results are given in the following Table:—

Wire No. I.

Temperature of coil.	Reading of bridge-scale.	Temperature of wire.
21°·1	888	21°·3
21·2	888	21·3
21·4	890	21·4
21·3	891	21·4
Length 1·5906 m.	Zero-point at 514·25.	Weight 2·9208 grammes.

No. II.

18·8	194	19·3
19·0	199	19·4
19·3	204	19·5
19·4	206	19·6
Length 1·6749 m.	Zero-point at 514·25	Weight 3·4419 grammes.

No. III.

18·6	840	18·2
18·8	855	18·8
19·3	870	19·2
19·8	880	19·5
Length 1·3692 m.	Zero-point at 513·7	Weight 2·1572 grammes.

Resistance of metre-gramme wire No. I. 1·0000.
" " No. II. 0·9991.
" " No. III. 0·9986.

COPPER.

Three copper wires were tried. The copper employed was electrotype copper, and it was drawn without previous fusion. The copper of wires Nos. I. and II. was prepared by Messrs. De la Rue & Co., that of No. III. wire as follows:—Sulphate of copper was made by dissolving electrotype copper in pure sulphuric acid, and twice recrystallizing: the copper was obtained from the sulphate thus prepared by electrolysis; it was precipitated on a greased platinum pole, the other pole being of electrotype copper.

Wire No. I.

Temperature of coil.	Reading of bridge-scale.	Temperature of wire.
23°·4	244	21°·2
23·6	246	21·3
23·7	248	21·3
23·8	250	21·4
Length 1·9324 m.	Zero-point at 514.	Weight 3·9867 grammes.

No. II.

20·1	198	19·9
20·5	217	20·2
20·8	221	20·4
20·8	223	20·45
Length 1181·05 m.	Zero-point at 514.	Weight 1·4908 gramme.

No. III.

Temperature of coil.	Reading of bridge-scale.	Temperature of wire.
21°·6	565	20°·8
21·8	570	21
21·8	573	21
22·0	572·5	21
Length 1·6187 m.	Zero-point at 514.	Weight 2·7151 grammes.

Resistance of metre-gramme of wire No. I. 1·0000
" " No. II. 1·0005
" " No. III. 1·0011

Gold.

Three gold wires were tried.

No. I. from Australian gold.
No. II. from English coin.
No. III. from English coin.

The metal was first dissolved in nitro-hydrochloric acid, the excess of acid was then evaporated off, and the salt largely diluted with water to precipitate the chloride of silver. After filtering the gold was precipitated by sulphurous acid, the precipitate collected in a small beaker, and washed four times with hydrochloric and nitric acid alternately. After drying it was fused with borax and nitrate of potassium and cast. It was again fused, and finally cast in the mould.

Wire No. I.

Temperature of coil.	Reading of bridge-scale.	Temperature of wire.
20°·2	849	18°·8
20·4	849·8	18·8
20·4	851·5	18·9
20·8	852·5	18·9
Length 0·8854 m.	Zero-point at 515·2.	Weight 2·2200 grammes.

No. II.

21·6	634·5	20·2
21·6	638	20·3
21·6	638	20·3
21·8	638	20·3
Length 0·9998 m.	Zero-point at 515·2.	Weight 2·9021 grammes.

No. III.

19·8	782	19·2
20·1	788	19·4
20·5	784·6	19·6
20·8	797	19·8
Length 1·0211 m.	Zero-point at 515.	Weight 2·9753 grammes.

Resistance of metre-gramme of wire No. I. 1·0000
" " No. II. 0·9998
" " No. III. 0·9995

Lead.

With lead very good results were obtained. Five wires were determined.

The wires were pressed at a gentle heat, the press being carefully bored and cleaned beforehand. As the wire came from the press it was received on a smooth board. It was then at once soldered on to the connexions and placed in the trough. The solder employed was Wood's cadmium alloy. After being cut from the connectors the wire was straightened by rolling between two boards with great care; it was then placed on the board beneath the beam-compass, adjusted to the groove below the line of motion of the cross wires of the telescope, and carefully measured and then weighed.

Wire No. I. was cut from a bar of commercially pure lead, prepared by Mr. Baker of Sheffield.

Wire No. II. made from lead obtained by heating the acetate thrice recrystallized. This specimen was kindly prepared by Mr. Mathews.

Wire No. III. from the acetate of lead of commerce twice crystallized.

Wire No. IV. from the acetate of lead of commerce three times crystallized.

Wire No. V. from the seventh recrystallization of acetate of lead. Kindly prepared by Professor Atkinson.

Wire No. I.

Temperature of coil.	Reading of bridge-scale.	Temperature of wire.
18°·1	355	17°·5
18·2	362	17·8
18·3	362	17·6
18·4	362	17·6
Length 0·4907 m.	Zero-point at 514.	Weight 2·0689 grammes.

No. II.

16·4	855	17·1
16·4	867	17·5
18·0	869	17·6
18·1	869	17·6
Length 0·5100 m.	Zero-point at 514·5.	Weight 2·1320 grammes.

No. III.

17·0	746	16·1
17·0	748	16·2
17·2	748	16·2
17·4	748	16·3
Length 0·4910 m.	Zero-point at 516.	Weight 1·9883 gramme.

No. IV.

17·2	525	15·3
17·6	529	15·3
17·7	530	15·3
17·8	535	15·4
Length 488·2 m.	Zero-point at 515·2.	Weight 1·9991 gramme.

No. V.

18·8	628	17·8
18·8	628	17·8
19·1	634	18·0
19·6	640	18·2
Length 0·4915 m.	Zero-point at 515·5.	Weight 2·0253 grammes.

Resistance of metre-gramme of wire No.		I. 1·00000
„	„	II. 1·00045
„	„	III. 1·00029
„	„	IV. 1·00054
„	„	V. 1·00026

Gold-silver alloy.

No. I. Part of the alloy formerly prepared for the experiments described in 'Phil. Mag.' Feb. 1861, and there described as wire No. I.

No. II. Part of No. VII. there described.

No. III. Part of No. VIII. there described.

No. IV. From the first three alloys mixed and refused and drawn.

No. V. Alloy reprepared from the pure metals.

Wire No. I.

Temperature of coil.	Reading of bridge-scale.	Temperature of wire.
17°·8	816·0	17°·8
18·4	821·6	18·2
19·8	831·5	20·2
20·0	845·0	21·2
Length 0·5374 m.	Zero-point at 517·6.	Weight 1·8607 gramme.

No. II.

18·4	481·0	18·6
18·8	482·4	18·8
19·6	486·0	19·4
20·0	497·6	21·3
Length 0·4263 m.	Zero-point at 517·3.	Weight 1·2082 gramme.

No. III.

19·2	594·0	17·8
20·0	600·2	19·2
18·6	595·4	18·2
19·0	596·0	18·5
Length 0·3709 m.	Zero-point at 517·4.	Weight 0·9052 gramme.

No. IV.

18·8	870	17·8
19·0	870	17·9
19·1	869	18·0
19·3	869	18·0
Length 0·5472 m.	Zero-point at 514·5.	Weight 1·9199 gramme.

No. V.

18·8	542·0	17·6
19·0	541·6	17·7
19·2	541·4	17·6
19·2	541·8	17·7
Length 0·6333 m.	Zero-point at 515.	Weight 2·6497 grammes.

Results for Gold-silver alloy.

Resistance of metre-gramme of wire	No. I. 1·00000
„ „	No. II. 0·99963
„ „	No. III. 1·00017
„ „	No. IV. 1·00036
„ „	No. V. 0·99996

Mercury.

Three tubes were filled with mercury and their resistance taken.

Tubes Nos. I. and II. with distilled mercury treated with nitric and sulphuric acid.

Tube No. III. with mercury distilledf rom a specimen which contained a small quantity of gold.

The lengths of the column are given below in their order.

Tube I.	Tube II.	Tube III.
mm.	mm.	mm.
383	291	245
384	288	242
390	289	240
386	287	240
389	288	242
384½	288	243
381	291	243
377	290	244
384½	292	246
392	288	246
399	288	248
405	289	248
407	288	252
407	288	253
406	288	254
413	290½	254
418	292	257
424	293	228
416·5	295	260
416	297	262
414		265
405		267
405		
$\log \frac{C'}{C}$	$\log \frac{C'}{C}$	$\log \frac{C'}{C}$
$=\bar{1}\cdot9995018$.	$=\bar{1}\cdot9998710$.	$=\bar{1}\cdot9995614$.

Several other kinds of mercury were tried in one and the same tube, and the resistances found to be the same within two or three hundredths per cent.

Sixteen tubes were obtained, picked from a great number, and of these the three best ones were taken. No. I. was not so good a tube as the others, as the outside was uneven, rendering it impossible to calibrate it with very great accuracy.

To calibrate a tube it was taken and carefully cleaned with pure nitric acid, and then with a solution of caustic potash. It was then well rinsed with distilled water, and dried by passing a current of hot air through a chloride-of-calcium bulb and then through the tube. A small column of mercury was put in the tube, and the length of column measured by the beam-compass. The column was shifted along the tube by sucking up or blowing through an india-rubber tube with a chloride-of-calcium tube inserted between it and the tube to be calibrated. By this arrangement the column could be adjusted with the greatest nicety to the place in the tube required. The lengths of the column were taken at equal intervals from one end of the tube to the other. The formula for correction used is given below.

Let C be conducting power of a tube of uniform bore and of length, capacity equal to that of tube considered; C′ observed conducting power. Then

$$C' = C\frac{n^2}{\Sigma\lambda\,\Sigma\frac{1}{\lambda}},$$

where n is the number of measurements made, λ the length of the column of mercury in any position, the summation extending to all the readings taken.

The ends of the tubes were ground by putting some emery powder and naptha on a slate table, holding the tube vertically upright with the left hand, and with the right hand rubbing the end of the tube in contact with the table round the circumference of a small circle. Thus the end of the tube was made slightly convex, the opening being at the apex of the convexity. To measure the tubes they were placed under the beam-compass, and a stout pin inserted partially into each end.

From the shape of the ends of the tube, the point where the pin emerged from the tube could be exactly seen and the measurement made with certainty. Many measurements were made turning the tube round its axis through a small angle before each measurement, and the mean of the lengths found taken for the true length. To find the weight of the tube full of mercury it was carefully cleaned, filled with mercury, and placed in a long narrow trough full of pure mercury. The tube was held down by iron weights, a thermometer inserted in the trough, and the apparatus allowed to stand until the temperature was constant. After the true temperature had been obtained the tube was taken out of the trough and the contents weighed.

This was managed in the following manner. One operator took hold of the tube by pressing a finger against each end and lifting from the trough; the little globules adhering to the outside of the tube were then rapidly removed by two assistants with brushes.

The mercury was then allowed to flow slowly out into a small porcelain crucible and weighed. In this way pretty consistent results were obtained if the tubes were cleaned before each filling.

To determine the resistance of the tubes they were placed in the water trough, with bent pieces of tubing fastened on to the ends with india-rubber tubing and reaching above the surface of the water.

The terminals were of copper, well amalgamated. They dipped into the bent tubes and came flat against the ends of the tubes the resistance of which was to be determined. In the calculation of the weight of mercury at 0° in the tube from the observed weight, Regnault's value for the expansion was used.

Connexions of amalgamated platinum were first used, but did not give good results. It was found that the amalgamation was imperfect. The mercury adhering to the platinum was rubbed off against the ends of the tube, and the resistance varied with the height of the mercury in the bent tubes. The platinum was amalgamated by dipping it into a mixture of mercury and sodium amalgam. The sodium was then oxidized and dissolved off by dipping the platinum in a little dish of water and hydrochloric acid. The terminal was then drawn through a dish of clean mercury, so that the water floated off. The platinum was then for the time beautifully amalgamated; but the mercury soon drained off when the plate was exposed to the air, and could be easily rubbed off even when the platinum was immersed in mercury.

Tube No. I.

Temperature of coil.		Reading of bridge-scale.	Temperature of wire.
18°·1		346·0	18°·2
18·4		349·2	18·6
18·4		349·2	18·8
18·8		348·0	18·0
	grms.	Zero-point 515	Length 0·9365 m
Wt., temp. 19°·6	24·7021		
„ 21·0	24·6930		
„ 20·8	24·6950		
Wts. reduced to 21°:—			
	24·6958		
	24·6930		
	24·6940		

Tube No. II.

19°·6		188·0	19°·0
19·9		186·8	19·05
20·0		186·5	19·1
20·0		186·5	19·1
	grms.		Length 0·6563 m.
Wt. at 21°·2	12·3140		
„ 21·6	12·3132		

Tube No. III.

19°·0		633·5	18°·9
19·0		633·1	18·8
19·1		631·7	18·8
19·15		631·3	18·8
	grms.		Length 0·5497 m.
Wt. at 22°·2	8·2894		
„ 22·2	8·2836		

Results.

Resistance of tube	No. I.	1·00000
„ „	No. II.	0·99849
„ „	No. III.	1·00000

An approximate table is subjoined of the resistances of a metre-gramme of the different metals in terms of the B. A. unit, 1864 :—

Copper	0·1469
Silver	0·1682
Gold	0·4150
Gold-silver alloy	1·668
Lead	2·257
Mercury	13·06

From the foregoing results we may draw these conclusions ;—

That with great care a unit may be reproduced with great accuracy by any of the metals or alloys above mentioned.

Of those tested it appears that lead is the most preferable on account of its easy purification, and because the presence of impurities, amounting to several per cent., produce no very disproportionate effect on its conducting power. For instance—

Conducting power of lead is	7·77
Of lead with 12·7 per cent. volume of tin is	8·13
„ 10·6 „ „ cadmium	8·38
„ 2·3 „ „ bismuth	7·0
„ 3·8 „ „ antimony	7·1
„ 2·3 „ „ silver	7·9

With the other metals and alloys tested a much greater difference is found in the conducting power when such impurity exists.

A few examples will show this.

The conducting power of pure copper is	100
Of copper with 1·6 per cent. in volume of silver	65
Of silver with 1·2 „ „ gold	59
Pure silver being taken as 100.	
Of gold with 1·2 per cent. of silver	73·8
The conducting power of pure gold being 78.	
If the conducting power of mercury is	10·9
That of mercury with 1·18 per cent. volume of lead is	11·5
„ 1·8 „ „ tin	11·8
„ 1·8 „ „ zinc	12·4
„ 0·7 „ „ gold	11·6
„ 1·2 „ „ silver	11·6

The manipulation with lead is rendered easier by its high resistance.

Mercury is easily purified, and good results are always obtained with it. It would, however, in reproducing a unit, be necessary to distill the mercury, because traces of such impurities as silver and gold, which may easily get into it when in use in a laboratory, cannot be removed by treat-

ment with nitric acid. The chief labour is in selecting and calibrating the tubes, and this is very great.

The results obtained with the gold-silver alloy, even when prepared by different persons, supposing great care used, give very accordant results; and for the easiness with which it can be made it may be recommended for producing a unit.

Electrotype copper would appear a good substance. The agreement of results obtained with commercial electrotype copper with those obtained with copper prepared from pure salts shows this.

The maximum difference of the conducting powers of electrotype copper, as observed with ordinary care, is 1·6 per cent. Copper is not, however, to be preferred, as great care and some practice is necessary to draw a good wire.

The purification and drawing of pure gold and silver would, in the hands of any one but a chemist, lead to no good results, in all probability. These facts being considered, we should prefer lead for the reproduction of a unit. No doubt it would be well to use two or three substances to check the results. For these auxiliary substances mercury and the gold-silver alloy may be recommended. The choice between these two will depend on the appliances of the individual observer. When thorough care is taken all the above means are equally good.

On forming an opinion on the difficulty of reproducing a unit by chemical means it must be remembered that if any thing like accuracy is wished for, not only expensive and delicate apparatus is required, but also very much time must be spent, and a great deal of experience in the manipulation is required. The experiments here described extended over about six months. Any person wishing to reproduce a unit should bear these considerations in mind, especially as it is the intention of your Committee to cause coils to be issued representing a known resistance. That copies of a given resistance can be made to a much greater accuracy than that to be obtained by chemical or other known means of reproduction, and that coils can be compared by different observers with different apparatus to almost any degree of accuracy (although this fact has been brought into question by a former experimenter), is proved by the following facts.

The two units which have come into our hands, made by Messrs. Siemens and Halske from copies of the coil used last year by your Committee for the determination of the absolute unit, were compared against the standard coil and found to agree with it within two-hundredths per cent. Again, copies of Weber's unit, one made of the gold-silver alloy, the other of German silver, were compared at the interval of two years by different observers with different apparatus, and the results found to agree to one half a hundreth per cent.

It is from the fact that copies can be produced with almost absolute accuracy, with a minimum of cost and labour as compared with chemical or mechanical means of reproduction, that we seem quite justified in recommending all who wish to obtain a standard to procure a copy of the British-Association unit, or any other in general use. As copies of the British-Association unit are being sold at a reasonable price by several of the leading instrument makers, which, we are given to understand, will agree together very closely, we confidently recommend the adoption of this unit.

And, in conclusion, we still adhere to the opinion, given in Appendix C of the Report of 1862, that the best means of reproducing a unit, for those who have not the opportunity of procuring a copy, and who cannot afford the

time and expense necessary to reproduce the unit with great care, is to procure a given length and weight of the gold-silver alloy, such as shall have been found equal to the unit adopted (the quantity required being very nearly 0·5995 of a metre of a wire, one metre in length of which would weigh a gramme) for the British-Association unit.

FOURTH REPORT—BIRMINGHAM, SEPTEMBER 6, 1865.

MEMBERS OF THE COMMITTEE:—Professor Williamson, Professor Wheatstone, Professor W. Thomson, Professor Miller, Dr. A. Matthiessen, Mr. Fleeming Jenkin, Sir Charles Bright, Professor Maxwell, Mr. C. W. Siemens, Mr. Balfour Stewart, Dr. Joule, and Mr. C. F. Varley.

THE Committee has the pleasure of reporting that the object for which they were first appointed has now been accomplished.

The unit of electrical resistance has been chosen and determined by fresh experiments; the standards have been prepared, and copies of these standards have been made with the same care as was employed in adjusting the standards themselves; seventeen of these copies have been given away, and sixteen have been sold.

The chief work of the Committee this year has been done by Dr. A. Matthiessen. Last year's Report announced the completion of the experiments determining the resistance in absolute measure of a certain coil of German-silver wire. Taking this coil as the basis, Dr. Matthiessen, assisted by Mr. C. Hockin, prepared ten standards, each expressing the British-Association unit of electrical resistance; two of these standards are coils of platinum wire, two are of platinum-silver alloy, two are coils of wire drawn from a gold-silver alloy, two are coils of wire drawn from a platinum-iridium alloy, and the remaining two are tubes of mercury.

The wires employed in the coils are from 0·5 millim. to 0·8 millim. diameter, and range from one to two metres in length. They are insulated with white silk, and are wound round a long hollow bobbin of brass. The wires are imbedded in solid paraffin, and enclosed in a thin brass case, which allows the coils to be plunged in a bath of water by which their temperature may be conveniently regulated and observed. Two short copper terminals project from the case and are forked at their ends, so that they may be connected with the Wheatstone's balance in the manner recommended by Professor W. Thomson, avoiding the error due to the possible resistance of connexions. The mercury standards consist of two glass tubes about three-quarters of a metre in length.

These ten standards are equal to one another and to the British-Association unit, at some temperature stated on the coil or tube, and lying between 14°·5 and 16°·5 C.

None of them, when correct, differ more than 0·03 per cent. from their value at 15°·5 C.

In the choice of the material of which the standards are constructed, the

Committee have been much assisted by the experiments on permanency made by Dr. Matthiessen.

Silver and copper were found to alter in their resistance simply by age. German silver was also found to alter in some cases.

These materials had therefore to be rejected. Gold appears constant; but owing to its low specific resistance a considerable length would have been required, unless a wire had been adopted of very small diameter. This was not thought desirable, for several reasons: any slight decay or injury in the surface of a small wire would cause much greater alteration in the resistance than the same injury to a large wire; a small wire would be more liable to mechanical injury, and would be much more rapidly heated by the passage of currents. The Committee having rejected small wires for these reasons, thought it unnecessary to incur the expense of a large and thick gold wire. The great change of resistance caused by a change of temperature furnished another reason for rejecting gold and other pure metals. One pair of standards, however, was made of platinum, which appeared the most suitable of all the pure metals. Platinum and the three alloys named appear all to be very constant—that is to say, their resistance is not altered by age, or even by being subjected to considerable heat and recooled.

These materials also possess considerable mechanical strength; they are not easily injured by chemical action, they have considerable specific resistance, and that resistance, in the case of the three alloys, changes little with a change of temperature.

It is of course impossible to say with certainty that their resistance will not vary with time; but it is most unlikely that the resistance of all will vary in the same ratio. If, therefore, as is hoped, the eight coils made of such different materials retain their relative values, some confidence may be felt in the permanence of the unit.

Some additional security is given by the power of reproducing the unit, if lost, by chemical means, or by fresh experiments on absolute electro-magnetic measure, although neither of these means at present appear to give such perfect accuracy as would be secured by the permanency of a material standard. Fresh absolute experiments of the kind described in previous Reports would hardly reproduce the same value much within one part in a thousand; and Dr. Matthiessen, as appears from last year's Report, is not very sanguine of obtaining a better result than this by chemical means. Thus a difference exists in Dr. Siemens's and Dr. Matthiessen's reproduction of a unit by means of mercury, as pointed out in last year's Report. It is of course probable that differences of this kind will in time disappear; and Mr. Siemens fairly points out that the discrepancy mentioned in last year's Report, between coils made from a very old and those made from a new determination of the mercury unit, affords no criterion of the accuracy with which mercury can now be used as a means of reproduction. Dr. Siemens was the first person who produced numerous sets of coils accurately adjusted; and although unable to recommend the adoption of his unit of resistance, the Committee once more take an opportunity of expressing their sense of the high value of Dr. Siemens's researches on the reproduction of units by means of mercury. Dr. Siemens is confident that a unit can be and has been reproduced by means of mercury with an accuracy of 0·05 per cent.; but, meanwhile, the chief security for the permanency of the unit consists in the preservation of standards constructed in various ways and of various materials.

The mercury tubes furnish an additional security. A molecular change may occur in the wires, that is to say, they may become of harder or softer

temper; they may be injured chemically in course of time by some action on their surface; it is just possible that the repeated passage of currents may alter them in some way, although we have no reason as yet to expect such an alteration.

Mercury is free from all these objections. Its temper cannot vary, and as it would be purified afresh on each occasion, it will be chemically uninjured.

On the other hand, some fresh dangers may occur in its use. The tubes themselves may alter in time, or the mercury may not always be absolutely pure. Absolute security cannot be had; but the choice of a variety of materials will probably prevent any serious alteration from occurring without detection.

The copies which have been issued are similar in form to the standard coils; but the terminals are simple thick copper rods, intended to be dipped in mercury cups. The security given by this mode of connexion is sufficient for all ordinary purposes, and it was feared that the use of the double terminals might not be everywhere understood. The platinum-silver alloy has been used in all the copies. Wire made of this alloy is very strong and ductile. It can, for instance, be drawn down to a diameter of 0·0002 inch. Its resistance is not permanently altered even by a great change of temperature, and even annealing hardly affects it. Moreover, the change in its resistance due to a variation of 1° Centigrade is at ordinary temperature only 0·032 per cent., being less than that of any other alloy tested. It is also a commercial alloy, which has been long used by dentists; and Dr. Matthiessen points out, as a curious coincidence, that many commercial alloys coincide with proportions indicating peculiar electrical properties. *Vide* Appendix A.

The copies of the standard have been supplied for £2 : 10*s*. in boxes, with small mercury cups for the connexion, and with a printed direction for use inside the box, stating the temperature at which that particular coil is equal to 1 B.A. unit.

A satisfactory proof of the accuracy with which these coils have been prepared was given by four independent observations, by practical electricians not belonging to the Committee, of the relative value of four distinct B.A. coils and four independent standards issued by Dr. Siemens.

These four observations gave 10456, 10455, 10456, and 10457 as the measures of Siemens's standard, in terms of the B.A. units, proving the accuracy both of Dr. Siemens's work and that of the Committee.

Twenty coils were to be distributed gratis, and seventeen have actually been given away to the following recipients:—

The Directors of Public Telegraphs in

France.	Spain.	Prussia.
Austria.	Italy.	Sweden and Norway.
Belgium.	Portugal.	Russia.
India.	Victoria.	
Queensland.	New South Wales.	

Also to Professor Kirchhoff, Dr. Joule, Professor Neumann, and Professor Weber.

Three remain for distribution. Sixteen have been sold. Dr. Faraday, on behalf of the Royal Institution, was the first purchaser.

In distributing the coils, it was thought best not to give them to institutions, where they would probably have laid on a shelf useless and unknown, but rather to distribute them widely, where they might become available to practical electricians.

The new unit has been actually employed to express the tests of the Atlantic Telegraph Cable. Mr. Varley promises that the unit shall in future be the basis of the coils used by the Electric and International Company.

Sir Charles Bright promises that the unit shall be exclusively used by the British and Irish Magnetic Telegraph Company.

A standard has been supplied to the Royal Engineers at their request. The head of the Telegraph Department in India has introduced the unit, and there is little doubt that the British Colonies generally will adopt it.

More time will certainly be required to introduce it on the Continent. The French Government has taken no steps to insure its introduction; but M. Blavier, the official editor of the 'Annales Télégraphiques,' has promised his cordial support to the Committee. The Austrian Government has promised to use the coils experimentally, and the German gentlemen to whom coils were given have promised to give their best assistance.

Coils have also been bought by the managers of two large telegraphic establishments in Switzerland, at Neuchatel and Zurich. There is therefore reason to hope that the unit may come into extensive use.

When standard galvanometers, Leyden jars, and electrometers are issued, all forming part of one coherent and necessary system, it is probable that the B.A. unit will be found so much more useful than any other as to supplant them entirely. Until these further issues take place, it will only be adopted either by men who can understand the advantage given by it in calculation, or by electricians who feel confidence in the recommendations of your Committee.

With a view to experiments which will allow of these further issues of electrical units, a large electrodynamometer has been designed and is nearly complete. Graduated Leyden jars, with air as the only dielectric, have also been designed and are nearly ready for use. An apparatus for the determination of the quantity called v in Appendix C of the 1863 Report is in the same condition. Prof. W. Thomson has for some time had ready apparatus for absolute measurements of electrical effects, but his connexion with the Atlantic Cable has suspended his work. Dr. Joule promises fresh measurements of the mechanical coefficient of heat, and has only been delayed by the want of experiments which other members of the Committee must previously complete.

In conclusion, the Committee are at last able to report one positive result, but they feel that much more remains to be done.

Appendix A.—*On the Construction of the Copies of the B.A. Unit. By* A. Matthiessen, *F.R.S., and* Mr. Charles Hockin.

The standard coil used in the experiments at King's College, described in the Report of your Committee for 1864, was put into our hands about last Christmas, in order that unit-coils representing a resistance equal to ten million metres per second in Weber's electro-magnetic system might be made from it.

Since that time several unit-coils have been made and issued.

We propose to state the method by which these coils were made, and the reasons for choosing the particular alloy which has been adopted for the conductor. The alloy referred to is composed of 66 per cent. of silver and 33 of platinum.

This alloy possesses many properties which fit it for the use to which it has been put.

As to its electrical properties:—

I. It alters less in electrical resistance with changes of temperature than any other known alloy.

The importance of this point needs hardly to be enforced on any one who has used resistance-coils.

The increment in the resistance of the alloy due to a change of temperature from 0° to 100° C. is only 3·2 per cent.

II. The conducting power of the alloy is very low, and is about one half that of German silver.

III. The conducting power of the alloy is not altered by baking, that is by exposing it to a temperature of about 100° C. for several days.

This is a property of great importance, for it has been observed that those conductors which do not alter by baking, do not alter by age either. The experiments by which this has been established have been published in former Reports.

IV. The conducting power of a wire of the alloy is little altered by annealing.

Further, the alloy does not oxidize by exposure to the air; it does not readily alloy with mercury; it makes a sufficiently pliable wire, and can be drawn to a very great degree of fineness. Dentists have made considerable use of it in consequence of its good chemical and mechanical properties*. Of this alloy twenty unit-coils have been made and sent to several leading electricians at home and abroad. The form of bobbin adopted for putting up the wire, and shown in Plate IV. fig. 1, has been found very convenient, as it can be immersed in water during an observation. The wire is twice coated with silk, and protected by being imbedded in solid paraffin.

Besides the coils already mentioned, ten unit-coils have been made, which will be deposited at the Kew Observatory.

Any one possessing a copy of the B.A. unit may have it compared at any future time against one of these coils for a small payment.

Of the coils to be sent to Kew, two are of the platinum-silver alloy, two of the gold-silver alloy, two of a platinum-iridium alloy, and two of commercially pure platinum. Two mercury units have also been prepared.

With so many coils for reference, made of such different metals, it appears quite improbable that the unit now proposed should be lost.

Along with the above-mentioned coils will be preserved the standard coil used in the experiments first referred to, the coil used in the similar experiments made by your Committee in 1863, and several copies of these coils.

Of the coil called "June 4th" in the Report of your Committee for 1863, two German-silver copies have been made. Of the other coil used in 1864, two German-silver, two gold-silver, and one platinum-silver copy have been made.

These coils have twice been recompared together at intervals of three months, and will be again compared; and if they are still found not to have altered, they will be deposited at the Kew Observatory for reference, their values being engraved on them.

The method adopted to obtain the unit from the standard which had at a certain temperature a resistance of 4·6677 B.A. units was this:—

Coils were made with the following approximate resistances, viz.:

Two coils nearly equal to	$\frac{1}{2}$ unit,	called	$\frac{1}{2}$ *a*	and $\frac{1}{2}$ *b*.
,, ,,	1 unit,	,,	1 *a*	,, 1 *b*.
One coil ,,	2 units,	,,	2.	
,, ,,	$2\frac{1}{2}$ units,	,,	$2\frac{1}{2}$.	

* *Messrs. Johnson* and Matthey inform us that this alloy has been in use for nearly *twenty years.*

The electrical balance used was that described in a paper on the reproduction of a unit by chemical means, in the Report of your Committee for 1864.

With this instrument two conductors, differing in resistance by not more than 3 per cent., could be directly compared, and the ratio found depended on to 0·0025 per cent.

Numerous comparisons were made by means of this balance between the following sets of coils, viz.:—

$\frac{1}{2}a$	was compared	with	$\frac{1}{2}b$.
$\frac{1}{2}a+\frac{1}{2}b$	„	„	$1a$.
$1a$	„	„	$1b$.
$1a+1b$	„	„	2.
$2+\frac{1}{2}a$	„	„	$2\frac{1}{2}$.
$2+2\frac{1}{2}$	„	„	standard.

By taking the mean of several very concordant observations, the value of the coil $1a$ was found in terms of the standard, and therefore of the unit, to a great degree of accuracy; and from this coil the first platinum-silver unit was constructed.

All the coils to be issued are recompared some weeks after they are made, and rejected if they are found to have altered in resistance by 0·01 per cent.

All the coils sent out are correct at the temperature written on them to *within* 0·01 per cent., and this temperature lies between 14·5 and 16·5 in all cases.

FIFTH REPORT—DUNDEE, SEPTEMBER 4, 1867.

Members of the Committee:—Professor Williamson, Professor Sir C. Wheatstone, Professor Sir W. Thomson, Professor Miller, Dr. A. Matthiessen, Mr. Fleeming Jenkin, Sir Charles Bright, Professor Maxwell, Mr. C. W. Siemens, Mr. Balfour Stewart, Mr. C. F. Varley, Professor G. C. Foster, Mr. Latimer Clark, Mr. D. Forbes, Mr. Charles Hockin, and Dr. Joule.

The Committee have much pleasure in reporting that during the past year considerable progress has been made, and that the principal instruments required by the Committee for experiments have been completed and are in use.

Experiments have been conducted by Dr. Joule, having for their object the determination of the mechanical equivalent of heat, by observing the heat generated in part of a voltaic circuit, the resistance of which was measured in absolute units by means of the standard of resistance issued by the Committee.

Last year preliminary experiments of this kind had been made by Dr. Joule, and the agreement which he then reported between his mechanical equivalent obtained by frictional experiments and that obtained by the electrical method was so close as to lead to a suspicion that it was partly fortuitous.

The experiments, which have this year been conducted with every possible care, give 783 as the value derived from the B.A. standard of resistance, while 772 is the well-known number derived from friction.

The details of the experiments are contained in an Appendix which accompanies this Report. Dr. Joule states his opinion that the electrical method has been carried out with greater accuracy than the frictional method, assuming the B.A. standard to be an exact decimal multiple of the absolute unit. The following extract from Dr. Joule's Report will show the laborious nature of the experiments. He says, "The last and most perfect series of experiments comprise thirty for the thermal effect of currents in the spiral, thirty for the effect of radiation &c., and thirty for the horizontal intensity of the earth's magnetism." Dr. Joule expresses himself willing to make a new determination by friction. Meanwhile the experiments already completed remove all fear of any serious error, either in the number hitherto used as "Joule's equivalent" or in the B.A. standard—a fear which hitherto, remembering the very discrepant results obtained by others, has been very naturally entertained even by the Subcommittee from whose experiments the standard was constructed.

In connexion with the measurement of resistances, Mr. C. W. Siemens has invented a simple and excellent contrivance, by which the measurement of resistances can be made by persons wholly unaccustomed to electrical experiments. They have only, after the necessary connexions are made, to turn a screw till a needle stands opposite a fiducial mark, when the resistance required may be read directly on a scale with considerable accuracy. Mr. Siemens proposes to apply this invention to pyrometers, where the resistance read will indicate the temperature, and the only electrical connexions required will be joining of the battery wires to two terminals. Other applications of this invention will doubtless arise, and extend the practical application of electrical measurements. A full description of the instrument is contained in the Appendix. Mr. Siemens reports very favourably of this instrument, which possesses considerable advantage in cheapness and portability. Mr. Siemens has constructed the instrument, and made the experiments entirely at his own expense.

An instrument similar in object, and suggested by the above, is also described by Mr. Jenkin in an Appendix.

Mr. Hockin has tested the constancy of the standard resistance-units, with satisfactory results, except in the case of one mercury tube. The exact result of Mr. Hockin's comparisons are appended. He suggests that lead-glass was used for the mercury tube, and that the glass may consequently have been injured by the nitric acid used to clean it.

Mr. Hockin has also made interesting experiments on the construction of large resistances by the use of selenium. He finds that resistances of one million units and upwards can be made of this material, and that these artificial resistances maintain a sensibly constant resistance at high temperatures, such as 100° C. It is hoped that these very high artificial resistances will be found useful in practice and much superior to those hitherto constructed of gutta percha or other insulators, which were of comparatively little use in accurate work, owing to absorption, change of resistance with temperature, and inconstancy when kept for any considerable time. These valuable experiments have not caused any expense to the Association.

The determination of a unit of capacity has occupied Dr. Matthiessen, Mr. Hockin, Mr. Foster, and Mr. Jenkin during the last two years.

Very considerable difficulties have been encountered, and are not yet wholly

overcome. The methods by which both the electrostatic and electromagnetic units can be determined, and multiples or submultiples prepared, are sufficiently simple in theory; but they assume that the condensers or Leyden jars compared have really a definite capacity, and that with a given electromotive force between the induction surfaces a definite quantity of electricity will be contained in the jar or condenser. This is very far from true with condensers of ordinary form. Whether the dielectric separating the plates be glass, mica, gutta percha, paraffin, ebonite, or any other known solid insulator, an absorption of electricity takes place; the longer the plates are charged, the more electricity the condenser will contain, and, conversely, it will continue to discharge itself for a very long period after the inner and outer armatures have been joined. With some of the best insulators the effect will continue for hours, if not for days. Condensers made with these solid dielectrics have therefore no definite measurable capacity. This capacity will differ according to the time during which they have been charged; and it may also vary with extreme variation in the electromotive forces employed, although this latter change has not been detected when the differences of potential are such as between one Daniell's cell and two hundred.

Only gaseous dielectrics appear free from this embarrassing peculiarity, called absorption, polarization, or residual charge. One object of the Subcommittee has therefore been to construct condensers in which air alone separates the induction-plates. But new difficulties arose in carrying this idea into practice. Some support for each plate was necessary, and then leakage occurred from one plate to another over the surface of any small insulating supports employed, such as glass balls or vulcanite stems. It was possible, by great care in drying the air, occasionally to make condensers of this type, which would remain insulated for a short time, or even for some months; but long experience has shown that an artificially dried atmosphere cannot be conveniently maintained in any instrument which is not hermetically sealed.

Dust also accumulated between the plates of the trial condensers; this altered their capacity and increased the leakage from plate to plate. Even a single filament of dust, by springing up and down between the two electrified surfaces, would occasionally bring them to the same potential with great rapidity, neutralizing the charge; moreover a condenser of this type could not be taken to pieces and cleaned, for no mechanical contrivances could ensure that the parts after cleaning would return to their original position so exactly as to constitute a condenser of the same capacity before and after the cleaning. It is therefore clear that an air-condenser can only be constructed in an hermetically sealed case, containing an artificially dried atmosphere; and even with these conditions, excluding the graduated and adjustable condensers which were first tried, the air-condenser is not easily constructed. For large capacities, which are alone useful in connexion with practical telegraphy, the plates require to be so numerous and large as to make the expense great and the bulk very inconvenient.

It is hoped by the use of tin plates soldered to metal rods, and supported on insulated stems inside a soldered metal case, that these objections may be partly avoided; but meanwhile practical men have introduced condensers of a more convenient form, overlooking the disadvantage which they all possess of ill-defined capacity.

These condensers consist of sheets of tinfoil separated by paraffin and paper, a preparation of gutta percha, or mica—three plans adopted by Mr. Varley, Mr. Willoughby Smith, and Mr. Latimer Clark respectively.

Condensers of this type have been made approximately equal to a knot

of some submarine cable; and the rough units thus introduced are gradually creeping into use, although all electricians have been anxious that the Committee should issue a more scientific standard. Under these circumstances, Mr. Jenkin has adjusted a mica-condenser, approximately equal to 10^{-14} absolute electromagnetic units. The capacity of this condenser is assumed as that which it possesses after electrification for one minute, and is measured by the discharge through a galvanometer, in the manner usually practised when testing the charge of a submarine cable. The formula for obtaining the measurement in absolute units from the throw of the needle is very simple, requiring only observations of the time of oscillation, of a resistance in absolute measure, and of a deflection of the galvanometer-needle. All of these observations can readily be made, so that their accumulated error cannot exceed one per cent.; and for the present purpose this accuracy is sufficient, inasmuch as, when using the condenser, small variations inevitably occur, arising from the residual discharge. While, therefore, the new provisional unit of capacity has no claim to a high scientific accuracy, it will supply a practical want and introduce a unit based on the principles adopted by the Committee, in place of the random measures supplied by a knot of Persian-Gulf or Atlantic cable.

No decision has yet been arrived at whether the new unit shall be issued by the Committee or on Mr. Jenkin's own responsibility, nor has the price been fixed.

The experiments by which it has been obtained are given in an Appendix.

The practical applications of the standard of capacity are important. It will allow the capacity of submarine cables to be universally expressed in comparable figures, and may lead to improvement by the diminution of the specific inductive capacity of the insulator, precisely as the introduction of units of resistance has assisted the improvement in insulation and conductivity.

The electromagnetic capacity standard will also, by comparison with the electrostatic standard about to be made, furnish one mode of determining the constant called v in previous Reports, a number of much importance in the theory of electricity.

The next unit or standard for consideration is that of the difference of potentials or electromotive force in absolute measure, concerning which the experiments have been wholly in Sir William Thomson's hands. He reports that he has at last succeeded in constructing a series of electrometers capable of measuring differences of potential ranging from $\frac{1}{400}$ of a Daniell's cell up to 100,000 cells, and that these measurements can all be reduced to absolute units by comparison with one instrument of the series.

This class of instruments has been created by Sir William Thomson, who year by year has produced electrometers each surpassing its predecessor, both in accuracy and delicacy; but although those who have had practical experience of the admirable results obtained by these have for the last two or three years believed that the limit of excellence has been reached, Sir William Thomson has not ceased to invent better and simpler forms, until the instruments now supplied surpass every expectation of practical electricians and furnish, indeed, a new engine for electrical research.

The chief difficulties encountered have been the insulation of the Leyden jar, which has formed an essential part of all the contrivances, its maintenance at a constant potential, and the reduction to absolute measurement. In the present instrument absolutely perfect insulation is no longer required; for by a new device for converting mechanical force into statical electricity

(first constructed by Mr. Varley in 1859) Sir William Thomson is able at any moment to replenish the jar by a few turns of a handle, and, by a gauge electrometer, he can insure that the same charge is constantly maintained in the instrument. The difficulty of the reduction to absolute units consists in the difficulty of comparing the extremely small forces produced by electrostatic attraction with the force of gravitation, and in the accurate measurement of the extremely small distances which separate the attracting surfaces. Sir William Thomson reports that these difficulties have been overcome in his opinion, and that he will be shortly in a position to construct and issue a simple pattern of an absolute electrometer or gauge of potential which will serve as a standard for general use.

Further experiments and tests are, however, required before this can be done, as any precipitation would only injure the interests of the Committee. It is right here to mention that the above experiments have been carried out almost entirely at the expense of Sir William Thomson.

The replenisher, which is founded on the principle of the electrophorus, may very possibly supersede the old form of electrical machine entirely; it has some analogy with the electromagnetic machines lately invented by Mr. C. W. Siemens and Professor Wheatstone, by which intense dynamic effects are evolved from the smallest initial trace of magnetism by the conversion of mechanical force into electric currents, and was, indeed, suggested by this invention to Sir William Thomson, who reinvented the plan patented by Mr. Varley*.

A modification of the same contrivance will allow the comparison of extremely minute quantities of electricity, such, indeed, as might be accumulated on a pin's head; by a series of rapid inductions a charge is accumulated on the electrode of an electrometer, which may be made equal in potential to that on the pin's head, but infinitely exceeding it in quantity; the effect of this charge in the electrometer can then be observed without difficulty, and any increase or diminution in the quantity of electricity on the pin's head or proof plane can be detected and the rate of loss or increase observed. The potentials to which various small bodies are charged can also be observed by the same method, the advantage of which consists in the fact that the original charge on the body tested is undisturbed by the test, whereas by any of the older tests the charge was altered by being touched by a proof plane or by the electrode of the electrometer.

A similar plan has already been proposed by Mr. Varley and Sir William Thomson, with a water-dropping arrangement, but the mechanical contrivance is in all ways preferable. No expense has been incurred by the Committee for these instruments or experiments.

Passing to the unit of current, the Committee regret that no experiments have yet been made with the large absolute electrodynamometer constructed with the funds granted by the Royal Society. Much difficulty has been experienced in finding a sufficiently solid foundation in London, and probably the instruments must be moved into the country for accurate use.

A portable electrodynamometer has been constructed which will be suitable for distribution as a standard instrument. It can be compared with the large absolute instrument, and can also be compared directly with the most sensitive astatic galvanometers yet made, as has been already proved by experiment. These instruments cannot be distributed until further experiments on their constancy have been made.

* A similar plan was proposed by Mr. Nicholson in 1785 (*vide* Phil. Trans.).

Sir William Thomson, at his own expense, has also constructed an electrodynamometer for absolute measure. His results will check those obtained in London, and the portable standard will also be tested by being sent backwards and forwards between Glasgow and London, to be compared alternately with the absolute instruments.

The determination of "v," the ratio between the electrostatic and electromagnetic units, is also an object pursued by the Committee. Sir William Thomson has made preliminary experiments, and has obtained numbers for this constant by the aid of the absolute electrodynamometer and the absolute electrometer already named. The number he has obtained differs so considerably from that hitherto received that he prefers to extend his experiments before publication. The same remark applies to the measurement of the electromotive force of a Daniell's cell made by the absolute electrometer.

It is hoped that the present Report contains satisfactory evidence that valuable work is being done by the Committee, and that the sums of money liberally granted by the Association have been expended on proper objects.

It will be seen that these grants have stimulated further expenditure on the part of more than one member; and thanks are also due to the Electric and International Telegraph Company, for the liberality with which they have lent large batteries, thereby saving much expense. The Committee are willing to be reappointed, and require no grant of money for the ensuing year.

APPENDIX.

I. *On a "Resistance-Measurer." By* C. W. Siemens, *F.R.S.*

For the measurement of small resistances the method formerly employed was that of the tangent galvanometer, which method is still valuable in the determination of resistances which are inseparable from a difference of electric potential, such, for instance, as a galvanic element.

In measuring wire-resistance more accurate and convenient methods have been devised, amongst which that of the common differential galvanometer and that known as Wheatstone's balance hold the most prominent places.

But both these systems have disadvantages which render them insufficient in a great many cases. For instance, in the first method a well-adjusted variable-resistance-coil is necessary, which, if the method is intended to be applicable between wide limits, will have impracticable large dimensions. The bridge method, though very beautiful, requires three adjusted coils, and frequently gives rise to calculations which renders it unavailable for unskilled operators. The sine method, which is the most suitable for measuring great resistances, requires even a superior amount of skill and mathematical knowledge on the part of the operator.

Many years' experience of these methods made me feel the want of an instrument which would, by its simplicity of construction and ease of manipulation, be capable of employment by an unskilled operator with a degree of exactness equal to that of the bridge method.

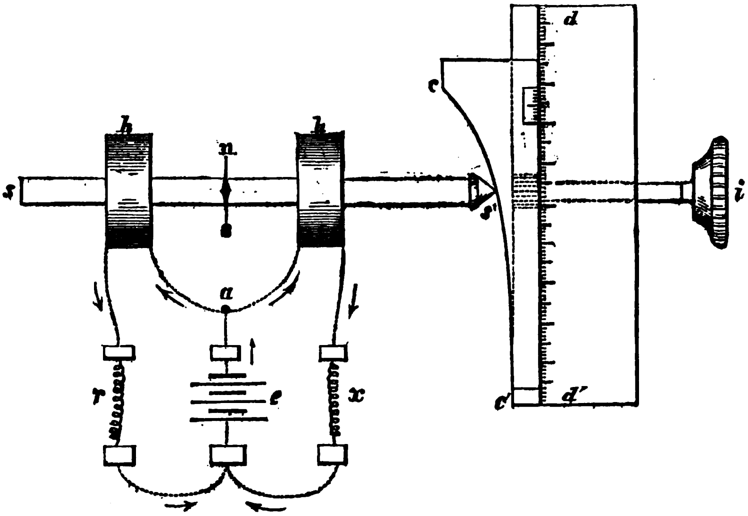
The conditions upon which such an instrument could be successful appeared to be the following:—

1. The employment of a zero method, by which the galvanometer-needle should always be brought to the direction of the magnetic meridian or the same given point upon the scale, and therefore be independent of the unknown function of the angle of deflection.

2. The readings to be made upon a simple lineal measure divided into equal parts signifying equal units of resistance.

3. The employment of a single and unalterable comparison-resistance.

The apparatus constructed to fulfil these conditions is represented by the following diagram:—



Two equal and parallel helices, h and h, are fixed upon the common slide $s\ s'$, which moves in the direction of its length between guide rollers. This motion is effected by the end s', armed by a facing of agate, which presses against the face of the metal curve $c\ c'$. The latter is fixed upon a slide moving in a groove in the rule $d\ d'$, at right angles in the direction $d\ d'$, by means of a milled head i, on the axis of which is a pinion gearing into a rack underneath the straight edge of the curve $c\ c'$. The rule $d\ d'$ is graduated in equal parts; and opposite to the divisions is a nonius up the straight edge and the curve, to divide each degree into ten parts. Whenever the milled head i', therefore, is turned, the position of the curve is altered; and as the point s' of the bobbin-slide is pressed against it by means of a spring, the bobbin follows it in all its movements.

The wires of the two bobbins are connected together, in the common point a, with the pole of a galvanic battery e, the other pole being connected with two resistances r, and through these with the remaining end of the galvanometer-helices. The resistance r is made constant, and adjusted so that when $x=0$ the index of the curve stands exactly opposite the zero of the graduated scale $d\ d'$, the unknown resistance being represented by x.

It is evident that, the resistance in the bobbins being equal, as also their dimensions and initial magnetic effects upon the needle suspended between them, if we make the resistance x equal to r, the current in the two branches will be equal, and the magnet-needle therefore balanced between them only when the helices are equally distant from it. Should, however, either of these resistances preponderate, the strength of current in that branch will be lessened; and in order to reestablish the balance it will be necessary to shift the bobbins, approaching the one in which the weaker current is circulating towards the suspended magnet.

The instrument is erected upon a horizontal metal table standing upon three levelling-screws. The bobbin, the suspended magnet, and dial plate for observing the zero of the pointer are contained in a glass case, supported by four brass pillars. The instrument is supplied with terminals for the battery-connexions, and a current-breaker for interrupting the battery-circuit. Opposite to these are four terminal screws for receiving the ends of the resistances r and x, with contact-plugs between them, in order to quickly establish a short circuit in case the operator should be in doubt towards which side he has to move the adjusting-curve. Two constant resistances accompany the apparatus r—that which is used during the measurement, and a, a resistance of known value, which is introduced between the terminals x in order to enable the operator for his own security to make a control measurement by which he may verify the accuracy of the instrument at any time. Another purpose of this resistance is to facilitate the readjustment of the zero-point, in case the galvanometer should at any time be cleaned or a new silk-fibre put in.

In constructing the sliding curve of this instrument, it might be determined by calculation from the formula given by Weber for the deflection produced by a circular current of known magnitude upon a magnetic point, and from the given distance of the coils from each other. I prefer, however, in practice to determine the curve of each separate apparatus empirically, because it is not possible to coil a helix mathematically true, or to set it when coiled absolutely at right angles to the plane of its horizontal motion.

In the determination of each curve I use a delicately adjusted rheostat or scale of resistances in the circuit of x, giving it varying values corresponding to the equal divisions of the engraved scale, and constructing the curve according to the position which it is found necessary to give to the point s' in order to arrive at the magnetic balance. With each instrument it would be possible to have two values of r—one expressed in mercury and the other in B.A. units; and in order to measure at pleasure in either of these units, it would only be necessary to insert the one or other between the terminal screws for r.

The instrument has been found to be very convenient for the measurement of the wire-resistances of overland lines, or for the reading of resistance-thermometers; it reduces the operation and the observation of the zero position of a needle, and the reading upon a graduated scale, which can be performed by a person of ordinary intelligence without experience in electrical measurement. In accuracy and range it equals the bridge method, while as regards portability and cheapness of apparatus the advantages are decidedly in its favour *.

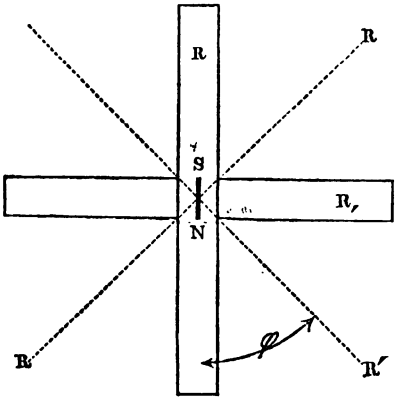
II. *On a Modification of Siemens's Resistance-Measurer.* *By* Fleeming Jenkin, *F.R.S.*

The following method of measuring resistances was suggested to Mr. Jenkin by the above invention of Mr. Siemens:—

Let two tangent galvanometer-coils of equal magnetic moment be fixed together at right angles, with a short magnet hung in their centre, having a long light index pointing at a fiducial mark when the needle is in the magnetic meridian. Let the battery and coils be so joined that the current shall divide in the ratio of the resistances in the two coils, and shall pass in such a direction as to tend to turn the needle in opposite directions.

* I have lately constructed the same instrument on this principle with a circular instead of a straight sliding-piece, which gives the advantage of a longer graduated scale in the form of a circle. The circular sliding curve is adjusted by radial set screws in a solid ring working in a V-groove round the galvanometer.

Let one coil with a resistance R at the beginning of the experiment stand in the magnetic meridian, and the other coil with a resistance R_1 in a plane



The dotted lines show the position of the coils when the current is passing.

perpendicular to the meridian; and when the current is passing in such a direction that R tends to turn N S in the direction of the arrow, let the coils be turned till the needle is again brought to the fiducial point and the coil R_1 makes an angle ϕ with the magnetic meridian, then we have $R = \tan \phi \, R_1$; for the force exerted by the coil R_1 to deflect the needle in the direction of the arrow will then equal $m \sin \phi$; the force exerted by the coil R_1 to deflect the needle in the opposite direction will be $m_1 \cos \phi$; and we have $m \sin \phi = m_1 \cos \varphi$, or $\frac{m_1}{m} = \tan \phi$, where m and m_1 are the couples experienced by the magnet under the action of the two coils; but as we have supposed these coils to have equal magnetic moments with equal currents, $\frac{m_1}{m} = \frac{R}{R_1}$, therefore $R = \tan \phi \, R_1$. R and R_1 need not be the resistances of the galvanometer-coils only, but may consist of two parts, $G + r$ and $G_1 + r_1$, where G and G_1 are the resistances of the galvanometer-coils, but r and r_1 are added resistances. Thus when G G_1 and r are known, r_1 can be obtained by a simple observation.

If $G + r$ be one, one hundred, or one thousand units, the resistance of r_1 will be equal to the tangent of ϕ, or to one hundred or one thousand times that tangent respectively minus in each case a constant $= G_1$.

If the range of the instrument were not required to be very great, the coils would be turned by the pushing of a straight slide, equal divisions on which would correspond to equal increments of the tangent of ϕ, and the scale would be numbered, so that the resistance r_1 should be read off directly, as in Mr. Siemens's instrument.

The tangent coils should be made of German-silver wire, and might be arranged as practised by Helmholtz and Gaugain. Theoretically, the range of each instrument would be infinite, *i. e.* any instrument would be capable of measuring an infinitely small or infinitely large resistance; but clearly the resistance of $G + r$ should be so arranged in each case that the angle observed was not very different from 45°. The range of the instrument may be further increased by the use of elements.

III. *Comparison of B.A. Units to be deposited at Kew Observatory.* *By* C. Hockin.

The following Table shows the value of the different copies of the B.A. units that have been made for preservation at Kew:—

Material of coil.	No. of coil.	Date of observation.	Temperatures at which coil has a resistance = $10^7 \frac{m}{s}$.	Observer.
Platinum-iridium alloy...	2	January 4, 1865	15·5 C.	C. H.
		June 6, 1865	16·0	A. M.
		February 10, 1867	16·0	C. H.
Platinum-iridium alloy...	3	January 4, 1865	15·3	C. H.
		June 6, 1865	15·8	A. M.
		February 10, 1867	15·8	C. H.
Gold-silver alloy	10	January 5, 1865	15·6	A. M.
		February 10, 1867	15·6	C. H.
Gold-silver alloy	58	April 10, 1865	15·3	A. M.
		June 6, 1865	15·3	A. M.
		February 10, 1867	15·3	C. H.
Platinum	35	January 7, 1865	15·7	C. H.
		August 18, 1866	15·7	A. M.
		February 10, 1867	15·7	C. H.
Platinum	36	January 7, 1865	15·5	C. H.
		August 18, 1866	15·5	A. M.
		February 10, 1867	15·7	C. H.
Platinum-silver alloy ...	43	February 15, 1865	15·2	C. H.
		March 9, 1865	15·2	A. M.
		February 10, 1867	15·2	C. H.
*Mercury	I.	February 2, 1865	16·0	A. M.
		July 18, 1866	16·0	A. M.
		February 11, 1867	16·7	C. H.
Mercury	II.	February 3, 1865	14·8	A. M.
		August 18, 1866	14·8	A. M.
		February 11, 1866	14·8	C. H.
Mercury	III.	February 11, 1867	17·9	C. H.

* The alteration of this coil, observed on February 11, 1867, is due, no doubt, to a defect observed in the glass tube.

The tube was of lead-glass. Perhaps the strong nitric acid used to clean the tube attacked the glass. A new mercury unit (No. III.) was made in consequence of this defect.

The apparent alteration in the platinum-iridium coils from the first value found, I believe to be owing to a clerical error. No alteration has been observed in them since the second observation made by Dr. Matthiessen in June 1865.

The values given in the above Table are deduced from the German-silver coil called B, used in your Committee's experiments in 1864. This coil was found (by comparison with copies made, in 1864, of gold-silver, German-silver, and platinum-silver) not to have altered. The coil B was also compared with the coil (June 4) used in 1863, and the ratio of the two coils was found not to have altered.

IV. *Experiments on Capacity.* *By* Fleeming Jenkin, *F.R.S.*

The capacity of a condenser made of mica and tinfoil was adjusted so as to be approximately equal to 10^{-14} electromagnetic absolute units, according to the following experiments. The capacity of any condenser can be directly

measured in absolute measure by the following formula, applying to the effect of a single discharge from the condenser through a galvanometer:—

$$S=2\frac{t\sin\frac{1}{2}i}{\pi R_1}$$

(*vide* Report, 1863, Appendix C, p. 144; *suprà*, p. 74), where R_1 is the resistance of a circuit in which the electromotive force used to charge the condenser would produce the unit deflection, while i is the angle to which the needle is observed to swing from a position of rest, and is half the period or time of a complete oscillation of the needle of the galvanometer under the influence of terrestrial magnetism alone.

This formula, which is analogous to that for any ballistic pendulum acted upon by a known impulse, supposes that the whole impulse is given in a time very short as compared with t, and it also supposes that the deflection i is unimpeded by friction.

I employed a Thomson's astatic reflecting galvanometer with double coils of German-silver wire. The oscillations, with the usual mirror and magnet, subside so rapidly that t cannot be measured with accuracy, and i is very sensibly affected by the resistance of the air; to obviate this I attached a brass ball to the lower magnet of the galvanometer, weighing 55 grains *.

A single floss-silk fibre can just support this weight, under which it continues to stretch sensibly for about three days. In order that the discharge from the condenser, electrified by from 20 to 30 cells, should have force to move this heavy ball through a sensible angle, the galvanometer was made highly astatic; and then I found that with even a single cocoon fibre the needle did not return to zero within three or four divisions of the scale for some minutes, exhibiting a kind of viscosity. The floss-silk fibre, though much weaker, gave a very constant zero. The value of t with the weighted needle seldom differed much from 20 seconds, and the times could be observed for 10 or 11 minutes, during which time t was found to remain sensibly constant. As there was no difficulty in observing the times of oscillation within one second, it may be said that the observed value of t was correct within one part in 500. Greater accuracy was not required, as the possible error from other sources considerably exceeds this. Twenty Daniell's cells were used to charge the condenser, and the discharge observed was about 180 divisions; but the observations were recorded within a quarter of a division: as this is done by estimating the position of the reflected spot stationary between the two black lines of the scale for an almost insensible time, it would not be right to assume that the deflection i is observed with greater accuracy than one part in 400. When the spot of light returned after making one complete oscillation, the diminution in the deflection was from 10 to 12 divisions; one quarter of this amount was therefore added as correction in each case to the deflection observed. The resistance of the whole circuit was composed of the battery resistance, that of German-silver resistance-coils, and of the German-silver coils in the galvanometer; no considerable variation could therefore occur except in the battery, which formed only a small portion of the total resistance. The coils (adjusted by Mr. Hockin) are probably correct within one part in a thousand, and the measurement of the galvanometer-coils is equally well known.

From what has been said, it might be expected that the capacity of any condenser could be obtained with an accuracy of one part in 400 or 500 at

* The ball, two magnets, mirror, and connecting bar, forming the whole suspended system, weighed 57½ grains.

least; but successive discharges were occasionally found to differ by as much as two divisions, though this amount of discrepancy was rare. It was due partly to the residual effect of former charges in the condenser (though great care was taken to avoid this), partly, it is believed, to slight changes in the electromotive force of the battery (which was not in very good order, the discharges being generally less toward the end of a set of experiments), and partly to slight motion of the needle at the moment of taking the discharge. This last source of error made it impossible to make the observations in London; even in the country the needle was seldom, if ever, absolutely still, though the oscillations were generally less than one division. The variation of the electromotive force and resistance of the battery when taking a permanent deflection was another source of error. Owing to the great inertia of the swinging parts, no observation could be taken until the current had been flowing for at least a minute, and often more; and, especially when small resistances were used, the deflections visibly diminished with time. Owing to all these causes, I do not depend on the results obtained as certainly accurate within less than one per cent. This is the less to be regretted, as the capacity of a mica condenser is very ill defined within wide limits, owing to absorption.

The condenser used consisted of 38 plates of mica, about 0·003 in. thick, and having a circular piece of tinfoil 3 in. in diameter cemented to each side of the mica, with a piece of each tinfoil projecting beyond the mica so as to join all the upper tinfoils and all the lower tinfoils together, and form the inner and outer armature of the condensers. This plan has for some time been practised by Mr. Latimer Clark and makes a very constant and well-insulated condenser, extremely easy to adjust roughly by altering the number of the mica plates, and for small corrections by cutting away portions of the tinfoil from the top plate. Mica, like all other solid dielectrics with which I am acquainted, apparently absorbs electricity to a very large extent, and continues to do so for a long time, discharging it at first rapidly, but at the last very slowly indeed, so that a complete discharge is not effected for hours. The total capacity of the condenser varies therefore as the time varies during which it is charged, and the apparent discharge varies with the time during which we measure it; for instance, if we merely observe the discharge due to a momentary contact, we shall obtain a different result from that given when we maintain the contact all the time the needle is swinging; the result will also vary in the latter case with the time of oscillation of the galvanometer-needle. If the needle oscillates slowly, it will be acted upon by a greater quantity of electricity than if oscillating rapidly. Thus, in one experiment, the deflection, when the discharging contact was permanently maintained, was 166 divisions; when a momentary contact was made by a blow it was only 156. When the contact was made for about 1·7 second the deflection was 161, and when the contact was maintained for 3·4 seconds the deflection was 164; the maximum deflection of 166 was reached after 5 seconds: these experiments show that when the needle had travelled two thirds of its maximum distance, the current being discharged exercised a very sensible influence on the deflection. The ballistic formula is therefore not strictly applicable to a case of this kind, and a different result would be obtained with a galvanometer oscillating either more or less quickly than the one I used. It seemed therefore unnecessary to take great precautions or to aim at any high degree of accuracy; and my object has simply been to provide a unit for cable-testing which shall be approximately equal to the ideal standard chosen by the Committee, and which can be used with at least as great accuracy as those copies of knots of Atlantic or Persian-Gulf cables hitherto used.

The value of R_1, in the formula given at the commencement, was found by two methods, which we will call the indirect and direct method. In the indirect method three sets of resistance-coils (a, b, c) were arranged as in fig. 1, with a battery, B, a galvanometer, G, and a shunt, Z, equal

Fig. 1.

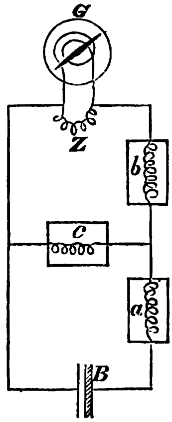
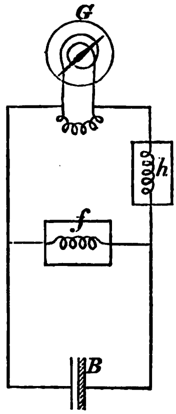


Fig. 2.



in resistance to $\frac{1}{999}$ of the galvanometer-coils. The resistance c was made equal to 1000 units, and the resistances a and b adjusted until a convenient deflection was obtained on the galvanometer; the resistance a was next changed to a_1, and b was then altered to b_1, so as to give the same deflection as before on the galvanometer G. Then calling d the deflection observed, G the resistance of the galvanometer, we have

$$R_1 = nd\left\{(a_1 - a)\frac{(b+c+\frac{1}{n}G)(b_1+c+\frac{1}{n}G)}{(b-b_1)c} - c\right\},$$

a formula for which the resistance of the battery need not be calculated ($n=1000$).

The second or direct method of obtaining R_1 was, first, to calculate the resistance of the battery B by the following formula (fig. 2):—h and f are variable resistances; g the resistance of the shunted galvanometer, $=47\cdot2$ in my experiments; break the circuit at f, and adjust h till a convenient reading is obtained; then join f, as shown in the sketch, and adjust f and h until the same deflection is obtained as before; then, calling h_1 the last resistance at h, we have

$$B = f\frac{h-h_1}{g=h_1}.$$

Secondly, a direct deflection d was obtained with a resistance k in circuit; then $R_1 = nd\,(k+B+g)$.

The following is a record of the experiments made in chronological order:—

September 22. *Discharge.*—Values of i after charging for one minute with 20 cells:—

1.	2.	3.	4.	5.	Mean.
167	167	166	165	165	166

Adding 2·5 to compensate for portion of air, $i=168\cdot5$; and the angle being very small, $\sin\frac{1}{2}i=84\cdot25$.

Test for insulation; discharge after one minute's insulation 154.

Times.—First four oscillations, the spot crossed the central point in the same direction at

0′ 35″, 0′ 55″, 1′ 14½″, 1′ 33″;

last four oscillations,

9′ 13″, 9′ 32″, 10′ 10″, 10′ 29″.

Total number of oscillations 31. Mean value of $2t$=19′ 15″.

Value of R_1. Indirect method:—

	a.	*a_1.*	*b.*	*b_1.*	*c.*	*d.*	R_1, Ohms.
1.	8000	10000	1000	649	1000	275½	$5·17\times10^9$
2.	6000	8000	1000	575	1000	354½	5·17 „
3.	8000	10000	1000	647	1000	274½	5·12 „
4.	6000	8000	1000	574	1000	355¼	5·18 „

Mean value of R_1 in absolute measure $5·16\times10^{16}$. Value of S, $99·53\times10^{-12}$.

Value of R_1. Direct method. Battery resistance:—

	f.	*h.*	*h_1.*	*g.*	B.
1.	2	18700	30	47	484
2.	10	18000	300	47	492

Mean value of B 488.

Deflection with variable resistance in circuit:—

	d.	*k.*	B.	*g.*	*n.*	R_1, Ohms.
1.	226½	22000	488	47	1000	$5·10\times10^9$
2.	310¼	16000	488	47	1000	$5·13\times10^9$

Mean value of $R_1=5·125\times10^{16}$ absolute units. Value of S from values of t and i as above, $100·21\times10^{-14}$.

September 24. *Discharge.*—sin $\frac{1}{2}$ i=84·75. R_1 from indirect method:—

	a.	*a_1.*	*b.*	*b_1.*	*c.*	*d.*	R_1, Ohms.
1.	6000	8000	1000	575	1000	354	$5·18\times10^9$
2.	8000	10000	1000	648	1000	275	$5·16\times10^9$

Mean value of R_1 in absolute measure $5·17\times10^6$. Assuming t as on September 22, $S=99·92\times10^{-12}$.

The box holding the condenser was now filled up with an insulating composition.

October 13. *Discharge.*—184 divisions, 12 divisions lost on return, sin $\frac{1}{2}$ i=93·5. Discharge after one minute's insulation 181 divisions.

Time.—First four oscillations,

0′ 30″, 0′ 51″, 1′ 11″, 1′ 31″;

last four oscillations,

10′ 4″, 10′ 23″, 10′ 43½″, 11′ 5″.

Total number of oscillations 31. Mean value of $2t$=20′ 47″.

R_1 by indirect method:—

a.	*a_1.*	*b.*	*b_1.*	*c.*	*d.*	R_1.
8000	10000	1000	646	1000	333	$6·19\times10^9$.

Value of $S=98·42\times10^{-12}$.

R_1 by direct method. Battery resistance;—

f.	*h.*	*h_1.*	*g.*	B.
10	17400	700	47	223½

Direct deflection:—

	d.	*k.*	B.	*g.*	*n.*	R_1.
1.	270½	22000	223½	47	1000	$6·01\times10^9$
2.	331	18000	223½	47	1000	$6·05\times10^9$

Mean value of $R_1=6·03\times10^{16}$ absolute units. Value of $S=101·03\times10^{-12}$.

October 15. *Discharge*:—

1.	2.	3.	4.	5.	6.
185	185	184½	184	184½	184¾

Mean 184·6, adding 3 for air, sin $\frac{1}{2}$ i=93·8.

Times.—First four, 0′ 23″, 0′ 42½″, missed, 1′ 24″; last four, 7′ 55″, 8′ 16″, 8′ 35″; 24 oscillations in all. Mean value of $2t$=20′ 56″.

Independent series of observations divided into triplets:—

first two, 0′ 22½, 1′ 24″, last two, 9′ 37½, 10′ 39″;

30 oscillations in all. Mean value of $2t=20' 55''$.

Value of R_1. Direct method. Battery resistance:—

1.	223
2.	216
Mean	219

Direct deflection:—

	d.	k.	B.	g.	n.	R_1, Ohms.
1.	278	22000	219	47	1000	$6{\cdot}19\times10^9$
2.	321½	19000	219	47	1000	$6{\cdot}19\times10^9$

Mean value of R_1 in absolute units $6{\cdot}19\times10^{16}$. Value of $S=99{\cdot}2\times10^{-13}$.

October 17. *Discharge*:—

1.	2.	3.	4.	Mean.
179	180	179	180	179·5

$\sin\frac{1}{2}i=91\frac{1}{4}$.

Times;—

0′ 55″, 1′ 56¼″, 10′ 7½″, 11′ 8¼″.

Total number of oscillations 30. Mean value of $2t=20' 46''$.

Value of R_1. Direct method. Battery resistance:—

1.	210
2.	223
Mean	215·5

Direct deflection:—

	d.	k.	B.	g.	n.	R_1, Ohms.
1.	268	22000	215½	47	1000	$5{\cdot}97\times10^9$
2.	329	18000	215½	47	1000	$6{\cdot}01\times10^9$

Mean value of $R_1=5{\cdot}99\times10^{19}$ absolute units. Value of $S=99{\cdot}25$.

The seven values obtained for S give a mean value of $\cdot9965\times10^{-14}$ as the capacity of the mica-plate condenser when charged for one minute, and measured by a discharge through a galvanometer, on the needle of which it acts for about 5 seconds. If we reject the two observations made on Oct. 15 (which were, indeed, only preliminary, and made with less care than all the others) we find the average to be $0{\cdot}9962\times10^{-14}$, and the approximation between this mean and any single results is 0·42 per cent. It is therefore probable that a unit copied from this preliminary standard will not be one per cent. wrong.

A tenfold multiple (10^{-13} absolute measure) of the condenser measured is a convenient magnitude as a practical unit of capacity for telegraphy; thus the capacity of the Atlantic cable per knot thus measured is 0·3535. Assuming that the practical unit of electromotive force will be chosen as that multiple which is most nearly equal to Daniell's cell, *i. e.* 10^5 electromagnetic units, then the capacity of the proposed practical unit is such that it contains with the unit E. M. F. the same quantity of electricity as would be passed in one second through a circuit of the resistance of one Megohm. Thus 10^5 E. M. F., acting on a circuit of 10^{13}, will pass in one second 10^{-8} absolute units of quantity; and, similarly, 10^5 E. M. F. will charge a condenser of absolute capacity equal to 10^{-13} with 10^{-8} absolute units of quantity. This practical series of units is that which, in the opinion of Mr. Latimer Clark and myself, is best adapted for practical use in telegraphy. Mr. Clark calls the unit of quantity thus defined (10^{-8}) one Farad, and similarly says that the unit of capacity has a capacity of one Farad, it being understood that this is the capacity when charged with unit electromotive force (10^5).

V. *Report on Electrometers and Electrostatic Measurements.* *By* Sir WM. THOMSON, *F.R.S.*

§ 1. AN electrometer is an instrument for measuring differences of electric potential between two conductors through effects of electrostatic force, and is distinguished from the galvanometer, which, of whatever species, measures differences of electric potentials through electromagnetic effects of electric currents produced by them. When an electrometer merely indicates the existence of electric potential, without measuring its amount, it is commonly called an electroscope; but the name electrometer is properly applied when greater or less degrees of difference are indicated on any scale of reckoning, if approximately constant, even during a single series of experiments. The first step towards accurate electrometry in every case is to deduce from the scale-readings numbers which shall be in simple proportion to the difference of potentials to be determined. The next and last step is to assign the corresponding values in absolute electrostatic measure. Thus, when for any electrometer the first step has been taken, it remains only to determine the single constant coefficient by which the numbers deduced from its indications as simply proportional to differences of potential must be multiplied to give differences of potential in absolute electrostatic measure. This coefficient will be called, for brevity, the absolute coefficient of the instrument in question.

§ 2. Thus, for example, the gold-leaf electrometer indicates differences of potential between the gold leaves and the solid walls enclosing the air-space in which they move. If this solid be of other than sufficiently perfect conducting material, of wood and glass, or of metal and glass, for instance, as in the instrument ordinarily made, it is quite imperfect and indefinite in its indications, and is not worthy of being even called an electroscope, as it may exhibit a divergence when the difference of potentials which the operator desires to discover is absolutely zero. It is interesting to remark that Faraday first remedied this defect by coating the interior of the glass case with tinfoil cut away to leave apertures proper and sufficient to allow indications to be seen, but not enough to cause these indications to differ sensibly from what they would be if the conducting envelope were completely closed around it; and that not till a long time after did any other naturalist, mathematician, or instrument-maker seem to have noticed the defect, or even to have unconsciously remedied it.

§ 3. Electrometers may be classified in genera and species according to the shape and kinematic relations of their parts; but as in plants and animals a perfect continuity of intermediate species has been imagined between the rudimentary plant and the most perfect animal, so in electrometers we may actually construct species having intermediate qualities continuous between the most widely different genera. But, notwithstanding, some such classification as the following is convenient with reference to the several instruments commonly in use and now to be described:—

I. Repulsion electrometers.
- Pair of diverging straws as used by Beccaria, Volta, and others, last century.
- Pair of diverging gold leaves (Bennet).
- Peltier's electrometer.
- Delmann's electrometer.
- Old-station electrometer, described in lecture to the Royal Institution, May 1860; also in Nichol's 'Cyclopædia,' article "Electricity, Atmospheric" (edition 1860), and in Dr. Everett's

paper of 1867, "On Atmospheric Electricity" (Philosophical Transactions).

II. Symmetrical electrometers.
  Bohnenberger's electrometer.
  Divided-ring electrometers.

III. Attracted disk electrometers.
  Absolute electrometer.
  Long-range electrometer.
  Portable electrometer.
  Spring-standard electrometer.

§ 4. Class I. is sufficiently illustrated by the examples referred to; and it is not necessary to explain any of these instruments minutely at present, as they are, for the present at all events, superseded by the divided-ring electrometer and electrometers of the third class.

There are at present only two known species of the second class; but it is intended to include all electrometers in which a symmetrical field of electric force is constituted by two symmetrical fixed conductors at different electric potentials, and in which the indication of the force is produced by means of an electrified body movable symmetrically in either direction from a middle position in this field. This definition is obviously fulfilled by Bohnenberger's well-known instrument *.

§ 5. My first published description of a divided-ring electrometer is to be found in the Memoirs of the Roman Academy of Sciences † about 1856; but since that time I have made great improvements in the instrument—first, by applying a light mirror to indicate deflections of the moving body; next, by substituting for two half rings four quadrants, and consequently for an electrified body projecting on one side only of the axis, an electrified body projecting symmetrically on the two sides and movable round an axis; and, lastly, by various mechanical improvements and by the addition of a simple gauge to test the electrification of the movable body, and a replenisher to raise this electrification to any desired degree.

§ 6. In the accompanying drawings, Plate V. fig. 1 represents the front elevation of the instrument, of which the chief bulk consists of a jar of white glass (flint) supported on three legs by a brass mounting cemented round the outside of its mouth, which is closed by a flat cover of stout sheet brass and a lantern-shaped cover standing over a wide aperture in its centre. For brevity, in what follows, these three parts will be called the jar, the main cover, and the lantern.

Fig. 5 represents the quadrants as seen from above; they are seen in elevation at *a* and *b*, fig. 1, and in section at *c* and *d*, fig. 2. They consist of four quarters of a flat circular box of brass, with circular apertures in the centres of its top and bottom. Their position in the instrument is shown in figs. 1, 2, & 6. Each of the four quadrants is supported on a glass stem passing downwards through a slot in the main cover of the jar, from a brass mounting on the outside of it, and admits of being drawn outwards for a space of about $\frac{3}{8}$ of an inch (1 centim.) from the positions they occupy when the instrument is in use, which are approximately those shown in the drawings. Three of them are secured in their proper positions by nuts (*e*, *e*, *e*) on the outside of the chief flat lid of the jar shown in fig. 4. The upper end of the stem, carrying the fourth, is attached to a brass piece (*f*) resting on three short legs

* A single gold leaf hanging between the upper ends of two equal and similar dry piles standing vertically on a horizontal plate of metal, one with its positive and the other with its negative pole up.

† Accademia Pontificia dei Nuovi Lincei.

on the upperside of the main cover, two of these legs being guided by a straight V-groove at g to give them freedom to move in a straight line inwards or outwards, and to prevent any other motion. This brass piece is pressed outwards and downwards by a properly arranged spring (h), and is kept from sliding out by a micrometer-screw (i) turning in a fixed nut. This simple kinematic arrangement gives great steadiness to the fourth quadrant when the screw is turned inwards or outwards and then left in any position; and at the same time produces but little friction against the sliding in either direction. The opposite quadrants are connected in two pairs by wires, as shown in fig. 5; and two stout vertical wires (l, m), called the chief electrodes, passing through holes in the roof of the lantern, are firmly supported by long perforated vulcanite columns passing through these holes, which serve to connect the pairs of quadrants with the external conductors whose difference of potentials is to be tested. Springs (n, o) at the lower ends of these columns, shown in figs. 1 & 2, maintain metallic contact between the chief electrodes and the uppersides of two contiguous quadrants (a & b) when the lantern is set down in its proper position, but allow the lantern to be removed, carrying the chief electrodes with it, and to be replaced at pleasure without disturbing the quadrants. The lantern also carries an insulated charging-rod (p), or temporary electrode, for charging the inner coating of the jar (§ 11) to a small degree, to be increased by the replenisher (§ 12), or, it may be, for making special experiments in which the potential of the interior coating of the jar is to be measured by a separate electrometer, or kept at any stated amount from that of the outer coating. When not in use this temporary electrode is secured in a position in which it is disconnected from the inner coating.

§ 7. The main cover supports a glass column (q, fig. 2) projecting vertically upwards through its central aperture, to the upper end of which is attached a brass piece (r), which bears above it a fixed attracting disk (s), to be described later (§ 13); and projecting down from it a fixed plate bearing the silk-fibre suspension of the mirror (t), needle (u), &c., seen in figs. 1 & 2, and fixed guard tubes (v, w), to be described presently.

§ 8. The movable conductor of the instrument consists of a stiff platinum wire (x), about 8 centimetres ($3\frac{1}{8}$ inches) long, with the needle rigidly attached in a perpendicular plane to it, and connected with sulphuric acid in the bottom of the jar by a fine platinum wire hung down from its lower end and kept stretched by a platinum weight under the level of the liquid. The upper end of the stiff platinum wire is supported by a single silk-fibre so that it hangs down vertically. The mirror is attached to it just below its upper end. Thus the mirror, the needle, and the stiff platinum stem constitute a rigid body having very perfect freedom to move round a vertical axis (the line of the bearing fibre), and yet practically prevented from any other motion in the regular use of the instrument by the weight of its own mass and that of the loose piece of platinum hanging from it below the surface of the liquid in the jar. A very small magnet is attached to the needle, which, by strong magnets fixed outside the jar, is directed to one position, about which it oscillates after it is turned through any angle round the vertical axis and then left to itself. The external magnets are so placed that when there is magnetic equilibrium the needle is in the symmetrical position shown in figs. 5 & 6 with reference to the quadrants*.

§ 9. The needle (u) is of very thin sheet aluminium cut to the shape seen in figs. 5 & 6, the very thinnest sheet aluminium that gives the requisite stiff-

* Recently I have made experiments on a bifilar suspension with a view to superseding the magnetic adjustment, which promise well.

ness being chosen. If the four quadrants are in a perfectly symmetrical position round it, and if they are kept at one electric potential by a metallic arc connecting the chief electrodes outside, the needle may be strongly electrified without being disturbed from its position of magnetic equilibrium; but if it is electrified, and if the external electrodes be disconnected and any difference of potentials established between them, the needle will clearly experience a couple turning it round its vertical axis, its two ends being driven from the positive quadrants towards the negative if it is itself positively electrified. It is kept positive rather than negative in the ordinary use of the instrument, because I find that when a conductor with sharp edges or points is surrounded by another presenting everywhere a smooth surface, a much greater difference of potentials can be established between them, without producing disruptive discharge, if the points and edges are positive than if they are negative.

§ 10. The mirror (t) serves to indicate, by reflecting a ray of light from a lamp, small angular motions of the needle round the vertical axis. It is a very light, concave, silvered glass mirror, being only 8 millimetres ($\frac{1}{3}$ of an inch) in diameter, and 22 milligrammes ($\frac{1}{3}$ grain) weight. I had for many years experienced great difficulty in getting suitable mirrors for my form of mirror galvanometer; but they are now supplied in very great perfection by Mr. Becker, of Messrs. Elliott Brothers, London. The focus for parallel rays is about 50 centimetres (20 inches) from the mirror, and thus the rays of the lamp placed at a distance of 1 metre (or 40 inches) are brought to a focus at the same distance. The lamp is usually placed close behind the vertical screen a little below or above the normal line of the mirror, and the image is thrown on a graduated scale extending horizontally above or below the aperture in the screen through which the lamp sends its light. When the mirror is at its magnetic zero position the lamp is so placed that its image is, as nearly as may be, in a vertical plane with itself, and not more than an inch above or below its level; so that there is as little obliquity as possible in the reflection, and the line traversed by the image on the screen during the deflection is, as nearly as may be, straight. The distance of the lamp and screen from the mirror is adjusted so as to give as perfect an image as possible of a fine wire which is stretched vertically in the plane of the screen across the aperture through which the lamp shines on the mirror; and with Mr. Becker's mirrors I find it easy to read the horizontal motions of the dark image to an accuracy of the tenth of a millimetre. In the ordinary use of the instrument a white paper screen, printed from a copper plate, is employed, and the readings are commonly taken to about a quarter of a scale-division; but with a little practice they may, when so much accuracy is desired, be read with considerable accuracy to the tenth of a scale-division. Formerly a slit in front of the lamp was used; but the wire giving a dark line in the middle of the image of the flame is a very great improvement, first introduced by Dr. Everett, in consequence of a suggestion made by Professor P. G. Tait, in his experiments on the elasticity of solids made in the Natural-Philosophy Laboratory of Glasgow University*.

§ 11. The charge of the needle remains sensibly constant from hour to hour, and even from day to day, in virtue of the arrangement according to which it is kept in communication with sulphuric acid in the bottom of the

* A Drummond light placed about 70 centimetres from the mirror gives an image, on a screen about 3 metres distance, brilliant enough for lecture-illustrations, and with sufficient definition to allow accurate readings of the positions on a scale marked by the image of a fine vertical wire in front of the light.

jar, the outside of the jar being coated with tinfoil and connected with the earth, so that it is in reality a Leyden jar. The whole outside of the jar, even where not coated with tinfoil, is in the ordinary use of the instrument, especially in our moist climate, kept virtually at one potential through conduction along its surface. This potential is generally, by connecting wires or metal pieces, kept the same as that of the brass legs and framework of the instrument. To prevent disturbance in case of strongly electrified bodies being brought into the neighbourhood of the instrument, a wire is either wrapped round the jar from top to bottom, or a cage or network of wire, or any convenient metal case, is placed round it; but this ought to be easily removed or opened at any time to admit of the interior being seen. When the instrument is left to itself from day to day in ordinary use, the needle, connected with the inner coating of the jar as just described, loses, of course, unless replenished, something of its charge; but not in general more than ½ per cent. per day when the jar is of flint glass made in Glasgow. On trying similar jars of green glass I found that they lost their charge more rapidly per hour than the white glass jars per month. I have occasionally, but very rarely, found white glass jars to be as defective as those green ones; and it is possible that the defect I found in the green jars was an accident to the jars tested, and not an essential property of that kind of glass.

§ 12. I have recently made the very useful addition of a replenisher to restore electricity to the jar from time to time when required. It consists of (1) a turning vertical shaft of vulcanite bearing two metal pieces called carriers (*b*, *b*, figs. 17 & 18, Plate V.); (2) two springs (*d*, *d*, figs. 16 & 18), connected by a metallic arc, making contact on the carriers once every half turn of the shaft, and therefore called connectors; and (3) two inductors (*a*, *a*) with receiving springs (*c*, *c*) attached to them, which make contact on the carriers once every half turn, shortly before the connecting contacts are made. The inductors (*a*, *a*, figs. 16 & 18) are pieces of sheet metal bent into circular cylindrical shapes of about 120° each; they are placed so as to deviate in the manner shown in the drawing from parts of a cylindrical surface coaxal with the turning-shaft, leaving gaps of about 60° on each side. The diameter of this cylindrical surface is about 15 millimetres (about ½ an inch). The carriers (*b*, *b*, figs. 17 & 18) are also of sheet metal bent to cylindrical surfaces, but not exactly circular cylinders, and are so placed on the bearing vulcanite shaft that each is rubbed by the contact springs over a very short space, about 1 millimetre beyond its foremost edge, when turned in the proper direction for replenishing. The receiving springs (*c*, *c*, figs. 17 & 18) make their contacts with each carrier immediately after it has got fairly under cover, as it were, of the inductor. Each carrier subtends an angle of about 60° at the axis of the turning-shaft. The connecting contacts are completed just before the carriers commence emerging from being under cover of the inductors. The carriers may be said to be under cover of the inductors when they are within an angle of 120° on each side of the axis subtended by the inductors. One of the inductors is in metallic communication with the outside coating of the jar, the other with the inside. Figs. 16, 17, & 18 illustrate sufficiently the shape of carriers and the succession of the contacts. The arrow-head indicates the direction to turn for replenishing. When it is desired to diminish the charge, the replenisher is turned backwards. A small charge having been given to the jar from an independent source, the replenisher when turned forwards increases the difference of potentials between the two inductors and the two coatings of the jar connected with them by a constant percentage per half turn, unless it is raised to so high a degree as to break

down the air-insulation by disruptive discharge. The electric action is explained simply thus:—The carriers, when connected by the connecting springs, receive opposite charges of induction, of which they deposit large proportions the next time they touch the receiving springs. Thus, for example, if the jar be charged positively, the carrier emerging from the inductor connected with the inner coating carries a negative charge round to the receiving spring connected with the outside coating, while the other carrier, emerging from the inductor connected with the outside coating, carries a positive charge round to the receiving spring connected with the inside coating. If the carriers are not sufficiently well under cover of the inductors during both the receiving contacts and the connecting contacts to render the charges which they acquire by induction during the connecting contacts greater than that which they carry away with them from the receiving contacts, the rotation, even in the proper direction for replenishing, does not increase, but, on the contrary, diminishes the charge of the jar. The deviations of the inductors from the circular cylinder referred to above have been adopted to give greater security against this failure. A steel pivot fixed to the top of the vulcanite shaft, and passing through the main cover, carries a small milled head (y, fig. 1) above, on the outside, which is spun rapidly round in either direction by pressing the finger on it; and thus in less than a minute a small charge in the jar may be doubled. The diminution of the charge, when the instrument is left to itself for twenty-four hours, is sometimes imperceptible; but when any loss is discovered to have taken place, even if to the extent of 10 per cent., a few moments' use of the replenisher suffices to restore it, and to adjust it with minute accuracy to the required degree by aid of the gauge to be described presently. The principle of the "replenisher" is identical with that of the "doubler" of Bennet. In the essentials of its construction it is the same as Varley's improved form of Nicholson's "revolving doubler."

§ 13. The gauge consists of an electrometer of Class III. The movable attracted disk is a square portion of a piece of very thin sheet aluminium of the shape shown at α in fig. 4. It is supported on a stretched platinum wire passing through two holes in the sheet and over a very small projecting ridge of bent sheet aluminium placed in the manner shown in the magnified drawing, fig. 3. The ends of this wire are passed through holes in curved springs, shown in fig. 4, and are bent round them so as to give a secure attachment without solder and without touching the straight stretched part of the wire. The ends of the platinum wire (β, β) are attached by cement to the springs, merely to prevent them from becoming loose, care being taken that the cement does not prevent metallic contact between some part of the aluminium wire and one or both of the brass springs. I have constantly found fine platinum wire rendered brittle by ordinary solder applied to it. The use of these springs is to keep the platinum wire stretched, with an approximately constant tension, from year to year and at various temperatures. Their fixed ends are attached to round pins, which are held with their axes in a line with the fibre by friction, in bearings forming parts of two adjustable brass pieces (γ, γ) indicated in fig. 4; these pieces are adjusted once for all to stretch the wire with sufficient force, and to keep the square attracted disk in its proper position. The round pins bearing the stretching-springs are turned through very small angles by pressing on the projecting springs with the finger. They are set so as to give a proper amount of torsion tending to tilt the attracted disk (a) upwards, and the long end of the aluminium lever (δ), of which it forms a part, downwards. The downward motion of the long end

is limited by a properly placed stop. Another stop (ϵ) above limits the upward motion, which takes place under the influence of electrification in the use of the instrument. A very fine opaque black hair (that of a small black-and-tan terrier I have found much superior to any hitherto tried) is stretched across the forked portion of the sheet aluminium in which the long arm of the lever terminates. Looked at horizontally from the outside of the instrument it is seen, as shown in fig. 7, Plate V., against a white background, marked with two very fine black circles. These sight-plates in the instruments, as now made by Mr. White, are of the same material as the ordinary enamel watch-dials with black figures on a white ground. The white space between the two circles should be a very little less than the breadth of the hair. The sight-plate is set to be as near the hair as it can be without impeding its motion in any part of its range; and it is slightly convex forwards, and is so placed that the hair is nearer to it when in the middle between the black circles than when in any other part of its range. It is thus made very easy, even without optical aid, to avoid any considerable error of parallax in estimating the position of the hair relatively to the two black circles. By a simple plano-convex lens (ϕ, fig. 2), with the convex side turned inwards, it is easy, in the ordinary use of the instrument, to distinguish a motion up or down of the hair amounting to $\frac{1}{5000}$ of an inch. With a little care I have ascertained, Dr. Joule assisting, that a motion of no more than $\frac{1}{50,000}$ of an inch from one definite central position can be securely tested without the aid of other magnifying-power than that given by the simple lens. The lens during use is in a fixed position relatively to the framework bearing the needle, but it may be drawn out or pushed in to suit the focus of each observer. To give great magnification, it ought to be drawn out so far that the hair and sight-plate behind may be but little nearer to the lens than its principal focus, and the observer's eye ought to be at a very considerable distance from the instrument, no less than 20 centimetres (8 inches), to get a good magnification; and a short-sighted person should use his ordinary concave eye-lens close to his eye. The reason for turning the convexity of the small plano-convex lens inwards is, that if the eye of the observer is too high or too low, the hair seems to him curved upwards or downwards, and he is thus guided to keep his eye on a level sufficiently constant to do away with all sensible effects of parallax on the position of the hair relatively to the black circles. The framework carrying the stretched platinum wire and movable attracted disk is above the brass roof of the lantern, in which a square aperture is cut to allow the square portion constituting the short arm of the aluminium balance to be attracted downwards by the fixed attracting disk (§ 7), to be presently described. A side view of the attracting plate, the brass roof of the lantern, the aluminium balance, the sight-plate, the hair, and the plano-convex lens is shown in section (fig. 2), also a glass upper roof to protect the gauge and the interior of the instrument below from dust and disturbance by currents of air, to which, without this upper roof, it would be exposed, through the small vacant space round the movable aluminium square. The fixed attracting disk is borne by a vertical screw screwing into the upper brass mounting (z, fig. 2) (§ 7), connected with the inner coating of the Leyden jar through the guard tubes, &c., and is secured in any position by the "jam nut," shown in the drawings at z, fig. 2. This disk (s) is circular, and about 38 millimetres ($1\frac{1}{2}$ inch) diameter, and it is placed horizontally with its centre under the centre of *the square aperture* in the roof of the lantern. Its distance from the lower

surface of the roof and of the movable attracted disk may be from $2\frac{1}{2}$ to 5 millimetres (from $\frac{1}{10}$ to $\frac{1}{5}$ of an inch), and is to be adjusted, along with the amount of torsion in the platinum wire bearing the aluminium balance-arm, so as to give the proper sensibility to the gauge. The sensibility is increased by diminishing the distance from the attracting to the attracted plate and increasing the amount of torsion. Or, again, the degree of the potential indicated by it when the hair is in the sighted position is increased by increasing the distance between the plates, or by diminishing the amount of torsion. If the electrification of the needle is too great, its proper position of equilibrium becomes unstable; or before this there is sometimes a liability to discharge by a spark across some of the air-spaces. The instrument works extremely well with the needle charged but little less than to give rise to one or both of these faults, and I adjust the gauge accordingly.

§ 14. The strength of the fixed steel-directing magnets is to be adjusted to give the desired amount of deflection with any stated difference of potentials maintained between the two chief electrodes, when the jar is charged to the degree which brings the hair of the gauge to its sighted position. In the instruments already made, the deflection* by a single cell of Daniell's amounts to about 100 scale-divisions (of $\frac{1}{40}$ of an inch each at a distance of 40 inches) when the magnetic directive force is such as to give a period of vibration equal to about 1·5 second. When the jar is discharged and the four quadrants are connected with one another and with the inner coating of the jar, lower degrees of sensibility may be attained better by increasing the magnetic directing-force than by diminishing the charge of the jar. Thus, for instance, when it is to be used for measuring and photographically recording the potential of atmospheric electricity at the point where the stream of the water-dropping collector† breaks into drops, the magnetic directing-force may be made from 10 to 100 times more than that just described. When this is to be done it may be convenient to attach a somewhat more powerful magnetic needle than that which has been made in the most recent instruments where a high degree of sensibility is desired. But it is to be remarked that in general the directing-force of the external steel magnets cannot be too strong, as the stronger it is the less is the disturbance produced by changing magnetic bodies in the neighbourhood of the instrument. In laboratory work, where numerous magnetic experiments are being performed in the immediate neighbourhood, and in telegraph factories, where there is constant disturbance by large moving masses of iron, the artificial magnetic field of the electrometer ought to be made very strong. To allow this, and yet leave sufficient sensibility to the instrument, the suspended magnetic needle has been made smaller and smaller, until it is now reduced to two small pieces of steel side by side, 6 millimetres ($\frac{1}{4}$ of an inch) long. For a meteorological observatory all that is necessary is, that the directing magnetic force should be so great that the greatest disturbance experienced in magnetic storms shall not sensibly deflect the luminous image‡.

§ 15. The sensibility of the gauge should be so adjusted that a variation in the change of the jar, producing an easily perceived change in the posi-

* That is to say, the number of scale-divisions over which the luminous image moves when the chief electrodes are disconnected from one another and put in metallic connexion with the two plates of a Daniell's battery.

† See Royal Institution Lecture, May 18, 1860 (Proceedings of the R. I.), or Nichol's 'Cyclopædia,' article "Electricity, Atmospheric" (edition 1860).

‡ All embarrassment from this source will be done away with if the bifilar plan be adopted (*vide* footnote to § 8).

tion of the hair, shall produce no sensible change in the deflection of the luminous image produced by the greatest difference of potentials between the quadrants, which is to be measured in the use of the instrument. I believe the instruments already made, when adjusted to fulfil these conditions, may be trusted to measure the difference of potentials produced by a single cell of Daniell's to an accuracy of a quarter per cent. It must be remembered that the constancy of value of the unit of each instrument depends not only on the constancy of the potential indicated by the gauge, but also on the constancy of the force in the field traversed by the suspended needle. As both these may be expected to decrease gradually from year to year (although very slowly after the first few hours or weeks), rigorous methods must be adopted to take such variations into account, if the instrument is to be trusted to as giving accurately comparable indications at all times. The only method hitherto provided for this most important object consists in the observation of the deflection produced by a measured motion of one of the quadrants by the micrometer-screw (*i*) when the four quadrants are put in metallic communication with one another through the principal electrodes—the force producing this deflection when the potential of the jar is constant; and therefore, the jar being brought to one constant potential by aid of the gauge, the amount of the deflection will show whether or not the force of the magnetic field has changed, and will render it easy at any time to adjust the strength of the magnets, if necessary, to secure this constancy. But to attain this object by these means, the three quadrants not moved by the micrometer-screw must be clamped by their fixing-screws so that they may be always in the same position.

§ 16. The absolute constancy of the gauge cannot be altogether relied upon. It certainly changes to a sensible degree with temperature, and to very different degrees, and even in different directions, as will be seen (§ 32) in connexion with the description of the portable electrometer to be given later. But this temperature variation does not amount in ordinary cases probably to as much as one per cent.; and it is probable that after a year or two any further secular variation of the platinum torsion spring will be quite insensible. It is to be remarked, however, that secular experiments on the elasticity of metals are wanting, and ought at least to be commenced in our generation. In the mean time it will be desirable, both on account of the temperature variation and of the possible secular variation in the couple of torsion, to check the gauge by accurate measurements of the time of oscillation of the needle with its appurtenances. The moment of inertia of this rigid body, except in so far as it may be influenced by oxidation of the metal, of which I have as yet discovered no signs, may be regarded as constant; and therefore the amount of the directing couple due to the magnets may be determined with great accuracy by finding the period of an oscillation when the four quadrants are put in connexion through the charging rod with the metal mounting bearing the guard plates, &c. I have not as yet put into practice any of the obvious methods, founded on the general principle of coincidences used in pendulum observations, for determining the period of the oscillation; but although not more than twenty or thirty oscillations can be counted, it seems certain that with a little trouble the period of one of them may be determined without much trouble to an accuracy of about $\frac{1}{10}$ per cent.

Absolute Electrometer.

§ 17. The absolute electrometer (fig. 11, Plate VI.) and the other instruments of Class III. are founded on a method of experimenting introduced by

Sir Wm. Snow Harris, and described in his first paper "On the Elementary Laws of Electricity" * thirty-four years ago. In these experiments a conductor, hung from one arm of a balance and kept in metallic communication with the earth, is attracted by a fixed insulated conductor, which is electrified, and, for the sake of keeping its electric potential constant, is connected with the inner coating of a Leyden battery. The first result which he announced is, that, when other circumstances remain the same, the attraction varies with the square of the quantity of electricity with which the insulated body is charged; but "it is readily seen that, in the case of Mr. Harris's "experiments, it will be so slight on the unopposed portions that it could "not be perceived without experiments of a very refined nature, such as "might be made by the proof plane of Coulomb, which is, in fact, with a "slight modification, the instrument employed by Mr. Faraday in the inves-"tigation. Now to the degree of approximation to which the intensity on "the unopposed parts may be neglected, the laws observed by Mr. Harris "when the opposed surfaces are plane may be readily deduced from the "mathematical theory. Thus let v be the potential in the interior of A, the "charged body, a quantity which will depend solely on the state of the inte-"rior coating of the battery with which, in Mr. Harris's experiments, A is "connected, and will therefore be sensibly constant for different positions "of A relative to the uninsulated opposed body B. Let a be the distance "between the plane opposed faces of A and B, and let S be the area of the "opposed parts of these faces, which will in general be the area of the "smaller, if they be unequal. When the distance a is so small that we may "entirely neglect the intensity on all the unopposed parts of the bodies, it "is readily shown, from the mathematical theory, that (since the difference "of the potentials at the surfaces of A and B is v) the intensity of the elec-"tricity produced by induction at any point of the portion of the surface of "B which is opposed to A is $\frac{v}{4\pi a}$, the intensity at any point which is not so "situated being insensible. Hence the attraction on any small element ω, "of the portion S of the surface of B, will be in a direction perpendicular to "the plane and equal to $2\pi\left(\frac{v}{4\pi a}\right)^2$ †. Hence the whole attraction on B is

$$\frac{v^2 S}{8\pi a^2}.$$

"This formula expresses all the laws stated by Mr. Harris as results of "his experiments in the case when the opposed surfaces are plane" ‡.

§ 18. After many trials to make an absolute electrometer founded on the repulsion between two electrified spherical conductors for which I had given a convenient mathematical formula in § 4 of the paper just quoted, it occurred to me to take advantage of the fact noticed by Harris, but easily seen as an immediate consequence of Green's mathematical theory, that the mutual attraction between two conductors used as in his experiments is but little influenced by the form of the unopposed parts; and in 1853, in a paper "On transient Electric Currents" §, I described a method for measuring differences of electric potential in absolute electrostatic measure founded on that idea. The "absolute electrometer" which I exhibited to the British Association

* Philosophical Transactions, 1834.
† See Mathematical Journal, vol. iii. p. 275.
‡ "On the Elementary Laws of Statical Electricity," Cambridge and Dublin Mathematical Journal, 1846; and Phil. Mag. July 1854.
§ Phil. Mag. June 1853.

at its Glasgow Meeting in 1855 was constructed for the purpose of putting these methods in practice. This instrument consists of a plane metal disk insulated in a fixed horizontal position, with a somewhat smaller fixed metal disk hung centrally over it from one end of the beam of a balance. In two papers entitled "Measurement of Electostatic Force produced by a Battery" and "Measurement of the Electromotive Force required to produce a spark in Air between parallel metal plates at different distances," published in the Proceedings of the Royal Society* for February 1860, I described applications of this electrometer, in which, for the first time, I believe, absolute electrostatic measurements were made. The calculations of differences of potentials in absolute measure were made according to the formula quoted above (§ 17) from my old paper on "The Elementary Laws of Statical Electricity."

§ 19. This formula is rigorous only if the distance between the disks is infinitely small in comparison with their diameters; and therefore, in my earliest attempt to make absolute electrostatic measurements, I used very small distances. I found great difficulty in securing that the distance should be nearly enough equal between different parts of the plates, and in measuring its absolute amount with sufficient accuracy; and found besides serious inconveniences in respect of sensibility and electric range: later I made a great improvement in the instrument by making only a small central area of one of the disks movable. Thus the electric part of the instrument becomes two large parallel plates with a circular aperture in one of them, nearly filled up by a light circular disk supported properly to admit of its electrical attraction towards the other being accurately measured in absolute units of force. The disk and the perforated plate surrounding it will be called, for brevity, the disk and the guard-plate. The faces of these two next the other plate must be as nearly as possible in one plane when the disk is precisely in the position for measuring its electric force, which, for brevity, will be called its sighted position. The space between the disk and the inner edge of its guard-ring must be a very small part of the diameter of the aperture, and must be very small in comparison with the distance between the plates; but the diameter of the disk may be greater than, equal to, or less than the distance between the plates.

§ 20. Mathematical theory shows that the electric attraction experienced by the disk is the same as that experienced by a certain part of one of two infinite planes at the same distance, with the same difference of electric potentials, this area being very approximately the mean between the area of the aperture and the area of the disk, and that the approximation is very good, even although the distance between the plates be as much as a fourth or fifth, and the diameter of the disk as much as three fourths of the diameter of the smaller of the two plates. This conclusion will be readily assented to when we consider that† the resultant electric force at any point in the air between the two plates is equal numerically to the rate of conduction of heat per unit area across the corresponding space in the following thermal analogue. Let a solid of uniform thermal conductivity replace all the air between and round the plates; and in place of the plates let there be hollow spaces in this solid. Let these hollow spaces be kept at two uniform temperatures, differing by a number of degrees equal numerically to the difference of potentials in the electric system, the space corresponding to the

* Phil. Mag. September and October 1860.

† "On the Uniform Conduction of Heat through Solid Bodies, and its connexion with the Mathematical Theory of Electricity," Cambridge Mathematical Journal, Feb. 1842, and Phil. Mag. July 1854.

disk and guard-ring being at one temperature, and that corresponding to the opposite plate at the other temperature; and let the thermal conductivity of the solid be unity. If we attempt to draw the isothermal surfaces between the hollow corresponding to the continuous plate on the one side, and that corresponding to the disk and guard-ring on the other side, we see immediately that they must be very nearly plane from very near the disk all the way across to the corresponding central portion of the opposite plate, but that there will be a convexity towards the annular space between the disk and guard-ring.

§ 21. Thus we see that the resultant electric force will, to a very close approximation, be equal to $\frac{V}{D}$ for all points of the air between the plates at distances from the outer bounding edges exceeding two or three times the distance between the plates, and at distances from the interstice between the guard-ring and disk any less than the breadth of this interstice. Hence if ρ denote the electric density of any point of the plate or disk far enough from the edges, we have

$$\rho=\frac{V}{4\pi D}.$$

But the outward force experienced by the surface of the electrified conductor per unit of area at any point is $2\pi\rho^2$; and therefore if F denote the force experienced by any area A of the fixed plate, any part of which comes near its edge, we have

$$F=\frac{V^2A}{8\pi D^2},$$

which will clearly be equal to the attraction experienced by the movable disk, if A be the mean area defined above. This gives $V=D\sqrt{\frac{8\pi F}{A}}$, the formula by which difference of potentials in absolute electrostatic measure is calculated from the result of a measurement of the force F, which, it must be remembered, is to be expressed in kinetic units. Thus if W be the mass in grammes to which the weight is equal, we have

$$F=gW,$$

where g is the force of gravity in centimetres per second.

The difficulty which, in first applying this method about twelve years ago, I found in measuring accurately the distance D between the plates and in avoiding error from their not being rigorously parallel, I now elude by measuring only *differences* of distance, and deducing the desired results from the difference of the corresponding differences of potentials. Thus let V′ be the difference of potentials between the plates required to give the same force F; when the difference of potentials is V′ instead of V, we have

$$V'-V=(D'-D)\sqrt{\frac{8\pi F}{A}}.$$

§ 22. The plan of proceeding which I now use is as follows:—Each plate (fig. 11, Plate VI.) is insulated; one of them, the continuous one, for instance, is kept at a potential differing from the earth by a fixed amount tested by aid of a separate idiostatic* electrometer; the other plate (the guard-ring and movable disk in metallic communication with one another) is alternately connected with the earth and with the body whose potential is to be

* See § 40 below.

measured. The lower plate is moved up or down by a micrometer-screw until the movable disk balances in a definite position, indicated by the hair (with background of white with black dots) seen through a lens, as shown in fig. 11. Before and after commencing each series of electrical experiments, the amount of weight to be placed on the upperside of the disk to bring the hair to its sighted position when there is no electric force is determined. This last condition is secured by putting the two plates in metallic communication with one another. For the electric experiments the weight is removed, so that when the hair is in the sighted position the electric attraction on the movable disk is equal to the force of gravity on the weight. The electric connexions suitable in using this instrument for determining in absolute electrostatic measure the difference of potentials maintained by a galvanic battery between its two electrodes are indicated in fig. 11. No details as to the case for preventing disturbance by currents of air, and for maintaining a dry atmosphere, by aid of pumice impregnated with strong sulphuric acid, are shown, because they are by no means convenient in the instrument at present in use, which has undergone so many transformations that scarcely any part of the original structure remains. I hope soon to construct a compact instrument convenient for general use. The amount of force which is constant in each series of experiments may be varied from one series to another by changing the position of a small wire rider on the lever from which the movable disk is hung.

The electric system here described is heterostatic (§ 40 below), there being an independent electrification besides that whose difference of potential is to be measured.

Portable Electrometer.

§ 23. In the ordinary use of the portable electrometer (figs. 8, 9, & 10, Plate VI.), the electric system is heterostatic and quite similar to that of the absolute electrometer, when used in the manner described above in § 22. But the balance is not adapted for absolute measure of the amount of force of attraction experienced by the movable disk; on the contrary, it is precisely the same as that described for the gauge of the quadrant electrometer in § 13 above, only turned upside down. Thus, in the portable instrument, the square disk (f) forming part of the lever of thin sheet aluminium is attracted *upwards* by a solid circular disk of sheet brass (g), thick enough for stiffness. Every part of the aluminium lever except this square portion is protected from electric attraction by a fixed brass plate ($h\ h$) with a square hole in it, as nearly as may be stopped by the square part of the sheet aluminium destined to experience the electric attraction, all other parts of the aluminium balance-lever being below this guard-plate. The aluminium lever ($i\ k$), as shown in figs. 8 & 10, is shaped so that when the hair (l) at the long end of its lever is in its sighted position, the upper surfaces of the fixed guard-plate (h) and movable aluminium square (f) are as nearly as may be in one plane. The mode of suspension is precisely the same as that described (§ 13) for the gauge of the quadrant electrometer. In the portable instrument, careful attention is given by the maker to balance the aluminium lever by adding to it small masses of shellac or other convenient substance, so that its centre of gravity may be in the line of its platinum-wire axis, or, more properly speaking, in such a position that the instrument shall give, when electrified, the same "earth-readings" when held in any positions, either upright, or inclined, or inverted (§ 30 below). Thus the condition of equilibrium of the balance, when the hair is in its sighted position, is that

the moment of electric attraction round the axis of suspension shall be equal to the moment of the couple of torsion, the latter being as constant as the properties of the matter concerned (platinum wire, brass stretching-springs, &c.) will allow.

§ 24. The guard-plate carrying, by the platinum-wire suspension, the aluminium balance, is attached to the bottom of a small glass Leyden jar (*m m*), and is in permanent metallic communication with its inside coating of tinfoil. The outside tinfoil coating of this jar is in permanent metallic communication with the outside brass-protecting case. The upper open mouth of this case is closed by a lid or roof, which bears on its underside a firm frame projecting downwards. This frame has two V notches, in which a stout brass tube (*o*) slides, kept in the Vs by a properly placed spring (*p*) giving it freedom to slide up and down in one definite line*. Firmly fixed in the upper end of this tube is a nut (*a*, fig. 8), which is caused to move up and down by a micrometer-screw. The lower end of the shaft of this screw has attached to it a convex piece of polished steel (*b*, fig. 8), which is pressed upon a horizontal agate plate rigidly attached to the framework above mentioned by a stiff brass piece projecting into the interior of the brass tube through a slot long enough to allow the requisite range of motion. This arrangement will be readily understood from the accompanying drawings. It has been designed upon obvious geometrical principles, which have been hitherto neglected, so far as I know, in all micrometer-screw mechanisms, whether for astronomical instruments or other purposes. The screw-shaft is turned by a milled head, fixed to it at its top outside the roof of the instrument, and the angles through which it is turned are read on a circle divided into 100 equal parts of the circumference (or $3^\circ{\cdot}6$ each) from a fixed mark on the roof of the instrument. The hole in the roof through which the screw-shaft passes is wide enough to allow the shaft to turn without touching it, and the lower edge of the graduated circle turning with the screw is everywhere very near the upperside of the roof, but must not touch it at any point. A second nut (*c*, fig. 8) above the effective nut fits easily, but somewhat accurately, in the hollow brass tube, but is prevented from turning round in the tube by a proper projection and slot. Thus the screw is rendered sufficiently steady, with reference to the sliding-tube; that is to say, it is prevented from any but excessively small rotations round axis perpendicular to the length of the screw-shaft; and when the nut is kept from being turned round its proper axis, it forms along with the sliding-tube virtually a rigid body. A carefully arranged spiral spring presses the two nuts asunder, and so causes the upperside of the thread of the screw-shaft always to press against the underside of the thread of the effective nut, thus doing away with what is technically called in mechanics "lost time." In turning the micrometer-screw, the operator presses its head gently downwards with his finger, to secure that its lower end bears firmly upon the agate plate. It would be the reverse of an improvement to introduce a spring attached to the roof of the instrument outside to press the screw-head downwards, inasmuch as however smooth the top of the screw-shaft might be made, and however smooth the spring pressing it down, there would still be a very injurious friction impeding the proper settlement of the sliding-tube into its Vs. A stiff fork (*q*) stretching over the graduated circle is firmly

* In consequence of suggestions by Mr. Jenkin, it is probable that the spring may be done away with, and the Vs replaced by rings approximately fitting round the tube, but leaving it quite free to fall down by its own weight. In consequence of the symmetrical position of the convex end of the screw over the centre of the attracted disk, slight lateral motions of the tube produce no sensible effect on the electric attraction.

attached to the roof outside, to prevent the screw from being lifted up by more than a very small space, perhaps not more than $\frac{1}{20}$ of an inch at most. In using the instrument, the observer should occasionally pull up the screw-head and press it down again, and give it small horizontal motions, to make sure that when he is using it it is pressed in properly to its Vs and down upon the agate plate. A long arm (*d*, figs. 8 and 9) (or two arms one above the other), firmly attached to the sliding-tube, carries a pointer which moves up and down with it. Two fixed guiding-cheeks on each side of this pointer prevent the tube from being carried round too far in either direction when the screw is turned: one of these cheeks is graduated so that each division is equal in length to the step of the micrometer-screw; this enables the operator to ascertain the number of times he has turned the screw. These two cheeks must never simultaneously press upon the sliding-pointer; on the contrary, they must leave it a slight amount of lateral freedom to move. If this does not amount to ·36 of a degree, the amount of "lost time" produced by it will not exceed $\frac{1}{10}$ of a division of the micrometer-circle, and will not produce any sensible error in the use of the instrument. A glass rod cemented to the lower end of the tube prolongs its axis downwards, and bears the continuous attracting-plate of the electrometer at its lower end.

The object aimed at in the mechanism just described is to prevent the nut and other parts rigidly connected with it from any other motion than parallel to one definite line, and to leave it freedom to move in this line, unimpeded by any other friction than that which is indispensable in the arrangement for keeping the sliding-tube in its Vs.

§ 25. If the inner tinfoil covering of the Leyden jar were completed up to the guard-plate bearing the aluminium balance, the long arm of this lever being in the interior of a hollow conductor would experience no electric influence and no force from the electrification of the Leyden jar, or from separate electrification of the upper attracting-plate, or, more strictly speaking, the electric density and consequent electric force on the long arm of the lever would be absolutely insensible to the most refined test we could apply, because of the smallness of the gap between the movable aluminium square and the boundary of the square aperture in the guard-plate. But to see the hair on the long end of the lever, and the white background with black dots behind it, a good portion of the glass under the guard-plate must be cleared of tinfoil outside and inside. Thus the electric potential of the inner coating of the Leyden jar will not be continued quite uniformly over the inner surface of the bared portion of the glass, and a disturbance affecting chiefly the most sensitive part of the lever will be introduced. To diminish this as much as possible without inconveniently impeding vision, a double screen of thin wire fences, in metallic communication with the inner tinfoil coating and the guard-plate, is introduced between the end of the lever and the glass through which it is observed.

§ 26. A very light spiral spring (*r*) connects the upper attracting-plate with a brass piece supported upon a fixed vertical glass column projecting downwards from the roof of the instrument. This brass piece bears a stout wire (*s*), called the main electrode, projecting vertically upwards along the axis of a brass tube open at each end, fixed in an aperture in the roof so as to project upwards and downwards, as shown in fig. 9.

§ 27. The top of the main electrode bears a brass sliding-piece (*t*), which, when raised a little, serves for umbrella and wind-guard without disturbing the insulation; and when pressed down closes the aperture and puts the electrode in metallic connexion with the roof of the instrument. When the instru-

ment is to be used for atmospheric electricity (unless at a fixed station) a steel wire, about 20 centimetres long, is placed in the hole on the top of the sliding brass piece just mentioned, and is thus held in the vertical position. A burning match is attached to its upper end, which has the effect of bringing the potential of the chief electrode and upper attracting-plate &c. all to the potential of the air at the point where the match burns*. The instrument is either held in the observer's hand, or it is placed upon a fixed support, and care taken that its outer brass case is in connexion with the earth. When the difference of potentials between two conductors is to be tested, one of these is connected with the brass case of the instrument, and the other with the chief electrode, the umbrella being kept up. If both of these conductors must be kept insulated from the earth, the brass case of the electrometer must be put on an insulating stand, and the micrometer-screw turned by an insulating handle.

§ 28. A lead cup (*e e*, fig. 8), supported by metal pillars from the roof and carrying pieces of pumice-stone, held in their place by India-rubber bands, completes the instrument. The inner surface of the glass must be clean, and particles of dust, minute shreds or fibres, &c. removed as carefully as possible, especially from the lower surface of the upper attracting-plate, and the upper surface of the guard-plate and aluminium square facing it from below. The pumice is prepared by moistening it with a few drops of strong pure sulphuric acid. Ordinary sulphuric acid of commerce should be boiled with sulphate of ammonia to free it from volatile acid vapours, and to strengthen it sufficiently by removing water if the acid be not of the strongest. There should not be so much acid applied to the pumice as to make it have the appearance of being moist, but there must be enough to maintain a sufficiently dry atmosphere within the instrument for very perfect insulation of the Leyden jar, which I find does not in general lose more of its charge than 5 per cent. per week when the pumice is properly acidulated. Thus there is no tendency of the liquid to drop out of the pumice; and the pumice being properly secured by the india-rubber bands, the instrument may be thrown about with any force, short of that which might break the glass jar or either of the glass stems, without doing any damage; but to ensure this hardiness the sheet aluminium of which the balance is made must be *very thin*. After several weeks' use the pumice may commence to look moist, and even slight traces of moisture may be seen on the outside of the lead cup, in consequence of watery vapour attracted by the sulphuric acid from the atmosphere; but the pumice should then be taken and dried. At all events this must be done in good time, before enough of liquid has collected to give any tendency to drop. In all climates in which I have hitherto tested the instrument, I have found the pumice effective for insulation and safe in keeping all the liquid to itself for two months. But it having been reported to me by Mr. Becker that many instruments have been returned to him in a ruinous condition from drops of sulphuric acid having become scattered through their metal work, I now cause to be engraved conspicuously on the outer case of the instrument "PUMICE DANGEROUS, IF NOT DRIED ONCE A MONTH;" also a frame carrying a card, on which the dates of drying are inscribed, to be placed in a convenient position on the roof of the instrument.

§ 29. To prepare the instrument for use, the inner coating of the Leyden jar must be charged through a charging-rod, insulated in a vulcanite or glass tube and let down for the occasion through a hole in the roof of the

* See Nichol's Cyclopædia, article "Electricity, Atmospheric," 2nd edition, 1860; or Royal Institution Lecture on Atmospheric Electricity, May 1860.

instrument, by aid of a small electrophorus, which generally accompanies the instrument, or by an electrical machine. I generally prefer to give a negative charge to the inner coating, as I have not found any physical reason, such as that mentioned in § 9 above, to prefer a positive charge to a negative charge; and the negative charge gives increased readings of the micrometer, in the ordinary use of the instrument, to correspond to positive charges of the principal electrode, as will be presently explained. Before commencing to charge the jar, the upper attracting-plate should be moved to nearly the highest position of its range by the micrometer-screw, otherwise too stiong a force of electric attraction may be put upon the aluminium square; and, besides, the jar will discharge itself between the upper plate and the extreme edge of the aluminium square, pulled as it is very much above the level of the guard-plate by the electric attraction. I have not found any injury or change of electric value of the scale-divisions to arise from any such rough usage; but still, to guard against such a possibility, I propose to add to the guard-plate checks to prevent the corners of the aluminium from rising much, if at all, above its level, and to conduct the discharge and protect the aluminium and platinum from the shock, in case of the upper plate being brought too near the lower. When the instrument is being charged, or when it is out of use at any time, the umbrella should always be kept down; but it must be raised to insulate the principal electrode, of course, before proceeding to apply this to a body whose difference of potential from a body connected with the case of the instrument is to be measured.

§ 30. In using the instrument the umbrella must very frequently be lowered, or metallic communication established in any other convenient way between the chief electrode and the outer brass case, the micrometer-screw turned until the hair takes its sighted position, and the reading taken, the hundreds being read on the interior vertical scale, and the units (or single divisions of the circle) on the graduated circle above. The number thus found is called the earth-reading; it measures the distance from an arbitrary zero position to the position in which the upper attracting-plate must be placed to give the amount of electric force on the aluminium square which balances the lever in its sighted position. A constant added to the earth-reading, or subtracted from it, gives (§ 1) a number simply proportional to the difference of potentials between the upper and lower plate; that is to say, between the two coatings of the Leyden jar. The vertical scale and micrometer-circle are numbered, so that increased distances between the plates gives increased readings; and the zero reading should correspond as nearly as may be to zero distance between them, although in the instruments hitherto made no pains have been taken to secure this condition, even somewhat approximately. If it is desired to know the constant, an electrical experiment must be made to determine it, which is done with ease; but this is not necessary for the ordinary use of the instrument, which is as follows.

§ 31. First an earth-reading is taken, then the upper electrode is insulated by raising the umbrella, or otherwise breaking connexion between the principal electrode and the outer metal case of the instrument. The principal electrode and the outer case are then connected with the two bodies whose difference of potential is to be determined, and the micrometer-screw is turned until the hair is brought to its sighted position. The reading of hundreds on the vertical scale and units on the circle is then taken. Lastly, the principal electrode is again connected with the case of the instrument and another earth-reading is taken. If the second earth-reading differs

from the first, the observer must estimate the most probable earth-reading for the moment when the hair was in its sighted position, with the upper plate and the metal case in connexion with the two bodies whose difference of potential is to be measured. The estimated earth-reading is to be subtracted from the reading taken in connexion with the bodies to be tested. This difference measures (§ 21) the required difference of potentials between them in units of the instrument. The value of the unit of the instrument ought to be known in absolute electrostatic measure; and the difference of reading found in any experiment is to be multiplied by this, which is called (§ 1) the absolute coefficient of the instrument, to give the required difference of potentials in absolute measure. It so happens that, in the portable electrometers of the kind now described which have been hitherto constructed, the absolute coefficient is somewhere about ·01, so that one turn of the screw, or 100 divisions of the circle, corresponds to somewhere about one electrostatic unit, with a gramme for the unit of mass, a centimetre for the unit of distance, and a second for the unit of time; but the different instruments differ from one another by as much as ten or twenty per cent. in their absolute coefficients. In all of these I have found between three and four Daniell's cells to correspond to the unit division; that is to say, between three hundred and four hundred cells to a full turn of the screw. With great care, the observer may measure small differences of potentials by this instrument to the tenth part of a division (or to about half a Daniell's cell). With a very moderate amount of practice and care, an error of as much as a half division may be avoided in each reading.

§ 32. But there are imperfections in the instrument itself which make it difficult or impossible to secure very minute accuracy, especially in measurements through wide ranges.

(1) In the first place, I am not sure that the end of the needle carrying the hair is protected sufficiently by the wire fences (§ 25) from electric disturbance to provide against any error from this source, which possibly introduces serious irregularities.

(2) In the second place, the capacity of the jar in the small portable instrument is not sufficient to secure that the potential of its inner coating shall not differ sensibly with the different distances to which the upper plate is brought to balance the aluminium lever with the hair in its sighted position. But on this point it is to be remarked that the electric density on the upper surface of the guard-plate is in its central parts always the same when the hair is in its sighted position; and it is therefore only the comparatively small difference of the quantity of electricity on this surface, towards the rim, corresponding to different distances of the attracted plate, that causes difference of potential in the inner coating of the jar. But if the upper attracting-plate be kept for several minutes at any distance, differing by a few turns of the screw, from that which brings the hair to its sighted position, the electricity creeps along the inner unconnected surface of the glass, so as to increase the charge of the inner metallic coating or diminish it, according as the distance is too great or too small. If then quickly the screw be turned and the earth-reading taken, it is found greater or smaller, as the case may be, than previously; but after a few minutes more it returns to its previous value very approximately. Error from this source may be practically avoided by taking care never to allow the hair to remain for more than a few minutes far from its sighted position—never so far, for instance, as above the centre of the upper, or below the centre of the lower dots.

(3) A third source of error arises from change of temperature influencing

the indications. In most of the instruments hitherto made I have found that the warmth of the hand produces in a few minutes a very notable augmentation of the earth-reading (as it were an increased charge in the jar); but in the last instrument which I have tested (White No. 18) I find the reverse effect, the earth-reading becoming smaller as the instrument is warmed, or larger when it is cooled. I have ascertained that these changes are not due to changes in the electric capacities of the Leyden jars; and I have found that the change, if any, of specific inductive capacity of glass by change of temperature is excessively small, in comparison to what would be required to account for the temperature errors of these instruments, which probably must be due to thermo-elastic properties of the platinum wire, or of the stretching-springs, or of the aluminium balance-lever, or to a combination of the effects depending on such properties; but I have endeavoured in vain, for several years, and made many experiments, to discover the precise cause. It surely will be found, and means invented for remedying the error, now that I have an instrument in which the error is in the opposite direction to that of most of the other instruments. It is of course much greater in some instruments than in others: in some it is so great that the earth-reading is varied by as much as twenty divisions by the warmth of the hand in the course of five or ten minutes after commencing to use the instrument, if it has been previously for some time in a cold place. Its influence may be eliminated, not quite rigorously, but nearly enough so for most practical purposes, by frequently taking earth-readings (§ 30) and proceeding according to the directions of § 31.

(4) A fourth fault in the portable electrometer is, that the diameter of the guard-plate and upper attracting-disk, which ought to be infinite, are not sufficiently great, in proportion to the greatest distance between them, to render the scale quite uniform in its electric value throughout. A careful observer, however, will remedy the greater part of the error due to this defect, by measuring experimentally the relative (or absolute) values of the scale-division in different parts of the range. There will, however, remain uncorrected some irregularity, due to influence of the distribution of electricity over the uncoated inner surface, in the instruments as hitherto made, in all of which the inner surface of the jar is coated with tinfoil only below the guard-plate, so that the upper surface of the guard-plate may be seen clearly, in order that the observer may always see that all is in order about the aluminium square and aperture round it; and particularly that there are no injurious shreds or minute fibres. But the irregular influence of the electrification of the uncoated glass, if found sensible, will be rendered insensible by continuing the tinfoil coating an inch above the upper surface of the guard-plate.

§ 33. All faults, except the temperature error, depend on the smallness of the instrument; and if the observer chooses to regard as portable an instrument of thirty centimetres (or a foot) diameter, with all other dimensions and all details of construction the same as those of the instrument described above, he may have a portable electrometer practically free from three of the four faults described. But it is scarcely to be expected that a small instrument ($12\frac{1}{2}$ centimetres high, and $8\frac{1}{2}$ centimetres in diameter) which may be carried about in the pocket can be free from such errors. They are, however, so far remedied as to be probably not perceptible in the large stationary instrument which I now proceed to describe.

STANDARD ELECTROMETER.

§ 34. This instrument (figs. 12, 13, & 14, Plate VI.) differs from the portable electrometer only in dimensions, and in certain mechanical details, which are arranged to give greater accuracy by taking advantage of freedom from the exigencies of a small portable instrument. It is at present called the standard electrometer, in anticipation of either remedying or of learning to perfectly allow for the temperature error, and of finding by secular experiments on the elasticity of metals that their properties used in the instrument are satisfactory as regards the permanence from year to year, and from century to century, of the electric value of its reading. It is an instrument capable of being applied with great ease to very accurate measurements of differences of potential, in terms of its own unit. The value of the unit for each such standard instrument ought, of course, to be determined with the greatest possible accuracy in absolute measure; and until confidence can be felt as to its secular constancy, determinations should frequently be made by aid of the absolute electrometer.

§ 35. The Leyden jar of the standard electrometer consists of a large thin white-glass shade coated inside and outside to within 6 centimetres of its lip, and placed over the instrument as an ordinary glass shade, to protect against dust, currents of air, and change of atmosphere. It may be removed at pleasure from the cast-iron sole of the instrument, and then the interior works are seen, consisting of:—

(1) A continuous disk of brass supported on a glass stem, in prolongation of a stout brass rod or tube sliding vertically in Vs, in which it is kept by a spring, and resting with its lower flat end on the upper end of a micrometer-screw shaft, shown in fig. 13, where the screw, graduated circle, and stout brass rod are as seen in the instrument; the perforated brass disk (which is intended to keep the round upper end of the screw-shaft in position) is shown in section in fig. 14.

(2) Resting on three glass columns, a guard-plate with a square aperture in its centre, and carrying on its upperside stretching-springs and thin platinum-wire suspension of an aluminium balance-lever, shaped like those of the gauge (§ 13) and the portable electrometer (§ 23) already described, but somewhat larger. The tops of the three glass columns are rounded; a round hole and a short slot in line with this hole are cut in the guard-plate and receive the rounded ends of two of the columns, which are somewhat longer than the third. The flat smooth lower surface of the guard-plate rests simply on the top of the third glass column. The diameter of the round hole and the breadth of the slot in the guard-plate may be about $\frac{1}{\sqrt{2}}$ of the diameter of curvature of the upper hemispherical rounded ends of the glass column, so that the bearing portions of the rounded ends in the round hole and in the slot respectively may be inclined somewhere about 45° to the plane of the plate. This well-known but too often neglected geometrical arrangement gives perfect steadiness to the supported plate, without putting any transverse strain upon the supporting glass columns, such as was almost inevitable, and caused the breakage of many glass stems, before mental inertia opposing deviations from the ordinary instrument-maker's plan (of screwing the guard-plate to brass mountings cemented to the tops of the glass columns) was overcome. It has also the advantage of allowing the guard-plate to be lifted off and replaced in a moment.

(3) Principal electrode projecting downwards through a hole in the sole

of the instrument, and rigidly supported from above by a brass mounting cemented to the top of a thick vertical glass column, connected by a light spiral spring with the lower attracting-plate moved up and down by the micrometer-screw. The aperture round the principal electrode may be ordinarily stopped by a perforated column of well-paraffined vulcanite projecting some distance above and below the aperture, which I find to insulate extremely well, even in the smoky, dusty, and acidulated atmosphere of Glasgow. When an extremely perfect insulation of the principal electrode and connected attracting-plate is required, the vulcanite stopper surrounding it may be removed, so that the only communication between the electrode and the case of the instrument may be along the two glass columns in the artificially dried interior atmosphere of the case; but from day to day, when the instrument is out of use, the aperture round the principal electrode should be kept carefully stopped, if not by a vulcanite insulator, by a perforated cork (although I find but little loss of insulation, either by the inner glass surface of the Leyden jar or by the three glass columns, when this precaution is neglected).

(4) Temporary charging-rod supported by a vertical perforated column of paraffined vulcanite, or a glass tube well varnished outside and thickly paraffined inside. The insulating column bearing this charging-rod is turned round till a horizontal spring projecting from its upper end touches the inner coating of the jar, when this is to be charged from an independent source, or when, for any other experimental reason, it is to be put in connexion with a conductor outside the case of the instrument.

(5) A small replenisher of the kind described for the quadrant electrometer (§ 12), but with much wider air-spaces to prevent discharge by sparks.

(6) A large glass or lead dish to hold as large masses of pumice as may be, which are to be kept sufficiently impregnated with strong sulphuric acid.

§ 36. A considerable portion of the jar above the guard-plate is left uncoated to allow the observer to see easily the hair and white background with black dots; also several other smaller parts of the glass above the guard-plate are left uncoated to admit light to allow a small circular level on the upper side of the guard-plate to be seen. The long arm of the aluminium balance-lever is very thoroughly guarded by double cages and fences of wire (§ 25), so that it can experience no sensible influence from electric disturbing forces when the covering jar is put in position and electric connexion is established between its inner coating and the guard-plate by projecting flexible wires or slips of metal.

§ 37. The aluminium square plate is somewhat larger and the platinum bearing wire somewhat longer in this instrument than in the portable electrometer, to render it sensible to smaller differences of potential. The step of the screw is the same as in the portable ($\frac{1}{50}$ of an inch), and one division ($\frac{1}{100}$ of the circumference of the screw-head) corresponds to a difference of potentials which, roughly speaking, is equal to about that of a single cell of Daniell's. The effective range of the instrument is about sixty turns of the screw, and therefore about 6000 cells of Daniell's; that of the portable electrometer is about 15 turns of the screw (equivalent to about 5000 cells). Neither of these instruments has sufficient range to measure the potential to which Leyden jars are charged in ordinary electric experiments, or those reached by the prime conductor of a powerful electric machine. The stationary instrument with its long screw and its large plates now described would go far towards meeting this want if its aluminium lever and platinum suspension were made on the same scale as those of the portable electrometer;

but for an instrument never wanted to directly measure differences of potentials of less than two or three thousand cells, the heterostatic (§ 40) principle is in general not useful; and therefore I have constructed the following very simple idiostatic (§ 40) instrument, which is adapted to measure with considerable accuracy differences of potential from 4000 cells upwards to about 80,000 cells.

Long-range Electrometer.

§ 38. In this (fig. 15, Plate VI.) the continuous attracting-plate is above, and the guard-plate with aluminium balance below, as in the portable electrometer; but, as in the standard stationary electrometer, the upper plate is fixed and the lower plate is moved up and down by a micrometer-screw. The mechanism of the screw and slide has all the simplicity and consequent accuracy of that of the standard electrometer. In the only long-range instrument yet constructed the step of the screw is the same as that of the others ($\frac{1}{50}$ of an inch). In future instruments it would be well either to have a longer step or to have a simple mechanism (which can be easily added) to give a quick motion, as in the use of the present instrument the turning of the screw required for great changes of the potential measured is very tedious. The guard-plate projects by more than an inch all round beyond the rim of the upper attracting-plate—partly to obviate the necessity of giving it a thick rim, which would be required to prevent brushes and sparks originating in it if it had only the same diameter as the continuous plate above, and partly to guard the observer from receiving a spark or shock in measuring the potential of an electric machine or of a Leyden battery, and to prevent the hair from being attracted to the upper plate. Thus the guard-plate is allowed to be no thicker than suffices for stiffness; and this allows the observer to see the hair at the end of the aluminium balance-lever without the lever being made of a dynamically disadvantageous shape, as would be necessary if the guard-plate were thick or had a thick rim added to it. No glass case is required for this instrument. The smallness of the needle and the greatness of the electric force acting on it are such that I find in practice no disturbance to any inconvenient degree by ordinary currents of air; although it and all these attracted disk instruments show the influence of sudden change of barometric pressure, such as that produced by opening or shutting a door. If not kept under a glass shade when out of use, the lower surface of the upper attracting-plate and the lower surface of the guard-plate and attracted aluminium square should be carefully dusted by a dry cool hand. Generally speaking, none of the vital electric organs of an electrometer should be touched by a cloth, as this is almost sure to leave shreds fatal to their healthy action.

§ 39. The effective range of this instrument is about 200 turns of the screw; rather greater force of torsion is given than in the portable electrometer, and a rather smaller attracted disk may be used, so that upwards of four cells may be the electric value of one division. The instrument in its present state measures nearly, but not quite, the highest potential I can ordinarily produce in the conductor of a good Winter's electric machine, which sometimes gives sparks and brushes a foot long.

§ 40. The classification of electrometers given above is founded on the shape and kinematic relations of their chief organic parts; but it will be remarked that another principle of classification is presented by the different electric systems used in them, which may be divided into two classes :—

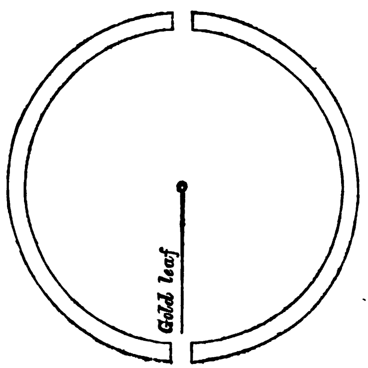
I. Idiostatic, that in which the whole electric force depends on the electrification which is itself the subject of the test.

II. Heterostatic, in which, besides the electrification to be tested, another electrification maintained independently of it is taken advantage of.

Thus, for example, the long-range electrometer (§§ 38, 39) is simply idiostatic and is not adapted for heterostatic use; but each of them may be used idiostatically. The absolute electrometer was at first simply idiostatic (§§ 17–21); more recently it has been used heterostatically, and is about to acquire (§ 22) special organs adapted for heterostatic use; as yet, however, no species of the absolute electrometer promising permanence has come into existence.

§ 41. It is instructive to trace the origin of various heterostatic species of electrometers by natural selection. A body hanging, or otherwise symmetrically balanced, in the middle of a symmetrical field of force, but free to move in one direction or the other in a line tangential to a line of force, moves in one direction or the opposite when electrified positively or negatively. Bohnenberger's arrangement of this kind has a convenient and approximately constant field of force; and his instrument was chosen in preference to others which may have been equally sensitive, but were less convenient and constant, and it became a permanent species.

§ 42. Bennet's gold-leaf electroscope, constructed with care to secure good insulation, electrified sufficiently to produce a moderate divergence, has been often used to test, by aid of this electrification, the quality of the electrification of an electrified body brought into the neighbourhood of its upper projecting electrode, causing, if its elasticity is of the same sign as that of the gold leaves, increase of divergence; if of the opposite sign, diminution. By connecting the upper electrode with the inner coating of a Leyden jar with internal artificially dried atmosphere, the charge of the gold leaves may be made to last with little loss from day to day; and by insulating Faraday's metal cage (§ 2) round the gold leaves and alternately connecting it with the earth and with a conducter whose difference of potentials from the earth is to be tested, an increase or a diminution of divergence is observed according as this difference is negative or positive, the gold leaves being positive. Hence (through Peltier's and Delmann's forms) the heterostatic stationary and portable repulsion electrometers, described in the Royal Institution Lecture on "Atmospheric Electricity" and in Nichol's Cyclopædia, article "Electricity, Atmospheric," already referred to, of which one species still survives in King's College, Nova Scotia, and in the Natural Philosophy Class-room of Edinburgh University. The same form of the heterostatic principle applied to Snow Harris's attracted-disk electrometer gave the portable and standard electrometers described above.



§ 43. A modification of Bohnenberger's electroscope, in which the two knobs on the two sides of the hanging gold leaf became transformed into halves of a circular cylinder, with its axis horizontal and the gold leaf hung on a wire insulated in a position coinciding with its axis, producing a species designed for telegraphic purposes, but which did not acquire permanence by natural se-

lection, and is only known to exist in one fossil specimen. In this instrument the wire bearing the gold leaf was connected with a charged Leyden jar, and the semicylinders with the bodies whose difference of potential was to be tested. But various modifications of the divided-cylinder or divided-ring class with the axis vertical and plane of motion horizontal have done some practical work, and one species, the new quadrant electrometer (§ 6), promises to become permanent.

§ 44. The heterostatic principle in one form or other is essential to distinguish between positive and negative. As remarked above (§ 42), the original type of this use of it is to be found in the old system of testing the quality of the charge taken by the diverging straws or gold leaves of the electroscopes used for the observation of atmospheric electricity, which was done by bringing a piece of rubbed sealing-wax into the neighbourhood, and observing whether this caused increase or diminution of the divergence. A doubt which still exists as to the sign* of the atmospheric electricity observed by Professor Piazzi Smyth on the Peak of Teneriffe, is owing to the imperfection of this way of applying the principle. It is, indeed, to be doubted in any one instance whether it is not vitreous electricity that the rubbed sealing-wax acquires; and, again (§ 2), it is not certain that the glass case enclosing the gold leaves, especially if very clean and surrounded by a very dry natural atmosphere, screens them sufficiently from direct influence of the piece of sealing-wax to make sure that the divergence due to vitreous electricity could not be increased by the presence of the resinously electrified sealing-wax if held nearer the gold leaves than the upper projecting stem.

§ 45. The heterostatic principle has a very great advantage as regards sensibility over any simple idiostatic arrangement, inasmuch as, for infinitely small differences of potential to be measured, the force is as the squares of the differences in any idiostatic arrangement, but is simply proportional to the difference in every heterostatic arrangement.

VI. *Determination of the Dynamical Equivalent of Heat from the thermal effects of Electric Currents.* *By* J. P. Joule, *D.C.L., F.R.S., &c.*

Sir W. Thomson, as long ago as 1851, showed that it was desirable to make experiments such as are the subject of the present paper. They have necessarily been delayed until a sufficiently accurate method of measuring resistance was discovered. Such a method having been described by Sir William, and carried out into practice by Professor C. Maxwell and his able coadjutors, the task assigned to me by the Committee of Electric Standards was comparatively simple.

My experiments were commenced nearly two years ago, and the apparent ease with which they could be executed gave promise of their early completion. It was, however, found essential that careful observations of the earth's horizontal magnetic intensity should be frequently made, and these required the construction of apparatus whereby this element could be determined with accuracy and rapidity.

The apparatus finally adopted for this purpose consists of a suspended horizontal flat coil of wire between two fixed similar coils. A current of electricity can be made to traverse all three, communication with the sus-

* Nichol's Cyclopædia, article "Electricity, Atmospheric," edition 1860.

pended coil being made by the suspending wires themselves according to Sir W. Thomson's plan. The strength of a current is found by observing the sum of the forces of attraction and repulsion by which the suspended coil is urged. The strength of a current can in this manner be determined in absolute measure; for the area of each of the three equal coils being called a, the weight required to counterpoise the force with which the suspended one is urged w, the force of gravity g, and the length of wire in each of the coils l, the current $c=\frac{1}{2l}\sqrt{\frac{agw}{\pi}}$ (1 + correction), the correction being principally due to the distance between the fixed coils. In my instrument, in which this distance is 1 inch, the diameter of the coils being 12 inches and their interior core 4 inches, this correction was proved by experiment to be ·1185.

There was, however, considerable difficulty in obtaining an exact measure of the distance between the fixed coils; and I therefore judged that the measure of the currents used in the experiments would be most accurately obtained by means of a tangent galvanometer, the above-described current-meter being employed to determine the horizontal intensity.

This determination was effected as follows:—Many careful observations of the horizontal intensity by an improved method on Gauss and Weber's system were made alternately with observations of the deflections of a tangent galvanometer and the weighings of the current-meter when the same currents traversed both instruments in succession. Then calling the horizontal intensity H, the angle of deflection θ, and the weighing w, there was obtained a constant $c=\frac{\text{H} \tan \theta}{\sqrt{w}}=\cdot 17676$. Hence with these instruments

$$\text{H}=\frac{\cdot 17676\sqrt{w}}{\tan \theta}.$$

The experiments for the determinations of horizontal intensity by the use of this formula could be effected in a few minutes, and did not require an alteration in the disposition of any part of the apparatus. It was satisfactory to find that, although the presence of masses of iron at only a few yards distance made the field in which I worked considerably more intense than that due to the latitude, and although I worked at different times of the day, the highest intensity, out of upwards of seventy observations distributed over a year, was 3·6853, and the lowest 3·6607, indicating a much greater degree of constancy than might have been expected.

The galvanometer above mentioned was that employed in the thermal experiments. It had a single circle of $\frac{1}{10}$-inch copper wire, the diameter of which, being measured in many places by a standard rule, gave a radius of ·62723 of a foot. The needle was half an inch long, and furnished with a glass pointer traversing a divided circle of 6 inches diameter. In the experiments the deflections were not far from 26° 34′, the angle at which the influence of the length of the needle within certain limits is inappreciable. It was easy by a magnifier, arranged so as to avoid parallax, to read to one minute. The torsion of the fibre gave only 3′·5 for an entire twist. The trifling correction thus required is applied to the recorded observations of deflection.

The calorimeter first used was a copper vessel upwards of a gallon in

capacity, filled with distilled water. It had a conical lid, attached by screws, in which were two tubulures, one for the introduction of a copper stirrer, the other for the thermometer, around the immersed stem of which a wire of platinum silver, having a resistance nearly equal to that of the Association unit, was coiled.

The resistance of the wire was found by comparing it with the Association unit, sent me by the Committee, using Ohm's formula, $R=\frac{C_2}{C_1}\left(\frac{C_3-C_1}{C_3-C_2}\right)$, where C_3, C_2, and C_1 are the tangents of deflection with the battery and connexions only with these and the unit and with the coil respectively. This, though by no means so delicate a method as that of the Wheatstone balance improved by Thomson, was able to give a final result certainly accurate to the two-thousandth part. The results for the resistance of the coil in the first series of experiments are as follow. They were obtained before and after those experiments. A large galvanic cell, consisting of cast iron and amalgamated zinc plunged in dilute sulphuric acid, was the source of electricity, which was measured by a galvanometer with a coil of nine turns, 17 inches in diameter.

C_3.	C_2.	C_1.	Temperature of unit.	Temperature of coil.	Resistance of coil in terms of unit.
tan 55° 6′·75	tan 28° 18′	tan 28° 1′·3	63°·7	62°·65	1·01901
tan 59 32·5	tan 32 39·6	tan 32 22	59·24	58·39	1·01825

The average resistance 1·01863 being reduced from the temperature 14°·5 Cent., at which the unit was adjusted, to 69°·9 Fahr., the average temperature of the calorimeter in the first series of experiments, becomes 1·0191, which, multiplied by 32808990, gives 33435640 as the resistance in British absolute measure.

A delicate thermometer was placed at a few inches distance from the calorimeter, for the purpose of registering the temperature of the air. In the Tables its indications are reduced to the scale of the instrument plunged in the calorimeter. A string attached the handle of the stirrer to a stick, so that the water could be effectually stirred without communicating the heat of the hand. A wooden screen separated the observer from the apparatus.

In the experiments of the first series a battery of five large Daniell's cells, arranged in series, transmitted the current through the coil for 40′ exactly, determined by chronometer. During this time twenty-eight observations of deflection were obtained, seven at each end of the pointer directed N.E. and S.W., and seven when it was directed N.W. and S.E. by reversing the current in the galvanometer for the latter half of the time. The water was stirred twenty-eight times. Its temperature was taken at the beginning, middle, and end of an experiment. There were also fourteen observations of the temperature of the air.

Immediately after each experiment the horizontal intensity of magnetic force was obtained by observing the deflection of the galvanometer and the weighing of the current-meter produced by the same current.

Before and after each experiment, two others were made in precisely the same manner, but excepting the current, in order to discover the influence of radiation and the conducting power of the atmosphere.

First Series of Thermal Experiments.

Date.	Deflection.	tan². Deflection.	Temperature of air.	Temperature of water.	Rise of temperature.	Horizontal intensity.
1866.	° ′					
Aug. 22 ...	32 46·86	·414719	492·36	497·42	23·55	3·6763
„ 23 ...	34 0·29	·455133	494·77	493·27	25·65	3·6815
Sept. 8 ...	32 24·83	·403156	400·4	401·8	22·8	
„ 10 ...	31 50·22	·385542	441·11	433·85	22·214	3·6737
„ 11 ...	31 31·02	·376024	367·0	392·89	18·51	3·6758
„ 12 ...	31 14·42	·367944	344·33	344·45	21·9	3·6656
„ 13 ...	30 57·51	·359850	361·54	358·47	20·95	3·6671
„ 15 ...	30 24·86	·344607	346·7	330·01	21·98	3·6638
„ 15 ...	30 20·51	·342610	381·41	367·56	21·07	3·6711
„ 18 ...	30 34·34	·348982	342·64	324·32	22·29	3·6607
Average ...	....	·379857	397·226	394·406	22·0914	3·67073

First Series of Radiation Experiments.

Date.	Temperature of air.	Temperature of water.	Rise of temperature of water.
1866.			
Aug. 22	495·93	469·14	2·88
„	502·22	477·83	3·15
Aug. 23	476·37	458·96	3·08
„	490·81	499·22	−0·55
Sept. 8	393·5	382·75	2·0
„	395·82	414·15	−1·7
Sept. 10	444·31	419·4	2·9
„	437·15	396·96	4·83
Sept. 11	373·07	384·72	−0·63
„	367·14	391·76	−1·75
Sept. 12	334·0	332·42	0·44
„	365·34	360·2	1·6
Sept. 13	352·82	343·11	1·83
„	366·65	369·16	−0·08
Sept. 15	330·78	315·41	2·78
„	381·47	347·14	3·72
„	378·93	350·67	3·34
„	381·05	379·51	0·22
Sept. 18	326·99	309·28	2·55
„	339·9	338·35	0·04
Average	373·058	364·686	1·3806

In applying the preceding Table for the purpose of correcting the results of the thermal experiments, it must be first observed that the external influences on the calorimeter are not zero when the temperature of the air-thermometer coincides with the indication of that immersed in the calorimeter. This might arise partly from the locality of the two instruments not being the same, but was, I found, principally owing to the different radiating and absorbing powers of the air-thermometer bulb and of the surface of the calorimeter. Taking, then, the number of instances in which the temperature of the air appeared to exceed that of the water, there are fifteen, with a

total excess of 259·63 and a resulting gain of temperature of 35·36; also those in which the air appeared to be colder than the water were five, giving a total deficiency of 65·5 with a loss of temperature 4·71. Hence $\frac{65{\cdot}5-5x}{4{\cdot}71}$ $=\frac{259{\cdot}63+15x}{35{\cdot}36}$, whence $x=4{\cdot}418$, which must be added to the indications of the thermometer registering the temperature of the air. After this correction has been made, it will be found that the effect of a difference of temperature between the air and water of 9·216 is unity.

4·418 added to 397·226 gives 401·644 for the corrected temperature of the air in the thermal experiments; and this being 7·238 in excess of the temperature of the calorimeter, the corrected thermal effect will be $22{\cdot}0914-\frac{7{\cdot}238}{9{\cdot}216}=21{\cdot}306$, which, after applying the needful correction for the immersed portion of the thermometer-stem, becomes ultimately 21·326.

The thermal capacity of the calorimeter was made up of 95525 grains of distilled water, 26220 grains of copper, equivalent to 2501 grains of water, and the thermometer and coil equivalent to 80 grains, giving a total capacity equal to 98106 grains of water. 12·951 divisions of the thermometer are equivalent to one degree Fahr.

The dynamical equivalent is the quotient of the work done by the thermal effect, or

$$\frac{\left\{\frac{k}{2\pi}\mathrm{H}\right\}^2 \tan^2\theta\,\mathrm{R}t}{\mathrm{T}}=$$

$$\frac{\left\{\frac{\cdot 62723}{6{\cdot}2832}\times 3{\cdot}67073\right\}^2\times\cdot 379857\times 33435640\times 2400}{\frac{21{\cdot}326}{12{\cdot}951}\times 98106}=25335.$$

It appeared to be desirable to diminish the atmospheric influence; I therefore commenced a second series, in which the calorimeter was covered with two folds of cotton wadding. The bulb of the air-registering thermometer was also placed in a small bag made of the same material. In this fresh series each experiment occupied one hour, as I had learned by experience that with my battery arrangement the current would be sufficiently uniform. In fact the highest reading in an experiment was not more than $\frac{1}{30}$ higher than the lowest. There were, evenly distributed through the hour, forty observations of deflection, twenty of the air, and three of the water-thermometer; and the water was stirred forty times. Two minutes were allowed for the complete equalization of temperature previous to the final thermometer reading. The experiments on radiation were also similarly extended.

The coil was the same as that used in the first series; it had a coat o shellac varnish. Five determinations of its resistance were made, using a single Daniell's cell with various resistances included in the circuit. The galvanometer had a coil 17 inches in diameter consisting of nine turns. The results are as follow:—

C_3.	C_2.	C_1.	Temperature of unit.	Temperature of coil.	Resistance of coil in terms of unit.
° ′	° ′	° ′	°	°	
tan 79 39·5	tan 52 33·3	tan 52 9·3	59·25	58·6	1·0192
tan 71 39·5	tan 47 17·06	tan 46 55·6	48·6	48·5	1·0198
tan 70 16	tan 46 18·11	tan 45 57·4	54·68	57·4	1·0194
tan 71 54·33	tan 47 7·66	tan 46 45·93	......	......	1·0198
tan 62 6	tan 41 30·43	tan 41 13·46	......	......	1·0187
Average......	......	......	......	......	1·01938

The average temperature of the calorimeter in the experiments being 13°·55 Cent., and that at which the unit was adjusted 14°·5, the resistance during the experiments must have been 1·01906, which is equal to 33434330 in British measure.

The correction to be applied to the thermometer immersed in air as deduced from the above Table is given by $\frac{123\cdot66-12x}{12\cdot74}=\frac{356\cdot65+18x}{30\cdot99}$, whence $x=-1\cdot1835$. It appears also that a difference between the temperatures of the calorimeter and air-registering thermometer so corrected, equal to 10·822, gives the unit effect on the former.

Hence the corrected indication of the air-thermometer in the second series of thermal experiments will be $349\cdot63-1\cdot1835=348\cdot4465$. This being 12·5345 in excess of the temperature of the calorimeter, the corrected thermal effect will be $25\cdot65-\frac{12\cdot5345}{10\cdot822}=24\cdot4917$, which, after a small further correction for the immersed stem, becomes 24·512.

The thermal capacity in this second series was made up of 95561 grains

Second Series of Thermal Experiments.

Date.	Deflection.	$\tan^2$. Deflection.	Temperature of air.	Temperature of water.	Rise of temperature.	Horizontal intensity.
1866.	° ′					
Sept. 21 ...	29 51·68	·329623	397·4	363·42	30·38	3·6668
„ 22 ...	28 58·4	·306585	362·51	348·06	26·95	3·6707
„ 25 ...	29 14·63	·313472	345·19	386·94	29·75	3·6724
„ 26 ...	29 51·46	·329525	370·84	350·64	29·92	3·6644
„ 27 ...	28 54·78	·305064	365·91	361·71	25·88	3·6665
Oct. 5 ...	29 5·05	·309393	380·66	387·57	24·90	3·6612
„ 6 ...	28 22·54	·291761	426·55	392·77	27·40	3·6688
„ 8 ...	28 8·74	·286198	338·49	335·54	24·04	3·6595
„ 19 ...	28 42·81	·300074	398·56	332·35	31·08	3·6659
„ 20 ...	27 40·13	·274910	395·18	361·90	26·08	3·6654
„ 22 ...	26 40·5	·252409	371·72	388·63	19·12	3·6702
„ 23 ...	27 28·1	·270252	320·07	318·09	22·55	3·6638
„ 25 ...	27 9·63	·263230	275·65	286·25	20·98	3·6620
„ 26 ...	27 42·56	·275855	249·75	257·54	22·15	3·6623
„ 27 ...	28 7·84	·285838	245·96	247·27	23·57	3·6641
Average ...	....	·292946	349·63	335·912	25·65	3·6656

Second Series of Radiation Experiments.

Date.	Temperature of air.	Temperature of water.	Rise of temperature of water.
1866.			
Sept. 21	378·84	344·95	3·0
"	390·13	381·34	0·32
Sept. 22	326·32	334·37	−0·43
"	360·71	361·13	−0·41
Sept. 25	330·67	287·94	4·05
"	347·56	326·13	1·59
Sept. 26	352·15	333·12	2·12
"	377·56	368·12	0·70
Sept. 27	355·81	347·9	0·74
"	388·0	375·69	1·31
Oct. 5	376·9	375·04	0·
"	385·8	396·95	−1·15
Oct. 6	402·94	376·47	2·13
"	433·28	411·33	1·52
Oct. 8	319·5	323·51	−0·29
"	356·02	347·79	0·33
Oct. 19	365·08	303·94	5·95
"	398·49	356·29	3·57
Oct. 20	357·9	344·01	1·61
"	395·66	377·40	1·43
Oct. 22	371·24	380·45	−0·95
"	362·7	392·44	−3·18
Oct. 23	297·96	305·0	−0·50
"	334·07	329·05	0·5
Oct. 25	261·67	277·01	−1·26
"	277·59	294·31	−1·86
Oct. 26	233·31	247·61	−1·40
"	264·37	265·97	−0·66
Oct. 27	237·05	234·85	0·1
"	251·15	257·24	−0·65
Average	343·011	335·245	0·6083

distilled water, copper as water 2501, thermometer and coil as water 80, and cotton-wool as water 200 grs., giving a total of 98342 grains.

The equivalent, as deduced from the second series, is therefore

$$\frac{\left\{\frac{\cdot 62723}{6\cdot 2832}\times 3\cdot 6656\right\}^2\times\cdot 292946\times 33434330\times 3600}{\frac{24\cdot 512}{12\cdot 951}\times 98342}=25366.$$

The equivalents obtained in the two foregoing series of experiments are as much as one-fiftieth in excess of the equivalent I obtained in 1849 by agitating water. I therefore instituted a strict inquiry with a view to discover any causes of error, so that they might be avoided in a fresh series. The most probable source of error seems to be insufficient stirring of the water of the calorimeter. Although agitated so frequently as forty times in the hour, there could be no doubt that, during any intervals of comparative rest, a current of heated water would ascend from the coil, and that if a thin stratum of it remained any time at the top, some loss of heat would result. I resolved therefore to use a fresh calorimeter, and to introduce into it a stirrer which could be kept in constant motion by clockwork.

Another source of error which, though it would be finally eliminated by frequent repetition of the experiments, it seemed to be desirable to avoid, was the hygrometric quality of the cotton-wool which enveloped the calorimeter in the second series of experiments. I therefore sought for a material which did not present that inconvenience. The plan finally adopted was to cover the calorimeter first with tinfoil, to place over that two layers of silk net (tulle), and to finish with a second envelope of tinfoil.

A third source of possible error was the circumstance that the silver-platinum alloy, when made positively electrical in distilled water, is slowly acted upon, an oxide of silver as a bluish-white cloud arising from the metal, while hydrogen escapes from the negative electrode. On this account the coil in the experiments of the last series, as well as the subsequent, was well varnished. But it was found at the conclusion of the experiments that the varnish had in a great measure lost its protecting power. This circumstance gave me considerable anxiety: I was, however, ultimately able, by the following facts arrived at after the thermal experiments were completed, to satisfy myself that no perceptible influence had been produced by it on the results:—

1st. The resistance of the coils, after long-continued use had deteriorated the varnish, was not sensibly less than it was after they had been freshly varnished.

2nd. The coil of the 3rd series was, in the unprotected state, immersed in distilled water, and compared with many hundred yards of thick copper wire, unimmersed, having nearly equal resistance. The result showed that the resistance to the current was sensibly the same whether a single cell or five cells of Daniell in a series were used. Now, had any considerable leakage by electrolytic action taken place, it would have been very much less in proportion in the former than in the latter instance.

3rd. When the coils of the second and third series, in the unprotected state, were placed in distilled water, and made the electrodes of a battery of five cells, the deflection was 40′ of a degree on a galvanometer with a coil of 17 inches diameter composed of 18 turns of wire. This deflection indicates a current of about $\frac{1}{400}$ of the average current in the thermal experiments. In this case the chemical action was distinctly visible, but quite ceased to be so when the electrodes were connected by a wire of unit resistance, so as to reduce the potential to that in the thermal experiments.

4th. The coil of No. 2 series being used as a standard, that of No. 3 series, in the unprotected condition, was immersed, first in water, then in oil. The resistance to the current of five Daniell's cells was found to be sensibly equal in the two cases.

Hence there could be no doubt that the loss of heat during the experiments by electrolytic action could not possibly in any instance have been so great as one-thousandth of the entire effect, and was probably not one quarter of that small quantity; whilst in the larger number of experiments, when the varnish was fresh, it must have been *nil*.

The coil used in the third series of experiments was made by bending four yards of platinum-silver wire double, and then coiling it into a spiral, which was supported and kept in shape by being tied with silk thread to a thin glass tube. The terminals were thick copper wires, and the whole was coated with shellac and mastic varnish. The following results were obtained for its resistance. In the first three trials the current was measured by a galvanometer with a circle of nine turns 17 inches diameter, and in the

last six with an instrument with eighteen turns of wire. In the first six there was an extra unit of resistance included in the circuit:—

Battery.	Unit.	C_3.	C_2.	C_1.	Temp. of unit.	Temp. of coil.	Resistance in terms of my unit.
ne cell, Daniell ...	Mine ...	tan 52° 53'	tan 37° 3'·15	tan 37° 10'·6	63°·27	62°·78	·98963
itto	,, ...	tan 52 24·12	tan 36 29·02	tan 36 37·27	59·03	60·07	·98823
itto	Jenkin's	tan 52 3·62	tan 36 6·45	tan 36 14·79	60·88	60·57	·98752
aniell's cell. Positive metal iron ...	,,	tan 50 25·8	tan 35 21·88	tan 35 29·27	59·78	60·46	·98818
itto	Mine ...	tan 49 48·12	tan 34 57·36	tan 35 5·62	60·03	60·30	·98754
itto	,, ...	tan 48 17·62	tan 34 5·48	tan 34 12·24	60·50	60·88	·98816
itto	,, ...	tan 75 28	tan 49 58·6	tan 50 11·98	61·27	61·08	·98863
itto	Jenkin's	tan 75 17·25	tan 49 44·93	tan 49 57·51	61·96	61·27	·98871
itto	Mine ...	tan 75 59·6	tan 49 18·97	tan 49 33·08	69·35	70·28	·98820
verage	......	....	....	....	......	......	·98831

Third Series of Thermal Experiments.

Date.	Deflection.	$\tan^2$. Deflection.	Temperature of air.	Temperature of water.	Rise of temperature.	Fall of weight.
1867.	° '					in.
June 28, 12.54 P.M.	28 18·25	·290024	488·660	494·17	25·1	30
,, 28, 5.36	30 56·37	·359310	534·155	524·214	32·08	26
,, 29, 1.30	28 55·45	·305345	509·172	490·13	27·82	27
July 1, 10.30 A.M.	29 41·1	·324949	428·81	425·67	28·52	27
,, 1, 4.24 P.M.	30 19·4	·342107	508·78	467·214	33·05	26
,, 2, 12.45	30 10·12	·337891	405·343	450·73	25·13	26
,, 2, 6.0	30 30·98	·347424	401·822	458·104	24·99	28
,, 4, 1.20	31 23·4	·372299	516·992	452·97	57·98	27
,, 20, 11.11 A.M.	30 21·72	·343170	385·622	394·0	28·98	28
,, 20, 3.45 P.M.	31 37·55	·379241	454·19	430·97	34·92	28
,, 22, 12.36	32 0·6	·390765	482·44	460·621	35·48	30·5
,, 22, 5.21	32 23·47	·402470	493·087	498·573	34·47	28·4
,, 23, 1.7	31 18·43	·369881	465·238	473·167	31·27	28·7
,, 24, 11.0 A.M.	31 4·75	·363299	430·688	448·043	30·24	27·9
,, 24, 4.5 P.M.	30 49·15	·355900	439·007	470·954	28·14	28·2
,, 25, 12.15	32 39·5	·410832	465·354	432·45	38·48	29·4
,, 25, 4.55	33 10	·427129	521·569	486·049	39·72	28·4
,, 26, 12.58	32 33·95	·407920	445·009	464·267	33·61	30
,, 27, 11.13 A.M.	33 1·6	·422590	391·0	419·21	34·46	30
,, 27, 4.14 P.M.	32 58·22	·420777	418·11	446·623	34·09	29·4
Aug. 2, 12.31	31 52·98	·386923	385·876	390·911	33·1	30
,, 2, 5.18	31 53·77	·387325	407·781	422·843	32·25	28
,, 3, 12.56	31 37·18	·379056	453·66	421·948	35·37	29·75
,, 6, 11.18 A.M.	26 34·35	·250162	439·906	435·699	22·32	29·7
,, 6, 3.55 P.M.	28 42·8	·300070	457·145	462·056	25·67	29·6
,, 8, 12.17	29 29·25	·319773	465·586	443·204	29·6	29·7
,, 8, 5.45	29 39·25	·324137	499·874	480·564	29·67	28
,, 9, 1.27	29 33·2	·321491	478·658	469·296	28·8	26·4
,, 10, 11.9 A.M.	29 12·65	·312625	468·344	455·304	28·21	27·4
,, 10, 3.56 P.M.	28 14·47	·288500	519·082	493·136	27·28	28·4
Average	......	·3547795	458·699	455·436	31·02666	28·362

The above average resistance, reduced to 18°·63 C., the mean temperature in the third series, is ·98953 of the Association unit, or in British measure 32465480.

In the third series, the experiments for the heat of the current, of radiation, and for horizontal magnetic intensity were alternated in such a manner that each class occupied the same portions of the day that the others did. I sought in this way to avoid the effects of any horary change in the humidity &c. of the atmosphere or in the magnetic force. Of the thirty experiments comprising each class, six were performed at about each of the several hours—11 A.M., 12¼ P.M., 1½ P.M., 4 P.M., and 5½ P.M.

The calorimeter, protected as already described, was supported on the edges of a light wooden frame. It was carefully guarded against draughts by screens coated with tinfoil placed at a foot distance. The stirrer consisted of a vertical copper rod, to which vanes, on the plan of a screw-propeller, were soldered at four equidistant places. The rod extended 2 inches above the calorimeter, and was there affixed to a light wooden shaft 2 feet long, attached at the upper end to the last spindle of a train of clock-wheels. The weight was 35 lbs., which, falling about 2 feet per hour, produced a continuous revolution of the stirrer at a rate of about 200 in the

Third Series of Radiation Experiments.

Date.	Temperature of air.	Temperature of water.	Rise of temperature.	Fall of weight.
1867.				in.
June 28, 10.38 A.M.	460·527	481·990	−1·48	31
„ 28, 3.53 P.M.	513·687	506·770	0·75	28·2
„ 29, 11.55 A.M.	493·088	473·930	1·82	28
„ 29, 4.40 P.M.	526·185	508·480	1·88	28·5
July 1, 1.23	469·368	442·114	2·46	27·5
„ 2, 10.58 A.M.	404·842	439·790	−2·82	27
„ 2, 4.5 P.M.	401·779	450·930	−4·1	28·5
„ 4, 11.46 A.M.	492·210	427·517	5·97	28
„ 4, 4.42 P.M.	541·007	484·927	5·1	26·5
„ 20, 1.0	416·237	409·044	1·03	28·75
„ 22, 11.5 A.M.	474·393	439·140	3·32	30
„ 22, 3.50 P.M.	486·267	480·106	0·8	28·75
„ 23, 11.41 A.M.	451·029	456·947	−0·1	28·4
„ 23, 4.49 P.M.	475·319	486·113	−0·65	28·5
„ 24, 12.54	441·677	460·780	−1·48	26·5
„ 25, 10.40 A.M.	435·863	410·237	2·43	28
„ 25, 3.27 P.M.	515·653	460·939	5·03	28·8
„ 26, 11.29 A.M.	441·256	447·526	−0·2	28·5
„ 26, 4.49 P.M.	435·776	472·503	−3·0	29
„ 27, 1.7	404·58	433·444	−2·28	29·8
Aug. 2, 10.55 A.M.	369·966	374·18	−0·15	29·75
„ 2, 3.50 P.M.	407·34	406·42	0·17	27·8
„ 3, 11.30 A.M.	435·813	401·187	3·24	28·6
„ 3, 4.33 P.M.	476·691	446·393	2·9	27
„ 6, 1.15	457·87	447·843	1·05	28·9
„ 8, 10.46 A.M.	442·403	426·304	1·68	29
„ 8, 4.17 P.M.	489·901	463·143	2·42	29·7
„ 9, 11.51 A.M.	466·428	453·149	1·27	26·5
„ 9, 5.37 P.M.	490·308	484·753	0·66	27·9
„ 10, 1.20	502·96	472·469	2·82	28·6
Average	460·6808	451·6356	1·018	28·498

minute. The action of the stirrer left nothing to be desired. It was started five minutes before an experiment commenced, and kept going until the last observation of the thermometer had been made.

Each experiment, as in the second series, lasted one hour, during which were made eight observations of the thermometer immersed in the calorimeter, twenty of the temperature of the air, and forty of the deflection of the galvanometer.

The correction to be applied to the air-registering thermometer, as deduced from the radiation experiments of this third series, is found from $\frac{217\cdot452-10x}{16\cdot26}$ $=\frac{488\cdot807+20x}{46\cdot8}$, whence x, the quantity to be added to the observed temperature of the air in the thermal experiments, $=2\cdot81$. The temperature of the air was therefore virtually 6·073 higher than that of the water. The results

Determinations of Horizontal Magnetic Intensity.

Date.	Galvanometer deflection, θ.	Weighing by current-meter, w.	$H=\frac{\cdot17676\sqrt{w}}{\tan\theta}$.
1867.	° ′	grs.	
June 28, 1.30 P.M.	37 21·42	253·04	3·68334
„ 29, 10.50 A.M.	26 43·06	109·28	3·67114
„ 29, 3.50 P.M.	25 12·56	96·04	3·67964
July 1, 12.25	38 23·56	272·35	3·68144
„ 1, 5.20	38 59·25	284·95	3·68634
„ 2, 1.40	38 49·94	280·9	3·68034
„ 4, 10.45 A.M.	26 24·55	106·25	3·66894
„ 4, 3.45 P.M.	26 10·55	104·99	3·68474
„ 20, 12 Noon.	39 18·9	289·875	3·67484
„ 20, 4.40 P.M.	41 11·35	332·825	3·68504
„ 22, 1.30	41 21·4	335·13	3·67594
„ 23, 10.45 A.M.	32 5·1	169·616	3·67194
„ 23, 3.45 P.M.	31 56·15	168·608	3·68224
„ 24, 11.51 A.M.	39 52·95	301·591	3·67364
„ 24, 5.0 P.M.	40 24·9	315·092	3·68474
„ 25, 1.10	41 27·95	338·391	3·67964
„ 26, 10.30 A.M.	34 40·45	206·658	3·67324
„ 26, 3.33 P.M.	33 25·5	188·675	3·67864
„ 27, 12 Noon.	43 19·55	386·0	3·68194
„ 27, 5.12 P.M.	42 48·53	372·658	3·68414
Aug. 2, 1.30	41 15·35	332·733	3·67584
„ 3, 10.25 A.M.	34 13·9	198·99	3·66464
„ 3, 3.33 P.M.	33 40·3	191·983	3·67628
„ 6, 12.12	35 9·8	214·117	3·67156
„ 6, 4.50	37 8·1	248·258	3·67784
„ 8, 1.11	37 44·55	259·867	3·68110
„ 9, 10.53 A.M.	31 23·65	160·708	3·67186
„ 9, 4.42 P.M.	30 43·4	152·75	3·67590
„ 10, 12.12	36 25·4	235·433	3·67557
„ 10, 4.50	34 49·5	209·608	3·67864
Average	....	....	3·67771

also show that the unit of effect on the calorimeter was produced by a difference of temperature of 11·645.

Hence $31\cdot0266-\frac{6\cdot073}{11\cdot645}=30\cdot5051$; and adding ·077 for the unimmersed

part of the thermometer-stem, the corrected thermal effect in the third series is found to be 30·5821.

The average capacity of the calorimeter was equal to that of 93859 grs. of water, being made up of 91531 grs. distilled water, 22364 grs. of copper, 486 grs. of tin (the weight of the coating next the calorimeter), 52 grs. silk net (half that employed), the thermometer, coil, and corks.

The equivalent deduced from the third series is therefore

$$\frac{\left\{\frac{\cdot 62723}{6\cdot 2832}\times 3\cdot 6777\right\}^2 \times \cdot 35478\times 32465480\times 3600}{\frac{30\cdot 5821}{12\cdot 951}\times 93859}=25217.$$

The equivalents above arrived at are:—

From Series 1. Average of 10, 25335.
From Series 2. Average of 15, 25366.
From Series 3. Average of 30, 25217.

The extra precautions taken in the last series entitle the last figure to be taken as the result of the inquiry. Reduced to weighings *in vacuo* it becomes 25187.

SIXTH REPORT—EXETER, AUGUST 1869.

The Committee consists of Professor Williamson, Professor Sir C. Wheatstone, Professor Sir W. Thomson, Professor W. A. Miller, Dr. A. Matthiessen, Mr. Fleeming Jenkin, Sir Charles Bright, Professor Maxwell, Mr. C. W. Siemens, Mr. Balfour Stewart, Mr. C. F. Varley, Professor G. C. Foster, Mr. Latimer Clark, Mr. D. Forbes, Mr. Charles Hockin, and Dr. Joule.

The Electrical Standard Committee have this year had comparatively few meetings, and the results of the experiments made by the individual Members do not call for a Report of any length. It is, however, thought desirable to print at once, as Appendices, the important results obtained by Professor Clerk Maxwell, in determining the ratio of the electromagnetic and electrostatic series of units, and also a description of Sir Wm. Thomson's experiments on the same subject.

Description of Sir Wm. Thomson's Experiments made for the Determination of v, *the Number of Electrostatic Units in the Electromagnetic Unit.* By W. F. King.

The two principal pieces of apparatus used in these experiments were the absolute electrometer and the electrodynamometer. The former of these instruments was described at the last Meeting of the Association, and a description of it is printed in the Report. Plate VII. illustrates the arrangements described in what follows.

The electrodynamometer consists of two large coils of fine copper wire, and a smaller coil of still finer wire. The two large coils are about 30 centims. diameter, and are placed vertical, in planes parallel to one another; the distance between the large coils is 15 centims. (equal to their radius). The smaller coil is suspended between the large coils by a copper wire of such a thickness as to give the coil a time of vibration such that it com-

pletes a period in about thirteen seconds. The upper end of the suspending wire is attached to a milled head, and this head can be turned round by the fingers. The lower end of the wire is firmly fixed to the coil, and is in metallic connexion with one end of it. To the other end of the coil is soldered a spiral of very fine platinum wire, which hangs directly below the coil, and its lower end is cemented to the dry woodwork of the instrument. To the fixed end of the spiral coil a copper wire is attached, whose other end is soldered to a binding-screw in an accessible position.

On one side of the small or movable coil is fixed a plane mirror, and in front of the mirror, at a distance of about 450 centims., the scale is fixed on which the observations are read. A paraffin-lamp wire, to give dark line in image of flame, and lens are used in the ordinary way for finding accurately the angle through which the coil turns. It is never greater than ·05. Its true amount can be determined to within $\frac{1}{10}$ per cent.

The connexions are not very intricate, and are traced thus:—Starting from one pole of the battery (the battery used was sixty sawdust Daniell's in series), the current goes in at one end of large coil No. 1, and from the other end of No. 1 the current goes to either end of the movable coil; and the end of the movable coil at which we suppose the current to be coming out is connected with the end of No. 2 large coil, similar to the end of No. 1, to which the battery was first attached, that is to say, the end which will make the current go round in the same direction in both the large coils. When the current leaves the extreme end of No. 2, it passes through a 10,000 B.A. resistance-box; the current is completed by connecting the other end of the resistance-box with the pole of the battery not already engaged.

The absolute electrometer is used in the ordinary way for measuring differences of potential, and its electrodes are connected, one to the end of the dynamometer coil No. 1, which is joined to the battery, and the other electrode is fixed to the end of the resistance-box, which is connected to the other pole of the battery. Thus the greatest difference of potential in the arrangement is measured by the absolute electrometer. An electrometer-key is used to reverse these connexions in the course of the experiments.

There is only one other part of the arrangement to be explained, and that is the method of observing the resistance of the dynamometer coils while the experiments are going on. This was done by means of the resistance-box in the circuit and an electrometer. At one time the standard electrometer was used for this purpose, but more lately the quadrant, rendered unsensitive, was employed. Both these instruments are described in the last Report.

To take the resistance of the coils, the electrodes of the electrometer were first placed on the extreme ends of the three coils, and the difference of potential was ascertained. The electrodes were then shifted to the ends of the resistance-box, and the difference of potentials of its two ends was found. This gives at once the resistance of the coils.

There are two things which have to be done before the experiments are commenced. One is the determination of the moment of inertia of the movable coil. This is done at the beginning and end of a long series of experiments, by comparing it with a ring whose moment of inertia is known. The other is done every day, and it is finding the time of vibration of the small coil after all the connexions have been made and the coil put into its place. This was done both with the current from the battery flowing through the coils and with no current flowing; but this variation was of very little consequence, as no difference could be detected in the time. When the dynamometer is set up, care is taken to neutralize the effects of the

earth's magnetism by a large number of magnets fixed at a great distance from the coils. If the adjustment of the magnets is perfect, there is no alteration of the position of the spot of light when the current is reversed through the coils by the battery-key. Up to the present time (May 1868) various causes have prevented the obtainment of as satisfactory results as the method described above allows us to expect. Eleven sets of experiments, made at various dates, from March 10 to May 8 of the present year, have indicated values for v, of which the greatest was 292×10^{8}, the smallest $275{\cdot}4\times10^{8}$, and the mean $282{\cdot}5\times10^{8}$ centimetres per second. Sir W. Thomson intends to continue the investigation, hoping to attain much greater accuracy.

[P.S. Nov. 1869. A new form of absolute electrometer has now been completed and brought into use, with good promise as to accuracy and convenience. A glass jar constituting the "Leyden battery" contains within it the "absolute electrometer" proper, the "idiostatic gauge," and the "replenisher." One observer can use it effectively; although it is more easily worked by two, one maintaining constant potential in the Leyden jar by aid of the idiostatic gauge and the replenisher, and the other attending to the absolute electrometer (main balance and micrometer-screw). The main balance, giving electric weighing in known weights, is as steady and as easily used as any of the "attracted disk" electrometers, whether portable or stationary, described in previous Reports.]

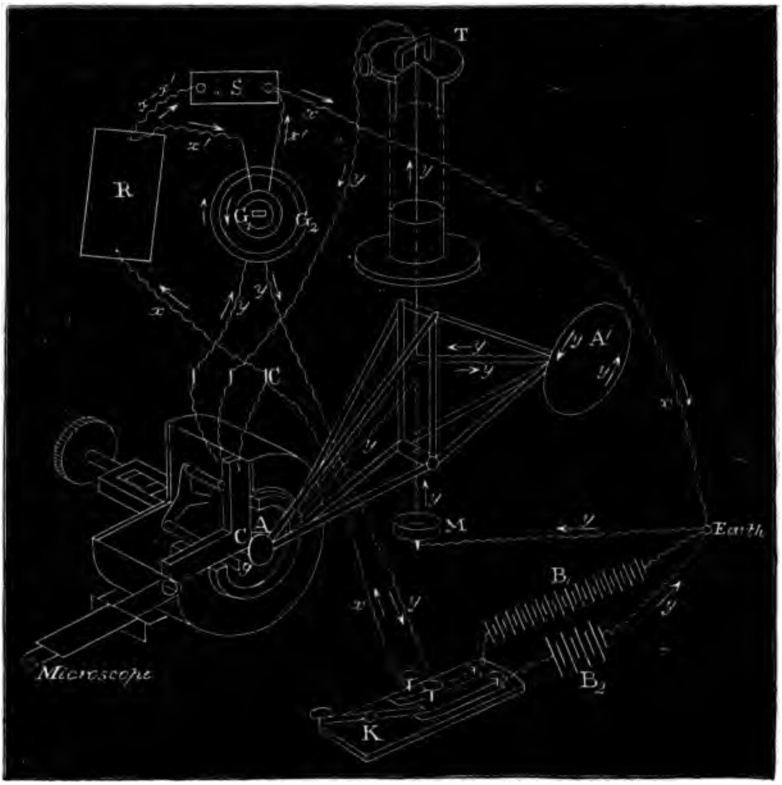
Experiments on the Value of v, *the Ratio of the Electromagnetic to the Electrostatic Unit of Electricity.* By J. Clerk Maxwell.

The experiments consisted in observing the equilibrium between the electrostatic attraction of two disks, at a certain difference of potential, and the electromagnetic repulsion of two coils traversed by a certain current. For this purpose one of the disks, with one of the coils at its back, was attached to one arm of a torsion-balance, while the other, with the other coil at its back, was capable of being moved to various distances from the suspended disk by a micrometer-screw. Another coil, traversed by the same current in the opposite direction, was attached to the other arm of the torsion-balance, so as to do away with the effect of terrestrial magnetism.

The fixed disk was larger than the suspended disk, and the latter, when in its zero position, had its surface in the same plane as that of a "guard-ring," as in Sir W. Thomson's electrometers. Its position and motion were observed by means of a microscope, directed to a graduated glass scale, connected with the disk. When the microscope was adjusted so that the image of the zero line on the glass scale coincided with the cross wires of the microscope, the very smallest motion of the scale could be easily detected, so that the observations were very rapid. The disk was brought to zero by the tangent-screw at the top of the suspension-wire, and its equilibrium was always observed at zero. The equilibrium, when the electrical forces were applied, was always unstable. This electrical balance was made by Mr. Becker. The experiments were made in the laboratory of Mr. Gassiot, who kindly gave the use of his great battery for the purpose. Mr. Willoughby Smith lent his resistance-coils, of 1,102,000 ohms; Messrs. Forde and Fleeming Jenkin lent a galvanometer, a resistance-box, a bridge and a key; and Mr. C. Hockin undertook the observation of the galvanometer, and the testing of the galvanometer, the resistances, and the micrometer-screw.

The difference of potentials of the disks was compared with the current in the coils as follows:—One electrode of the great battery was connected with the fixed disk, and the other with the case of the instrument and the guard-ring and the suspended disk. They were also connected through the great resistance R, and the primary coil of the galvanometer G_1, shunted with a resistance S.

A small Grove's battery was employed to send a current through the three coils and the secondary coil of the galvanometer G_2.



A. Suspended disk and coil.
A′. Counterpoise disk and coil.
C. Fixed disk and coil.
B_1. Great battery. B_2. Small battery.
G_1. Primary coil of galvanometer.
G_2. Secondary coil.
R. Great resistance. S. Shunt.
K. Double key. *g*. Graduated glass scale.
C. Electrode of fixed disk.
x. Current through R.
x′. Current through G_1. $x-x'$. Current through S.
y. Current through the three coils and G_2.
M. Mercury cup. T. Torsion head and tangent screw.

One quarter of the micrometer-box, disks, and coils is cut away to show the interior. The case of the instrument is not shown. The galvanometer and shunts were 10 feet from the electric balance.

Equilibrium of the electric balance was obtained by working the micrometer, and so adjusting the distance of the disks. At the same time equili-

brium of the galvanometer was obtained by altering the resistance of the shunt S.

The simultaneous values of the micrometer and the shunt formed the result of each experiment. It was necessary also to ascertain the ratio of the magnetic effects of the two coils of the galvanometer immediately after each set of experiments.

The experiments are described at greater length in the 'Philosophical Transactions' for 1868, p. 643.

The method of experimenting appeared capable of considerable accuracy; but some difficulties arose from want of constancy in the batteries, from leakage of electricity, &c., so that many of the experiments were known to be faulty. Twelve experiments, however, against which nothing could be proved at the time of making them, in which the distance of the disks ranged from $\frac{1}{4}$ to $\frac{1}{2}$ an inch, and the power of the battery from 1000 to 2600 cells, gave values of v of which the least was 28·4 and the greatest 29·4 ohms; and in nine of these the values lay between 28·68 and 28·91. The mean of the 12 was—

$$
\begin{aligned}
v &= 28{\cdot}798 \text{ ohms.} \\
&= 288{,}000{,}000 \text{ metres per second.} \\
&= 179{,}000 \text{ statute miles per second.}
\end{aligned}
$$

This result is much lower than that of MM. Weber and Kohlrausch, which was v=310,740,000 metres per second, but agrees, I believe, more nearly with values recently obtained by Sir W. Thomson, whose method, as well as mine, depends on the B.A. unit. Weber's method depends on the measure of capacity. It is to be hoped that this important physical quantity may soon be determined by methods founded on capacity, and disembarrassed from the phenomena of "electric absorption," which occurs in all solid condensers, and which would tend to give too high values of v.

REPORT

ON THE NEW UNIT OF ELECTRICAL RESISTANCE PROPOSED AND ISSUED BY THE COMMITTEE ON ELECTRICAL STANDARDS APPOINTED IN 1861 BY THE BRITISH ASSOCIATION.

By FLEEMING JENKIN, Esq.

Sir Humphry Davy, in 1821 *, published his researches proving a difference in the conducting power of metals and the decrease of that power as their temperature rose. This quality of metals was examined by Snow Harris, Cumming, and E. Becquerel, whose table of conducting powers, compiled by the aid of his differential galvanometer, and published in 1826 †, is still frequently quoted, and is indeed remarkable as the result of experiments made before the publication by Ohm, in 1827 ‡, of the true mathematical theory of the galvanic circuit.

The idea of resistance as the property of a conductor was introduced by Ohm, who conceived the force of the battery overcoming the resistance of the conductors and producing the current as a result. Sir Humphry Davy, on the contrary, and other writers of his time, conceived the voltaic battery rather as continually reproducing a charge, somewhat analogous to that of a Leyden jar, which was discharged so soon as a conductor allowed the fluid to pass. The idea of resistance is the necessary corollary of the conception of a force doing some kind of work §, whereas the idea of conducting power is the result of an obvious analogy when electricity is conceived as a fluid, or two fluids, allowed to pass in different quantities through different wires from pole to pole. When submitted to measurement, the qualities of conducting power and resistance are naturally expressed by reciprocal numbers; and the terms are used in this sense in the early writings of Lenz (1833) ||, who, with Fechner ¶ and Pouillet **, established the truth of Ohm's theory shortly after the year 1830.

The conception of a unit of resistance is implicitly contained in the very expression of Ohm's law; but the earlier writers seem to have contented themselves with reducing by calculation the resistance of all parts of a heterogeneous circuit into a given length of some given part of that circuit, so as to form an imaginary homogeneous conductor, the idea of which lies at the basis of Ohm's reasoning. These writers, therefore, generally speak of the resistance as the "reduced length" of the conductor, a term still much used

* Phil. Trans. 1821, vol. cxi. p. 425.

† Ann. de Chim. et de Phys. vol. xxxii. 2nd series, p. 420.

‡ Die galvanische Kette, mathematisch bearbeitet, 1827; also Taylor's Scientific Memoirs, vol. ii. p. 401.

§ The writer does not mean by this that electrical and mechanical resistance are truly analogous, or that a current truly represents work.

|| Pogg. Ann. vol. xxxiv. p. 418.

¶ Maassbestimmungen, etc. 1 vol. 4to. Leipzic, 1831.

** Elémens de Physique, p. 210, 5th edition; and Comptes Rendus, vol. iv. p. 267.

in France (*vide* Daguin, Jamin, Becquerel, De la Rive, and others). The next step would naturally be, when comparing different circuits, to reduce all resistances into a length of some one standard wire, though this wire might not form part of all or of any of the circuits, and then to treat the unit length of that standard wire as a unit of resistance. Accordingly we find Lenz (in 1838*) stating that 1 foot of No. 11 copper wire is his unit of resistance, and that it is 19·9 times as great as the unit he used in 1833†, which was a certain constant part of the old circuit. In the earlier paper the resistances are treated as lengths, in the later as so many "units."

Lenz appears to have chosen his unit at random, and apparently without the wish to impose that unit upon others. A further advance is seen when Professor Wheatstone, in his well-known paper of 1843‡, proposes 1 foot of copper wire, weighing 100 grains, not only as a unit, but as a standard of resistance, chosen with reference to the standard weight and length used in this country. To Professor Wheatstone also appears due the credit of constructing (in 1840) the first instruments by which definite multiples of the resistance-unit chosen might be added or subtracted at will from the circuit‡. He was closely followed by Poggendorff§ and Jacobi‖, the description of whose apparatus, indeed, precedes that of the Rheostat and Resistance-coils, although the writer understands that they acknowledge having cognizance of those inventions. Resistance-coils, as the means of adding, not given lengths, but given graduated resistances to any circuit, are now as necessary to the electrician as the balance to the chemist.

In 1846 Hankel¶ used as unit of resistance a certain iron wire; in 1847 I. B. Cooke** speaks of a length of wire of such section and conducting-power as is best fitted for a standard of resistance. Buff†† and Horsford‡‡ in the same year reduce the resistance of their experiments to lengths of a given German-silver wire, and as a further definition they give its value as compared with pure silver. To avoid the growing inconvenience of this multiplicity of standards, Jacobi§§ (in 1848) sent to Poggendorff and others a certain copper wire, since well known as Jacobi's standard, desiring that they would take copies of it, so that all their results might be expressed in one measure. He pointed out, with great justice, that mere definition of the standard used, as a given length and weight of wire, was insufficient, and that good copies of a standard, even if chosen at random, would be preferable to the reproduction in one laboratory of a standard prepared and kept in another. The present Committee fully indorse this view, although the definition of standards based on weights and dimensions of given materials has since then gained greatly in precision.

Until about the year 1850 measurements of resistance were confined, with few exceptions, to the laboratory; but about that time underground telegraphic wires were introduced, and were shortly followed by submarine cables, in the examination and manufacture of which the practical engineer soon found the benefit of a knowledge of electrical laws. Thus in 1847 the officers of the Electric and International Telegraph Company used resistance-coils made by Mr. W. F. Cooke, apparently multiples of Wheatstone's original standard, which was nearly equal to the No. 16 wire of commerce; and Mr.

* Pogg. Ann. vol. xlv. p. 105.
† Pogg. Ann. vol. xxxiv. p. 418.
‡ Phil. Trans. 1843, vol. cxxxiii. p. 303.
§ Pogg. Ann. vol. lii. p. 511.
‖ Pogg. Ann. vol. lii. p. 526, vol. liv. p. 347.
¶ Pogg. Ann. vol. lxix. p. 255.
** Phil. Mag. New Series, vol. xxx. p. 385.
†† Pogg. Ann. vol. lxxiii. p. 497.
‡‡ Pogg. Ann. vol. lxx. p. 238, and Silliman's Journ. vol. v. p. 36.
§§ Comptes Rendus, 1851, vol. xxxiii. p. 277.

C. F. Varley* states that, even at that date, he used a rough mode of "distance testing." In 1850, Lieut. Werner Siemens † published two methods for determining, by experiments made at distant stations, the position of "a fault"—that is to say, a connexion between the earth and the conducting-wire of the line at some point between the stations. In one of these plans a resistance equal to that of the battery is used, and the addition of resistances is also suggested; and Sir Charles Bright, in a Patent dated 1852 ‡, gives an account of a plan for determining the position of a fault by the direct use of resistance-coils. Since that time new methods of testing for faults and of examining the quality of materials employed, and the condition of the line, have been continually invented, almost all turning, more or less, on the measurement of resistance; greater accuracy has been continually demanded in the adjustment of coils and other testing-apparatus, until we have now reached a point where we look back with surprise at the rough and ready means by which the great discoveries were made on which all our work is founded.

The first effect of the commercial use of resistance was to turn the "feet" of the laboratory into "miles" of telegraph wire. Thus we find employed as units, in England the mile of No. 16 copper wire §, in Germany the German mile of No. 8 iron wire, and in France the kilometre of iron wire of 4 millimetres diameter. Several other units were from time to time proposed by Langsdorf ||, Jacobi ¶, Marie-Davy **, Weber ††, W. Thomson ‡‡, and others, with a gradually increasing perception of the points of chief importance in a standard; but none of these were generally accepted as the one recognized measure in any country. To remedy the continually increasing evils arising from the discrepancies invariably found between different sets of coils, Dr. Werner Siemens (in 1860 §§) constructed standards, taking as unit the resistance of a column of chemically pure mercury 1 metre long, having a section equal to 1 millimetre square, and maintained at the temperature of 0° Centigrade || ||. Dr. Siemens supposed that this standard could be reproduced without much difficulty where copies could not be directly obtained. Mercury had been proposed before as a fitting material for a standard by Marie-Davy and De la Rive; but Dr. Siemens merits especial recognition, as the coils and apparatus he issued have been made with great care, and have materially helped in introducing strict accuracy ¶¶.

The question had reached this point when (in 1861) the British Association, at the suggestion of Professor W. Thomson, appointed a Committee to determine the best standard of electrical resistance. This Committee, aided by a grant from the Royal Society, has now issued a new standard, the subject of the present paper.

* Letter to writer, 1865.

† Pogg. Ann. vol. lxxix. p. 481.

‡ Patent No. 14,331, dated Oct. 21, 1852.

§ A size much used in underground conductors, and equal in resistance to about double the length of the common No. 8 iron wire employed in aërial lines.

|| Liebig's Ann. vol. lxxxv. p. 155.

¶ Pogg. Ann. vol. lxxviii. p. 173.

** Ann. Chim. et Phys. 3rd series, vol. ix. p. 410.

†† Pogg. Ann. vol. lxxxii. p. 337.

‡‡ Phil. Mag., Dec. 1851, 4th ser. vol. ii. p. 551.

§§ Pogg. Ann. vol. cx. p. 1.

|| || Dr. Siemens, while retaining his definition, has altered the value of his standard about 2 per cent. since the first issue; and it is doubtful whether even the present standard represents the definition truly: his experiments were made by weight; and in reducing the results to simple measurements of length he has used a specific gravity for mercury of 13·557 instead of 13·596 as given by Regnault, 13·595 by H. Kopp, and 13·594 by Balfour Stewart. (1873. The error due to this cause has since been corrected.)

¶¶ Many of the different units described above were represented by resistance-coils in the International Exhibition of 1862: *vide* Jury Report, Class XIII. p. 83, where their relative values are given: *vide* also Appendix A to present paper.

The writer has hitherto described those units only which are founded on a more or less arbitrary size and weight of some more or less suitable material; but measurements of resistance can be conceived and carried out entirely without reference to the special qualities of any material whatever. In 1849 Kirchhoff* had already effected a measurement of this kind; but it is to W. Weber† that we owe the first distinct proposal (in 1851) of a definite system of electrical measurements, according to which resistance would be measured in terms of an absolute velocity. This system of measures he called absolute electromagnetic measure, in analogy with Gauss's nomenclature of absolute magnetic measure. The Committee have decided that Weber's proposal is far preferable to the use of any unit of the kind previously described. Setting aside the difficulties in the way of their reproduction, which are by no means contemptible, arbitrary material standards, whether of mercury, gold, silver, platinum, or any other material, would be heterogeneous isolated units without any natural connexion with any other physical units. The unit proposed by Weber, on the other hand, forms part of a symmetrical natural system, including both the fundamental units of length, time, and mass, and the derived electrical units of current quantity and electromotive force. Moreover it has been shown by Professor W. Thomson‡, who accepted and extended Weber's proposal immediately on its appearance, that the unit of absolute work, the connecting link between all physical forces, forms part of the same system, and may be used as the basis of the definition of the absolute electromagnetic units.

The full grounds of the choice of the Committee could only be explained by a needless repetition of the arguments given in the reports already made to the British Association. It will be sufficient here to state that, in the absolute electromagnetic system, the following equations exist between the mechanical and electrical units:—

$$W=C^2Rt, \quad \ldots \ldots \quad (1)$$

where W is the work done in the time t by the current C conveyed through a conductor of the resistance R. This equation expresses Joule and Thomson's law.

$$C=\frac{E}{R}, \quad \ldots \ldots \quad (2)$$

where E is the electromotive force. This equation expresses Ohm's law.

$$Q=Ct, \quad \ldots \ldots \quad (3)$$

expressing a relation first proved by Faraday, where Q is the quantity of electricity conveyed or neutralized by the current in the time t. Finally, the whole system is rendered determinate by the condition that the unit length of the unit current must produce the unit force on the unit pole (Gauss) at the unit distance. If it is preferred to omit the conception of magnetism, this last statement is exactly equivalent to saying that the unit current conducted round two circles of unit area in vertical planes at right angles to each other, one circuit being at a great distance D above the other, will cause a couple to act between the circuits of a magnitude equal to the reciprocal of he cube of the distance D. This last relation expresses the proposal made by Weber for connecting the electric and magnetic measure. These four re-

* Pogg. Ann. vol. lxxvi. p. 412. † Ibid. vol. lxxxii. p. 337.
‡ Phil. Mag., Dec. 1851, 4th series, vol. ii. p. 551.

lations serve to define the four magnitudes R, C, Q, and E, without reference to any but the fundamental units of time, space, and mass; and when reduced to these fundamental units, it will be found that the measurement of R involves simply a velocity, *i. e.* the quotient of a length by a time. It is for this reason that the absolute measure of resistance is styled $\frac{\text{metre}}{\text{second}}$ or $\frac{\text{foot}}{\text{second}}$, precisely as the common non-absolute unit of work involving the product of a weight into a length is styled kilogrammetre or foot-pound. The Committee have chosen as fundamental units the second of time, the metre, and the mass of the Paris gramme. The metrical rather than the British system of units was selected, in the hope that the new unit might so find better acceptance abroad, and with the feeling that while there is a possibility that we may accept foreign measures, there is no chance that the Continent will adopt ours. The unit of force is taken as the force capable of producing in one second a velocity of one metre per second in the mass of a Paris gramme, and the unit of work as that which would be done by the above force acting through one metre of space. These points are very fully explained in the British Association Report for 1863, and in the Appendix C to that Report by Professor J. Clerk Maxwell and the writer.

The magnitude of the $\frac{\text{metre}}{\text{second}}$ is far too small to be practically convenient, and the Committee have therefore, while adopting the system, chosen as their standard a decimal multiple 10^{10} times as great as Weber's unit $\left(\text{the } \frac{\text{millimetre}}{\text{second}}\right)$, or 10^7 times as great as the $\frac{\text{metre}}{\text{second}}$. This magnitude is not very different from Siemens's mercury unit, which has been found convenient in practice. It is about the twenty-fifth part of the mile of No. 16 impure copper wire used as a standard by the Electric and International Company, and about once and a half Jacobi's unit*.

It was found necessary to undertake entirely fresh experiments in order to determine the actual value of the abstract standard, and to express the same in a material standard which might form the basis of sets of resistance-coils to be used in the usual manner. These experiments, made during two years with two distinct sets of apparatus by Professor J. C. Maxwell and the writer, according to a plan devised by Professor W. Thomson, are fully described in the Reports to the British Association for 1863 and 1864.

The results of the two series of experiments made in the two years agree within 0·2 per cent., and they show that the new standard does not probably differ from true absolute measure by 0·1 per cent.† It is not far from the mean of a somewhat widely differing series of determinations by Weber.

In order to avoid the inconvenience of a fluctuating standard, it is proposed that the new standard shall not be called "absolute measure," or described as so many $\frac{\text{metre}}{\text{seconds}}$, but that it shall receive a distinctive name, such as the B. A. unit, or, as Mr. Latimer Clark suggests, the "Ohmad"‡; so that, if hereafter improved methods of determination in absolute measure are discovered or better experiments made, the standard need not be changed, but a small coefficient of correction applied in those cases in which it is necessary to con-

* This last number may be 30 per cent. wrong, as the writer has never been in possession of an authenticated Jacobi standard, and has only arrived at a rough idea of its value by a series of published values which afford an indirect comparison.

† *Vide* Appendix B.

‡ *Note*, 1873. The name Ohm has been adopted.

vert the B. A. measure into absolute measure. Every unit in popular use has a distinctive name; we say feet or grains, not units of length or units of weight; and it is in this way only that ambiguity can be avoided. There are many absolute measures, according as the foot and grain, the millimetre and milligramme, the metre and gramme, &c. are used as the basis of the system. Another chance of error arises from the possibility of a mistake in the decimal multiple used as standard. For all these reasons, as well as for convenience of expression, the writer would be glad if Mr. Clark's proposal were adopted and the unit called an Ohmad.

Experiments have been made for the Committee by Dr. Matthiessen, to determine how far the permanency of material standards may be relied on, and under what conditions wires unaltered in dimension, in chemical composition, or in temperature change their resistance. Dr. Matthiessen has established that in some metals a partial annealing, diminishing their resistance, does take place, apparently due to age only. Other metals exhibit no alteration of this kind; and no permanent change due to the passage of voltaic currents has been detected in any wires of any metal—a conclusion contrary to a belief which has very generally prevailed.

The standard obtained has been expressed in platinum, in a gold-silver alloy, in a platinum-silver alloy, in a platinum-iridium alloy, and in mercury. Two equal standards have been prepared in each metal; so that should time or accident cause a change in one or more, this change will be detected by reference to the others. The experiments and considerations which have led to the choice of the above materials are fully given in the Report to the British Association for 1864. The standards of solid metals are wires of from 0·5 millim. to 0·8 millim. diameter, and varying from one to two metres in length, insulated with white silk wound round a long hollow bobbin, and then saturated with solid paraffin. The long hollow form chosen allows the coils rapidly to assume the temperature of any surrounding medium, and they can be plunged, without injury, into a bath of water at the temperature at which they correctly express the standard. The mercury standards consist of two glass tubes about three quarters of a metre in length. All these standards are equal to one another at some temperature stated on each coil, and lying between 14°·5 and 16°·5 C. None of them, when correct, differ more than 0·03 per cent. from their value at 15°·5 C.

Serious errors have occasionally been introduced into observations by resistance at connexions between different parts of a voltaic circuit, as perfect metallic contact at these points is often prevented by oxide or dirt of some kind. Professor Thomson's method of inserting resistances in the Wheatstone balance (differential measurer) has been adopted for the standards; but in the use of the copies which have been issued it has been thought that sufficient accuracy would be attained by the use of amalgamated mercury connexions.

In the standards themselves permanence is the one paramount quality to be aimed at; but in copies for practical use a material which changes little in resistance with change of temperature is very desirable, as otherwise much time is lost in waiting till coils have cooled after the passage of a current; moreover large corrections have otherwise to be employed when the coils are used at various temperatures; and these temperatures are frequently not known with perfect accuracy. German silver, a suitable material in this respect, and much used hitherto, has been found to alter in resistance, in some cases, without any known cause but the lapse of time, since the change has been observed where the wires were carefully protected against mechanical

or chemical injury. A platinum-silver alloy has been preferred by the Committee to German silver for the copies which have been made of the standard. These have been adjusted by Dr. Matthiessen so as to be correct at some temperature not differing more than 1° from 15°·5 C. The resistance of platinum silver changes about 0·031 per cent. for each degree Centigrade within the limits of 5° above and below this temperature; this change is even less than that of German silver. The new material seems also likely to be very permanent, as it is little affected by annealing. The form of the copies is the same as that of the standard, with the exception of the terminals, which are simple copper rods ending in an amalgamated surface. Twenty copies have been distributed gratis, and notices issued that others can be procured from the Committee for £2 10*s*. The Committee also propose to verify, at a small charge, any coils made by opticians, as is done for thermometers and barometers at Kew.

Dr. Matthiessen reports, with reference to the question of reproduction, that given weights and dimensions of several pure metals might be employed for this purpose *if absolute care were taken*. The reproduction, in this manner, of the mercury unit, as defined by Dr. Siemens, differs from the standards issued by him in 1864 about 8·2 per thousand if the same specific gravity of mercury be used for both observations*. Each observer uses for his final value the mean of several extremely accordant results. It is therefore to be hoped that the standard will never have to be reproduced by this or any similar method. On the other hand, four distinct observers, with four different apparatus, using four different pairs of standards issued respectively by Dr. Siemens and the Committee, give the B. A. unit as respectively equal to 1·0456, 1·0455, 1·0456, and 1·0457 of Siemens's 1864 unit†. It is certain that two resistances can be compared with an accuracy of one part in one hundred thousand—an accuracy wholly unattainable in any reproduction by weights and measures of a given body, or by fresh reference to experiments on the absolute resistance. The above four comparisons, two of which were made by practical engineers, show how far the present practice and requirements differ from those of twenty and even ten years ago, when, although the change of resistance due to change of temperature was known, it was not thought necessary to specify the temperature at which the copper or silver standard used was correct. The difficulty of reproducing a standard by simple reference to a pure metal, further shows the unsatisfactory nature of that system in which the conducting power of substances is measured by comparison with that of some other body, such as silver or mercury. Dr. Matthiessen has frequently pointed out the discrepancies thus produced, although he has himself followed the same system pending the final selection of a unit of resistance. It is hoped that for the future this quality of materials will always be expressed as a specific resistance or specific conducting power referred to the unit of mass or the unit of volume, and measured in terms of the standard unit resistance, that the words conducting power will invariably be used to signify the reciprocal of resistance, and that the vague terms good and bad conductor or insulator will be replaced, in all writings aiming at scientific accuracy, by those exact measurements which can now be made with far greater ease than equally accurate measurements of length.

There is every reason to believe that the new standard will be gladly accepted throughout Great Britain and the colonies. Indeed the only obstacle

* If Dr. Matthiessen uses the sp. gr. of 13·596, as given by Regnault, the difference from Dr. Siemens's standard is 5 per thousand.

† 1873. The value now adopted is 1·0486.

to its introduction arises from the difficulty of explaining to inquirers what the unit is. The writer has been so much perplexed by this simple question, finding himself unable to answer it without entering at large on the subject of electrical measurement, that he has been led to devise the following definitions, in which none but already established measures are referred to.

The resistance of the absolute $\frac{\text{metre}}{\text{second}}$ is such that the current generated in a circuit of that resistance by the electromotive force due to a straight bar 1 metre long moving across a magnetic field of unit intensity*, perpendicularly to the lines of force and to its own direction, with a velocity of 1 metre per second, would, if doing no other work or equivalent of work, develop in that circuit in one second of time a total amount of heat equivalent to one absolute unit of work—or sufficient heat, according to Dr. Joule's experiments, to heat 0·0002405 gramme of water at its maximum density 1° Centigrade.

The new standard issued is as close an approximation as could be obtained by the Committee to a resistance ten million times as great as the absolute $\frac{\text{metre}}{\text{second}}$. The straight bar moving as described above in a magnetic field of unit intensity would require to move with a velocity of ten millions of metres per second to produce an electromotive force which would generate in a circuit of the resistance of the new standard the same current as would be produced in the circuit of one $\frac{\text{metre}}{\text{second}}$ resistance by the electromotive force due to the motion of the bar at a velocity of one metre per second. The velocity required to produce this particular current† being in each case proportional to the resistance of the circuit, may be used to measure that resistance; and the resistance of the B. A. unit may therefore be said to be ten millions of metres per second, or $10^7 \frac{\text{metres}}{\text{second}}$.

It is feared that these statements are still too complex to fulfil the purpose of popular definitions; but they may serve at least to show how a real velocity may be used to measure a resistance by using the velocity with which, under certain circumstances, part of a circuit must be made to move in order to induce a given current in a circuit of the resistance to be measured. That current in the absolute system is the unit current, and the work done by that unit current in the unit of time is equal to the resistance of the circuit, as results from the first equation stated above.

Those who from this slight sketch may desire to know more of the subject will find full information in the Reports of the Committee to the British Association in 1862, 1863, and 1864. The Committee continue to act with the view of establishing and issuing the correlative units of current, electromotive force, quantity, and capacity, the standard apparatus for which will, it is proposed, be deposited at Kew along with the ten standards of resistance already constructed with the funds voted by the Royal Society.

* Gauss's definition.

† This current is the unit current, and, if doing no other work or equivalent of work, would develop, in a circuit of the resistance of the B. A. unit, heat equivalent to ten millions of units of work, or enough to raise the temperature of 2405 grammes of water at its maximum density 1° Centigrade.

Resistance.

Bréquet.	Swiss.	Matthi
0·03123	0·02924	0·02
0·03279	0·03071	0·02
0·06520	0·06106	0·04
0·09416	0·08817	0·06
0·09775	0·09149	0·07
0·1024	**0·0959**	**0·07**
0·9491	0·8889	0·68
1·000	0·9365	0·71
1·068	1·000	0·76
1·391	1·303	1·00
2·622	2·456	1·88
5·882	5·509	4·22

s's 1864 issue has been corrected f

Appendix B.

The following Table shows the degree of concordance obtained in the separate experiments used to determine the unit. The determinations were made by observing the deflections of a certain magnet when a coil revolved at a given speed, first in one direction, and then in the opposite direction. The first column shows the speed in each experiment; the second shows the value of the B. A. unit in terms of $10^7 \frac{\text{metres}}{\text{second}}$, as calculated from the single experiments. A difference constantly in one direction may be observed in the values obtained when the coil revolved different ways. This difference depended on a slight bias of the suspending thread in one direction. The third column shows the value of the B. A. unit calculated from the pair of experiments; the fourth shows the error of the pair from the mean value finally adopted. In the final mean adopted, the 1864 determination was allowed five times the weight allowed to that of 1863.

1. Time of 100 revolutions of coil, in seconds.	2. Value of B. A. unit in terms of $10^7 \frac{\text{metres}}{\text{second}}$, as calculated from each experiment.	3. Value from mean of each pair of experiments.	4. Percentage error of pair of observations from mean value.
17·54 17·58	1·0121 0·9836	0·9978	−0·22
77·62 76·17	1·0468 0·9613	1·0040	+0·28
53·97 54·53	0·9985 0·9998	0·9992	−0·08
41·76 41·79	0·9915 0·9936	0·9925	−0·75
54·07 53·78	0·9961 0·9886	0·9924	−0·76
17·697 17·783	0·9878 1·0136	1·0007	+0·07
17·81 17·78	0·9952 1·0174	1·0063	+0·63
17·01 16·89	1·0191 0·9895	1·0043	+0·43
21·35 21·38	1·0034 1·0011	1·0022	+0·22
21·362 21·643	0·9968 1·0096	1·0040	+0·40
11·247 16·737	1·0424 0·9707	0·9981	−0·19

Probable error of R (1864).......... =0·1 per cent.
Probable error of R (1863).......... =0·24 „
Difference in two values 1864 and 1863 = 0·16 „
Probable error of two experiments.... =0·08 „

CANTOR LECTURES.

On Submarine Telegraphy. *By* Fleeming Jenkin, *Esq., C.E., F.R.S.*

[Delivered before the Royal Society of Arts.]

LECTURE I.

(Monday, January 29, 1866.)

The Insulated Conductor and its Properties.

The lecturer stated that in the lectures he was about to deliver he should aim rather at spreading more widely the knowledge possessed by those practically acquainted with submarine telegraphy than at announcing the latest discoveries or most novel theories.

1. *Terms used:—Conductor, Insulator, Battery, Earth, Circuit, Current.*—Some elementary explanations were given with the view of explaining these terms. The action of a current on a magnetic needle, the simplest form of galvanometer and electromagnet were shown, with their application to practical telegraphy. The two sources of failure, viz. want of continuity in the conductor and want of insulation forming a short circuit, were explained. The reflecting galvanometer was exhibited as a means of indicating a feeble current.

The following is a more detailed abstract of the rest of the lecture:—

2. *Component parts of Submarine Cable.*—These are:—1st, the conducting wire, generally formed of copper; 2nd, the insulator, surrounding the conductor, generally india-rubber or gutta-percha; 3rd, the outer covering, intended to give strength, and generally formed of a hempen serving, surrounded by iron wires, laid, as in the rope, round and round the core.

3. *Conductor.*—(*a*) *Mechanical Properties.*—The conductor is almost universally made of copper; but a solid copper wire is apt to be brittle, breaking after being bent a few times; interruptions occurred from this cause in early cables: this defect is wholly removed by the use of a strand of several wires, generally three or seven. The tensile strength of copper wire is in some books given as 60,000 lbs. per square inch. That used for submarine cables, being selected for electrical rather than mechanical qualities, will only bear from 35,000 lbs. to 39,000 lbs. per square inch. Copper stretches so much (10, 11, 12, or 15 per cent.) before breaking that its full strength can seldom be made use of. This extensibility is, as will be seen, a very valuable property, preventing the interruption of the circuit until the strengthening part of the cable be fairly broken. The following are convenient approximate formulæ:—A copper strand will bear $1\frac{1}{2}$ lb. per pound weight per knot before breaking; it will stretch one per cent. with 1 lb., and will not stretch at all with 0·75 lb. per pound per knot: thus a strand weighing three hundred pounds per knot will barely support 450 lbs., will stretch one per cent. with 300 lbs., and will not stretch at all with 225 lbs. The weight of copper in lbs. per knot can be calculated from the diameter d in inches by the use of the following constants:—weight$=18500\ d^2$ for solid wire, or $15100\ d^2$ for strand. The joint of the conductor is made with great care: a scarf joint is made by soldering together two filed and fitted ends; this joint is wrapped round with fine copper wire to give it strength, and solder is again run round this wire; a second wrapping of fine copper is then applied and left without solder. The joint is necessarily less extensible than the rest of the strand; if forcibly torn asunder the last wrapping of copper maintains the

electrical connexion, being simply pulled out like a spiral spring. No interruption from breakage at joints has ever occurred since this system was adopted.

(*b*) *General Electrical Properties.*—Copper is what is called a good conductor, offering small resistance to the passage of the electric current; that is to say, a much less powerful current would be sent by any given battery through a long iron or lead wire than through a copper wire of equal length and diameter. Table I. gives the relative electrical resistance of several substances, compiled from Dr. Matthiessen's experiments. The lower the number the better the conductor.

TABLE I.

Relative resistance of materials at 0° C. Wires of equal length and diameter.

Part I.—*Conductors.*

Silver, Hard	1·00
Copper "	1·00
Gold "	1·28
Iron	5·94
Tin	8·09
Lead	12·02
Brass	4·50
Gold-Silver alloy	6·65
German Silver	12·82
Platinum-Silver alloy	14·93
Mercury	58·15

Part II.—*Insulators.*

Gutta-percha at 75° Fahrenheit 60,000,000,000,000,000,000	or 6×10^{19}
Glass not less than 600,000,000,000,000,000,000,000,000	or 6×10^{26}

Conduction takes place through the mass, and not along the surface of the wire. A strand and solid wire of equal weights are equally good conductors; but owing to what is termed lateral induction, to be hereafter explained, the strand is at a slight disadvantage for rapid speaking through long submarine cables. Messrs. Bright and Clark, to avoid this defect, used in the Persian-Gulf cable a segmental strand, built up of six wires fitting one another and drawn through a tube; they hoped thus to combine the advantages of the strand with those of the solid wire. Mr. Thomas Allan surrounds his copper conductor with fine steel wires, to give strength and avoid the use of heavy external protection. In a sample given to the lecturer, the resistance of the conductor so formed was about 30 per cent. more than that of a simple copper conductor of equal weight. Taking induction into account, Mr. Allan's cable would be about 50 per cent. inferior in speaking power to a cable with simple copper conductor of equal weight, and covered with an equal amount of insulating material. This inferiority is not a fatal defect if the cost of the outer protection is avoided. The general merits or defects of this plan will be spoken of in a future lecture; although the danger of decay where iron and copper meet is known, Mr. Allan's proposal deserves serious consideration.

(*c*) *Chemical Properties of Copper Wire.*—A current flowing from the copper end or pole of the battery through a hole in the insulator to the sea causes the formation of chloride of copper, a soluble salt. The copper is thus

gradually eaten away, until metallic continuity is interrupted, and the cable ceases to transmit messages. The current from the zinc pole does not produce this effect, but only a deposit of soda in the fault, which, however, then allows a greater leakage, tending to enlarge the hole in the gutta-percha. Mr. C. F. Varley has proposed to twist up a fine platinum wire with the copper strand of long cables. This wire would maintain the communication at any point where the copper might be eaten away.

4. *Insulator.*—(*a*) *Gutta-percha and Chatterton's Compound.*—Gutta-percha is pressed out, while warm and plastic, through a die round the conductor; several successive coatings or tubes are thus applied, till the desired thickness is obtained. The first coating is attached to the strand by a substance known as Chatterton's compound, which is also used between each layer of gutta-percha, and between the separate wires of the strand, to prevent the percolation of water along the interstices, in case any part of the copper should be accidentally immersed in water.

(*b*) *Mechanical Properties.*—Gutta-percha has considerable tensile strength, bearing about 3500 lbs. per square inch of section; but, owing to its great extensibility, it does not add more than about one third of its whole strength to the copper strand. Roughly, it may be said to add in small wires 20 per cent. and in larger cases 30, 40, or even 50 per cent. to the strength of the copper strand; it will stretch 50 or 60 per cent. or more without breaking, but almost always fails as soon as the copper inside gives way. It will bear ill-usage, such as knotting, squeezing, or stretching, without injury, but can be pierced with a sharp instrument or cut by a knife without much difficulty. Uniform pressure, such as it sustains under water, improves its electrical qualities, augmenting its insulation resistance, according to Mr. Siemens's experiments, about 60 per cent., at 24° C., for every ton pressure per square inch, corresponding nearly to 1000 fathoms depth of water. It becomes soft at about 100° Fahrenheit, and should, after manufacture, never be heated beyond 90°. The joints required are made by heating the two ends of the covered conductors after the copper is joined, and applying by hand successive coatings of warmed and plastic gutta-percha. The separate layers of gutta-percha are also cemented by Chatterton's compound; thus the joint is, when sound, very similar to the rest of the core. Extreme cleanliness and much skill are required in making these joints. Some years since the joints frequently failed, not always when just made, but after some months, becoming hard and brittle, and shrinking, so as to leave a gap between the old and new materials. The process is now thoroughly understood, and is a safe one in skilled hands, but in skilled hands only.

(*c*) *India-rubber.*—This material is applied in many ways; most commonly tapes of masticated or bottle-rubber are wrapped round and round the conductor until the required thickness is reached. At first these tapes were, as it might be termed, gummed together with solvents; but these caused decay, and have been abandoned: heat is now the common agent for effecting the adhesion. Mr. Siemens, who applied his tapes longitudinally, like two long half-tubes, used simple pressure to join the two halves together. He employed most ingenious machinery to cut the tapes the instant before they were applied to the copper, as the material only reunites if quite freshly cut; several successive coatings could be applied in this way at one operation. Some manufacturers considered that none of these methods were fully successful, and vulcanized the india-rubber, converting it into various materials of different degrees of flexibility according to the process employed. This material was also criticised; and Mr. Hooper has covered conductors with

pure india-rubber next the copper, followed by a coating of oxide of zinc and rubber, and enclosed by a vulcanized jacket. In the process of baking the core to vulcanize the jacket, a little sulphur penetrates the india-rubber, and the whole mass becomes remarkably compact and durable. Mr. Hooper heats the core to 250° Fahrenheit, and bakes it for four hours. The mechanical properties of these different materials vary greatly; they are all, however, very extensible, and do not add sensibly to the tensile strength of the conductor; they will bear considerable ill-usage, but are mostly softer than gutta-percha, and the pure rubber will not bear continued pressure even by a blunt surface, but gradually yields. The joints in each form are now made so as to imitate, as far as possible, the main core. Mr. Hooper bakes his joints two hours in a steam jacket.

(*d*) *Chemical Properties and Permanency.*—When dry and exposed to light, gutta-percha becomes dry and brittle, losing all its valuable qualities, and is said to be oxidized. Under the same circumstances the various forms of india-rubber decay in various ways; some become treacly, some brittle, some almost friable. Mr. Hooper's hard-covered seems to last best of all in air. When in water gutta-percha is, so far as fifteen years' experience can show, absolutely permanent. Many thousands of miles have been laid down, and many hundred of miles picked up after laying in the sea in various parts of the world, in deep and shallow waters, for many years, and not one single yard of material has been found which had under those circumstances decayed or lost its insulating properties. The importance of this fact cannot be over-estimated. The experience as to india-rubber is the very opposite to this: little has been employed, and a great deal of that little has been found to decay, so as to be utterly useless. No doubt improvements are continually introduced, and possibly some of the forms now made may answer better; but till the subject is more thoroughly understood it would be lost time to reproduce all the theories by which the various failures are explained. Out of five specimens supplied lately to the Indian Government, one only, Mr. Hooper's, proved durable even for a year. The lecturer's own experience confirmed this experiment. It must, in justice, be said that considerable lengths of india-rubber-covered wire are successfully used on land, supplied by Messrs. Silver and their descendant the India-rubber and Gutta-percha Telegraph Construction Company, and by Messrs. Wells and Hall. The Indian Government has ordered about 100 miles of wire covered by Mr. Hooper's material, which will, therefore, now be subjected to a thorough practical test. India-rubber stands heat much better than gutta-percha.

(*e*) *General Electrical Properties.*—Gutta-percha is a very good insulator; all insulators conduct a little, but the figure written after gutta-percha in Table I. will show the relative resistance to conduction with equal bulks of copper and gutta-percha. A better idea of the vastness of the number will be obtained by observing that light would take two thousand years to travel through the number of feet which the number would express. The practical result of this degree of insulation with the Atlantic core is that more than 99½ per cent. of the current leaving England would reach America if the cable were but laid; any improvement in insulation will, therefore, only go to diminish this half per cent. loss, in itself of no consequence whatever. India-rubber has a higher resistance still; the chief advantage to be obtained from this high resistance is the facility it gives for detecting faults. India-rubber is, however, superior to gutta-percha in another electrical property, called its inductive capacity. More words per minute, in the proportion of 4 to 3 at least, could be sent through an Atlantic or other long cable insulated with

india-rubber than if insulated with gutta-percha, the weight of insulator and conductor remaining the same. This point will be more definitely treated of hereafter.

(*f*) *Absorption of Water.*—Mr. Fairbairn long since stated the superiority of gutta-percha to india-rubber for deep-sea cables, owing to the comparatively small quantity of water which it absorbs. Probably the newer forms of india-rubber may have improved in this respect; but Mr. Siemens found that pure india-rubber absorbed 25 per cent., vulcanized rubber 10 per cent., and gutta-percha $1\frac{1}{3}$ per cent. of their weight in pure water; these quantities were reduced to 3, 2·9, and 1 per cent. respectively in salt water. The absorption continued for three hundred days: it was eight times greater for india-rubber at 120° of Fahrenheit than at 39°, but for gutta-percha it was only doubled by the rise in temperature. Mr. Siemens considered that pressure affected the absorption very little. The amount absorbed by gutta-percha in no way damages it. This is proved by thousands of miles of submerged cables; for instance, the tests of the Malta-Alexandria cable, laid four years since, under Mr. Forde's superintendence, by Messrs. Glass and Elliott. Part of this cable supports about half a ton per square inch pressure.

5. *Mechanical Properties of Completed Core.*—Few persons are aware of the great strength of the common gutta-percha-covered wire. An experiment was shown by the lecturer in which 500 cwt. was hung from the slender-looking core of the New Atlantic cable; it stretched some 10 per cent. under this weight, and was then taken down, knotted, squeezed, and cut open, when the copper conductor appeared quite undisturbed in the centre of the gutta-percha, which exhibited no trace of injury. Before the application of Chatterton's compound, the wire was liable to start out of the cable after it had been stretched and cut or softened, owing to the unequal elasticity of copper and gutta-percha; but with Chatterton's compound considerable force must be used to drag out the copper wire, even when the core has been stretched and is cut open. Table II. shows the strains which various wires can support.

TABLE II.

	No stretch.	One per cent. stretch.	Breaking-strain.
	lbs.	lbs.	lbs.
Atlantic core	340	414	660
No. 14 copper-covered to No. 1, 107 lbs. copper, 166 lbs. gutta-percha	134	162	218
No. 16 copper-covered to No. 4, 73 lbs. copper, 93 lbs. gutta-percha	100	120	150

Table III. gives the dimensions of the cores in some of the most important cables laid. It is noteworthy that 300 miles of the very smallest core practically in use, laid without any outer protection whatever, maintained our connexion with the army for nine months during the Crimean war.

TABLE III.

Dimensions of Cores of important Cables.

Name of Cable.	Copper conductor		Gutta-percha.		Approximate ratio $\frac{D}{d}$.	$\log_e \frac{D}{d}$.
	Total weight in lbs. per knot.	Total diameter of conductor $= d$.	Weight in lbs. per knot.	Diameter in inches $=$ D.		
Red-Sea Cable	180	0·105	212	0·34	3·4	1·224
Malta-Alexandria Standard	400	0·162	400	0·457	2·95	1·082
Persian Gulf.................	225	0·109	275	0·38	3·48	1·249
First Atlantic	107	0·083	260	0·38	4·1	1·57
Second Atlantic	300	0·144	400	0·464	3·28	1·19
England and Holland	143	0·095	223	0·34	3·47	1·244
Toulon and Algiers	107	0·083	223	0·34	4·26	1·45
Varna, Balaclava	73	0·062	166	0·3	4·84	1·58
French Atlantic	400	0·159	400	0·463	2·92	1·872

LECTURE II.

(Monday, February 5.)

SHALLOW AND DEEP-SEA CABLES.

THE lecturer first alluded to the omission from the first lecture of any mention of the new insulators—balata, Parkesine, collodion, Mr. Macintosh's material, and others. This omission was an oversight, due possibly to the fact that, as he has been unable to procure a specimen of any one of these materials for examination, he had formed no opinion as to their merits. The value of a new, good, and cheap insulator would be very great. The following is an abstract of the second lecture, under the heads in the syllabus:—

1. *Serving and Worming.*—Strands of hemp or jute are commonly laid or spun round the insulated core to serve as a pad or protection against pressure from the iron wires afterwards applied, and also, in some cases, to form a larger heart, allowing larger and more wires to be applied than could lie round the small insulated wire. This covering of hemp or jute is called the "serving" of the cable. When several insulated wires, to transmit distinct simultaneous messages, are included in one cable, as for short distances is frequently the case, these insulated wires are laid in a long strand, with hemp between them, to form a circular core. This hemp s called the "worming." The worming and serving were formerly tarred for their preservation against decay in water; but Mr. Willoughby Smith showed that the tar temporarily mended small faults of insulation, and might, therefore, conceal an accidental injury to the core: but tar was not so good an insulator as permanently to mend the fault, so that tar might lead to the submersion of a fault which would otherwise have been discovered and repaired before submersion. To avoid this risk tanned hemp is now used, and is often applied wet to increase the chance of at once detecting any accidental injury to the gutta-percha. Hemp under wires is remarkably durable, and jute also answers well as a cheaper substitute. When hemp is exposed in water it soon decays, and jute decays still more rapidly; both are liable to be eaten

by animals where exposed, but not where covered by iron. A specimen was shown where a small quantity of hemp exposed by a kink, at a depth of 800 fathoms in the Mediterranean, had been attacked by a species of *Teredo*; the part immediately adjacent, covered by iron wire, was intact. These animals exist in the Mediterranean even in depths of 1200 and 1600 fathoms. In applying the covering, care must be taken that the insulated wire be not overstrained. The simplicity of the work has sometimes led to the use of imperfect machines, which might cut the gutta-percha, and to the employment of boys too young to be careful.

2. *Iron Sheathing.*—The served core is commonly protected by iron wires laid round and round in a long helix, and abutting one against another, so as to present the appearance of a simple iron wire rope. This sheathing is frequently called a spiral covering; but the wires lie in a helix, not a spiral, which is a curve like that formed by a watch-spring, not that formed by a corkscrew. There is a popular impression that this form of cable must necessarily be very easily extended or stretched; but this impression is wholly erroneous. The single helix stretches by becoming more nearly a straight line, and by gradually closing so as to include a smaller and smaller cylindrical space; if this closing be prevented (for instance, if the wire be wrapped round a solid core), the helix will not stretch more than a solid wire: the closing is prevented in the ordinary cables by the arrangement of the outer wires, which abut each upon its neighbour, so that a cross section of the cable shows a compact iron ring. The tube formed by the wires cannot diminish in diameter, and consequently the helix cannot stretch more than a solid wire; this is proved by the experiments of Messrs. Gisborne, Forde, and Siemans in the 'Report of the Joint Committee on Submarine Cables,' 1861. Some extracts from their results are given in Table IV. The stretch of the Atlantic, Red-Sea, and Malta-Alexandria cables before breaking is, as will be seen, hardly more than the stretch of a single iron wire (part 2, Table IV.); the slight excess is owing to a slight diminution in the diameter of the cable, due to the more perfect closing of the wires one upon another when the strain is applied. Owing to the perfect iron ring formed by the wires, the inner core is not sensibly compressed. A helix may elongate by untwisting as well as by closing in the manner described, and sometimes this defect has been alleged as the only serious one. The total elongation which could arise from this cause is the difference of length between the wire as it lies round the cable and when stretched out straight; this is about $1\frac{1}{2}$ per cent. in the Malta-Alexandria cable: but no sensible untwisting ever does occur; about forty or fifty turns are, at most, taken out per mile, and this would elongate such a cable eighteen inches per mile, or about 0·03 per cent. When cables are recovered from great depths, no sensible change in the lay is found to have taken place; it cannot be seen that they have in any way been untwisted or stretched. Specimens of cables thus recovered were exhibited, and the following experiments shown to enforce the reasoning:—First, half a ton was hung on a light iron cable of the usual form, and it was seen that no stretch occurred, although less than half the weight would have stretched the core inside 20 per cent., and finally have broken it. This proved that the strain was really borne by the rigid helical iron wires outside, not by the core inside. Secondly, weights were hung on a single wire, outside a core of hemp and gutta-percha; this stretched a very little. Lastly, an experiment was tried which to all appearance resembled the first; but on the weights being taken off, the rope was bent and opened, and shown to consist of a mere hollow shell of iron wires, without any core whatever in-

TABLE IV.—STRENGTH AND ELONGATION OF CABLES AND MATERIALS.

Part 1.—CABLES.

(The Specifications are given at the end of the Abstract of this Lecture.)

Cables.	Breaking-strain in cwts.	Corresponding length in water. Fathoms.	Per cent. of elongation with one half breaking-weight per cent.	Per cent. of elongation with breaking-weight per cent.	Weight per knot in air. cwts.	Weight per knot in water. cwts.	Remarks and authorities.
1st Atlantic	80	4,979	0·24	0·8	21·70	16·30	Report of Joint Committee, App. 10.
Red Sea	65 to 87·5	3,806 to 5,112	0·16 to 0·34	0·56 to 1·16	21	17·30	Ditto.
Malta-Alexandria	147 „ 157	4,565 „ 4,874	0·2 „ 0·36	0·5 „ 0·86	42·70	32·73	Ditto.
2nd Atlantic	154	11,000	......	2·57 „ 4·65	35·75	14·0	Unpublished experiments by Mr. Fairbairn.
Steel - and - Hemp-coated Gibraltar	102·5 „ 147·5	7,928 „ 11,407	0·62 to 1·24	1·87 „ 4·06	25·47	13·11	Report of Joint Committee, App. 10.
Iron-and-Hemp-coated	67·5 „ 75	5,346 „ 6,000	0·26 „ 0·77	1·80 „ 3·10	24·87	12·65	Ditto.
Siemens's Copper-covered Cable	50	6,250	......	0·8	18·61	7·97	Mr. C. W. Siemens's unpublished information.
Allan's Cable............	18·37	7,500	......	1·0	8·0	2·5	Mr. Allan's unpublished information.
Ratan and Stretched Hemp...............	15·75	8,500	0·52	1·56	7·73	1·86	Messrs. Forde and Jenkin's unpublished information.

Part 2.—MATERIALS.

Copper strand, Malta-Alexandria............	5·75	......	0·22	8·5	3·57	3·125	Report of Joint Committee, Appendix 10.
Core, Malta-Alex.......	5·75 to 7·5	2,260 to 2,826	0·28	22 to 25	7·15	3·36	Ditto.
*Iron Wire, 0·079 in.	4·18 „ 4·5	5,600 „ 6,040	0·12 to 0·18	0·46 „ 0·72	96 lbs.	83·5	„
*Steel Wire, 0·079 in.	8·00 „ 8·50	10,600 „ 11,200	0·28 „ 0·34	1·00 „ 1·80	97 lbs.	84·7	„
Hemp and Iron	5·00 „ 7·43	......	0·16 „ 0·32	1·04 „ 2·46	141 lbs.	......	„
Steel and Hemp	10·87 „ 11·75	......	0·37 „ 0·51	2·28 „ 2·70	142 lbs.	......	„
Hemp alone	2·87	......	......	......	45 lbs.	......	„

* Other specimens of iron and steel would be found to stretch differently. Some iron and some steel would stretch considerably more, and very hard specimens would stretch less. The above results seem to be taken from fair samples.

side for eighteen inches of its length. This proved that the iron wires do not press injuriously on the core. In all these experiments the rope was free to untwist, but did not do so sensibly. The experiments were simple illustrations of facts well known to all practically acquainted with telegraphic cables. It may therefore be assumed that the common form of cable is not liable to stretch; but another defect, the liability to kink, has been urged against it. A kink is a loop drawn tight, or a twist in a rope concentrated at one point. Specimens of kinks were shown. A kink may be produced in any form of cable, with or without helical covering, inasmuch as a loop or a twist may be produced in any form by mismanagement. A rope coiled round a drum with one side out may be wound off and rolled round another drum, or paid out into the sea, without receiving any twist; but if, by mismanagement, the rope were pulled off the end of the drum, it would be twisted or kinked. Similarly, if coiled in a tank, with one side always uppermost, although apparently without twist, it would be twisted or kinked when pulled straight out of the hold. In practice these plans are not adopted; the cable is carried down into tanks from a drum with one side always turned in one direction: let one side of a straight cable be marked black, and let it be coiled into the hold so that the black side shall always be north, then this black mark will, on the north side of the tank, be turned from the centre; at the east and west side it will be uppermost and undermost respectively. The rope thus coiled in will have one twist in it for every turn round the tank: in a spun rope this twist will twist the rope tighter, or untwist it according to the direction in which the rope is coiled; but, in either case, when the rope is drawn out of the coil it comes out as it was put it—straight and without twist. The extra turn or twist is caused by coiling, and removed by uncoiling. There is one simple, universal, and sufficient rule to prevent the occurrence of a permanent twist. The cable must be taken out of the tank or off the drum in the same manner as it is put in or on; the opposite course will always put a permanent twist into a cable, and this twist, concentrated at one point, produces a kink. These points were illustrated by elementary experiments with a piece of india-rubber tubing to represent a cable; one side of the tube was painted so that a twist could readily be seen. When a cable is properly coiled in the tank it is possible, by a severe jerk, so to mismanage the uncoiling as not to take out the twist regularly; and kinks have thus been caused by several turns being caught up at once out of the hold. This now very seldom happens. Not one kink occurred during the paying out of the Malta-Alexandria and Persian-Gulf cables, or during the late Atlantic expedition, in all about 3500 knots. Even when a kink does occur it seldom injures the cable. A specimen was shown, cut from the Dover and Calais cable, containing six insulated wires, through which, kinked as they were, messages had for years been transmitted between England and France. The common form of cables affords a good mechanical protection against injury.

3. *Iron and Steel Wire.*—The tensile strength of a cable is the sum of the strength of the wires composing it. A cable covered with good iron should bear a strain equal to two tons per pound of iron wire per fathom. Thus a cable with 3750 lbs. of iron per knot, or 3·75 lbs. per fathom, in the sheathing should bear $7\frac{1}{2}$ tons. This rule corresponds to a strength of about 41 tons per square inch. The larger gauges and inferior qualities of iron cannot be expected to bear so high a strain as this. Best best is the quality most usually specified; but charcoal wire seems to be more permanent than the inferior brands. The wire should in no case be hard or brittle. Bright

wire is generally used for the smaller gauges, and black wire for the larger gauges, unless the wire be galvanized. Table V. gives the relative weights per knot of the different gauges, according to Messrs. Johnson, of Manchester. The weight of a wire per knot in lbs. is nearly equal to the square of the diameter in inches, multiplied by 16100 or, say, $16100d^2$. The wires are joined by welding and the cables by splicing. These operations require no special description. Welds should not be allowed in two wires of a cable at the same point, or near the same point.

TABLE V.

Showing Weights of Iron Wire of Different Gauges.

B.W.G.	lbs. per knot.
00	2066·68
0	1716·48
1	1393·92
2	1212·20
3	1048·32
4	872·80
5	748·80
6	622·08
7	529·92
8	449·28
9	368·64
10	305·82
11	241·99
12	184·32
13	144·00
14	109·44
15	86·40
16	65·66
17	50·68
18	39·16

4. *Sheathing-machines.*—These apply the wire with a constant tension, and so as not to twist it, keeping one side always uppermost; so that if it faces the core below the cable it will be turned away from the core at the top. To do this, each bobbin holding the wire moves parallel to itself. The effect of this arrangement was experimentally shown with the india-rubber tube to represent a wire. The effect of the other arrangement, in which the bobbin moves round the cable fast on a disk, as the arm of a wheel moves round the nave, or as the moon round the earth, was also shown. This arrangement twists the wire. Cables made with twisted wire are weaker and less manageable than those made with untwisted wire.

5. *Permanency of Cables.*—The wires of cables may rust or be chafed through on rocks, or be eaten through by some chemical action other than simple rusting, or they may be broken by anchors. Any motion in the water round a cable much accelerates the rusting away, and chafing depends wholly on this cause. In some bottoms, even in still water or great depths, decay does occur very rapidly; and this must be due to some other cause than simple rusting. Large wires are the natural protection to injury from the causes enumerated. Galvanizing also protects the wires from simple rust. In some situations the simple unprotected wires remain wonderfully unaltered;

but protection, where possible, should always be given. Bright and Clark's silicated bituminous compound applied over the wires affords the best protection yet known. The Persian-Gulf Cable is coated with it from end to end. To ensure permanency, cables in shallow seas were now laid weighing as much as ten tons per mile, with shore ends weighing nearly twenty tons to resist anchors (*vide* England-Holland Cables, Appendix). Many heavy shore ends were covered with strands of wire instead of simple wires. Mr. Siemens proposed to apply a covering of hemp outside the iron wires and to wrap this round with a zinc armour.

6. *Statistics of Cables in Shallow Seas.*—The total failures of all kinds, in shallow water, excluding the cables which had no proper outer iron protection, did not amount to 100 miles. About 2350 miles have been laid, which worked for some time, but are now abandoned. Of these, 1400 miles weighed less than one ton per mile, a weight which, for shallow seas, is now known to be absurdly insufficient; these worked for about two years upon an average. 950 miles weighed more than one ton, but not more than two tons per mile: the average life of these cables was five years. 5000 miles are now certainly at work, possibly more; they have already worked upon an average four years and a half: they include one cable which has worked for 15 years, and several 13 years old; but the average is lowered by the long Malta-Alexandria and Persian-Gulf cables, only lately laid. Every one of these cables, except the Malta-Alexandria, not originally designed for shallow seas, weigh more than two tons per mile. The interruptions on the lighter cables are somewhat frequent: on the Malta-Alexandria they have averaged four days per 100 miles per annum; even this is not worse than the best land-lines in India, and is ten times better than the worst land-lines in India.

7. *Maintenance and Returns from Cables in Shallow Seas.*—The average cost of maintaining cables of the Submarine and Electric Telegraph Companies has been for some years from £8 to £9 per mile, excluding the cost of total renewals, which should be provided for by a reserve fund. The expense of the Malta-Alexandria repairs is not known. This line has earned as much as £3000 in one week, or at the rate of £117 per knot per annum. In one year the average earnings during the time it was open were at the rate of more than £90,000, or £68 per knot per annum; allowing for interruptions, the maximum earnings in one year were £64,000, or £48 per knot. The Persian-Gulf cable is said to be earning at the rate of more than £100,000 per annum, or £85 per knot per annum. Neither cable has yet worked under favourable conditions; the former ends in a *cul-de-sac*, and the land-lines connected with the latter are so badly worked as to cause extreme delay and uncertainty. Such cables can be laid for sums varying from £300 to £400 per knot. The receipts on the Submarine Company's lines seem lately to have been at the rate of about £85 per knot of cable, or £26 per knot of insulated wire.

8. *Deep-Sea Cables.*—Cables laid in less than 1000 fathoms would now hardly be considered deep-sea cables; but formerly a depth of 300 or 400 fathoms was thought sufficient to entitle a cable to be put in this class, and the old classification has been adhered to in preparing the statistics of shallow-sea cables. A cable to be laid in a deep sea must, of course, be strong both absolutely and relatively to its weight in water; it must be light, or the great lengths required cannot be conveniently carried; it must not be liable to stretch, and it must coil well and be paid out easily. At first, light specimens of the form already described as used for shallow seas were generally

Table VI.

Compiled from App. 10 to the Report of the Joint Committee on Submarine Cables (Gisborne, Forde, and Siemens).

Materials.	Breaking-strain, cwts.			Elongation in percentage of length.			
	Max.	Min.	Mean.	Max.	Min.	Mean.	No. of Experiments.
Steel wire 0·079 in. diameter	8·50	8·00	8·20	1·80	1·00	1·41	5
Iron wire 0·079 in. diameter	4·50	4·18	4·35	0·72	0·46	0·55	7
Steel wire with 4 strands of Manilla hemp ...... ¾ in. lay	9·25	9·25	9·25	1·77	1·77	1·77	3
1¼ in. lay	13·00	12·12	12·59	3·12	2·32	2·63	6
Steel wire with 4 strands of Russian hemp ...... ¾ in. lay	10·0	9·50	9·70	2·18	1·80	1·76	3
1¼ in. lay	11·75	10·87	11·42	2·70	2·28	2·45	5
Iron wire with 4 strands of Manilla hemp ...... ¾ in. lay	4·75	4·50	4·62	0·79	0·47	0·63	3
1¼ in. lay	8·50	8·12	8·28	2·80	2·56	2·62	4
Iron wire with 4 strands of Russian hemp ...... ¾ in. lay	5·00	4·75	4·87	0·92	0·66	0·82	3
1¼ in. lay	7·43	5·00	6·20	2·46	1·04	1·82	6
Manilla hemp weighing 0·05 lb. to 0·0615 lb. per fathom	3·87	3·62	3·75	...	...	2·62	2
Russian hemp weighing 0·41 lb. to 0·45 lb. per fathom	3·50	2·25	2·87	1·60	1·00	1·30	2

employed. The Red-Sea cable is a fair sample. The first Atlantic cable is very similar; but the simple outer wires were replaced by strands of still smaller wires. The examination of Table IV. will show how far these cables met the above requirements. They could support from 4000 fathoms to 5000 fathoms of themselves hanging vertically from the ship. They could be laid, and about 7000 miles of this class were laid, in depths approaching or exceeding 2000 fathoms; and these cables have even, for a few miles, been hauled back from these depths. They seldom broke while being laid, but they were not permanently successful. Communication generally ended within a year from the time it was established, and the outer covering was then too much rusted to allow of repairs. The causes of failure were many,—bad gutta-percha joints, bad copper joints, injuries to the insulator before the cable was laid, high battery-power burning small faults into big ones and eating away the copper; lightning, from which they were often unprotected. These may be instanced as known causes of failure. It is also said some cables were laid too light, and sprung asunder when the iron wires rusted. It may be conjectured that when these wires rusted the gutta-percha could not bear the cable if suspended across a hollow; these are less probable causes of failure, but it is certain that the rusting of the outside and the failure of the cable generally coincided as to time. The failure was seldom gradual; it was almost, if not always, accompanied by a total fracture or interruption in the copper. When any of these injuries did occur they were irremediable. The first important modification of the common form was to adopt steel wires instead of iron, reducing their number, and enveloping them in hempen strands, so as to produce a cable which ex-

ternally looks like a hempen rope. Many excellent experiments were made on this form of cable (which was subsequently chosen for the second Atlantic) by Messrs. Gisborne and Forde, aided by Mr. Siemens. These experiments are given in full in Appendix 10 to the 'Report of the Joint Committee on the Construction of Submarine Cables,' published by government in 1861. The great strength, both absolute and relative, of this form may be seen from Table IV., showing that these hemp and steel cables will support 11000 fathoms of themselves hanging vertically in water. The mass of steel required to cover the core is diminished by the use of hemp; but as hemp is no lighter than water, it does not buoy up the wire. A steel wire simply wrapped up in hemp weighs much the same in water as a bare wire, and therefore wires, whether simply wrapped in hemp or bare, will support equal lengths of themselves in water; but the hemp may be so applied as to add all its strength to that of the steel, although the extensibility of the two materials is different. To do this the hemp must be spun round the steel with a definite lay, to be ascertained in each case by experiment. Table VI. shows the strength of iron and steel strands wrapped with Russian and Manilla hemp, and with $\frac{3}{4}$ lay and $1\frac{1}{4}$ lay respectively; also the strength and stretch of the separate materials. It will at once be seen that a difference of lay produces an extraordinary augmentation in the breaking-strain and in the elongation. The stretch of a wire when approaching its breaking-strain is concentrated nearly at one point, where it rapidly diminishes in diameter. The effect of the hemp is to support the wire at a number of successive weak spots of this kind, and thus greatly to augment the elongation before breaking; but it will further be observed that the breaking-strain of the combined materials is actually greater in some cases than the sum of the strengths of the separate materials. Thus the sum of the Manilla and steel, taking the mean strength, is a little less than 12 cwt.; but the mean of the combined strand is more than $12\frac{1}{2}$ cwt. With Russian hemp the sum of the separate strengths is 11·07 cwt., but the combined strand supported 11·42 cwt. The results with iron do not show this anomaly; but the apparent paradox with steel wire has been fully confirmed by independent experiments made for the Atlantic Telegraph Company. The explanation is, that when tested separately we have the strength of the weakest points or smallest sections of the wires and strands; but these materials are never uniform, and when combined, as it is most improbable that the two weakest points should coincide, we obtain the sum of their mean sections or strengths. The cables formed by these hemp-covered steel wires are very strong. Table IV. shows that the Atlantic cable, relatively and absolutely, is the strongest cable yet made, bearing more than twice as great a length of itself as the old iron cable. The new form stretches more than the old. The hemp may be eaten off or decay from the wires, weakening the cable, and the hemp affords less mechanical protection against injury; but the stretch is never such as to endanger the core, as has been proved by repeated experiments, and the most serious defect of the cable is probably its expense.

9. *Proposed forms of Deep-Sea Cables.*—Rowett's hempen rope could certainly be laid. The lecturer has had no sample of it, but fears it would be extensible. Allan's cable could also be easily laid, so far as its strength is concerned. It is said to coil badly; but the lecturer has not seen this tested. The proximity of the copper and steel inside the cable might cause the steel to rust and burst the core. Still it is desirable that this form should be practically tested. Mr. Siemans's cable, also mentioned in Table IV., will be found described in the Appendix to this lecture. The stretched

hemp has great strength and elongates little, but has to carry an immense load of copper, which does not add to the strength of the cable. The phosphorized copper sheathing would probably be very permanent. This cable has actually been laid and is now working. A trial of it was not successful in deep water, but a piece was recovered from 1600 fathoms. Duncan's cable, covered with plaited ratan, is too extensible; and the ratan, though durable in water, does not add much tensile strength. The lecturer has had a sample of cable made, in which he used Siemens's stretched hemp covered with Duncan's plaited cane. Its properties are given in Table IV., and its specifications in the Appendix. This cable combines great strength, small elongation, lightness, and cheapness. A bare gutta-percha core could be laid easily, but could not be recovered from great depths.

10. *Statistics of Deep-Sea Cables.*—Excluding the 1000 miles in abeyance under the Atlantic, and the cable lost in the first experimental trips in the Atlantic, only some 500 or 600 miles of cable have been lost during laying. About 9000 miles have been laid and worked a little while, but are no longer working. From 700 to 850 miles are now at work; but much of this is in no great depth. The Barcelona Mahon cable, believed still to work, although faulty, is included in this list. There is but one quite sound cable lying at work in more than 1000 fathoms, viz. that between Sardinia and Sicily, 243 miles long. One section of the Malta-Alexandria cable is in 420 fathoms, and has never shown any deterioration. The probable causes of failure have already been enumerated.

11. The general conclusions to be drawn from the statistics given in this lecture seem to be:—that in shallow seas, by laying heavy strong cables, we can ensure, and have obtained, success, both from an engineering and commercial point of view; that in deep seas we have hitherto failed, but that success is not unattainable, and may probably be reached by various methods. The lecturer believes that while in shallow seas, where repairs are possible, cables can hardly be laid too heavy or at too great an expense, in deep seas, where repairs will always be precarious, they can hardly be laid too light or too cheap.

Appendix I.—*Specification of Cables in Tables.*

1. First Atlantic.—Core (*vide* Table III., p. 205) covered with 18 strands of 7 bright best charcoal wires 0·028 in. diameter, called No. 22. Total diameter 0·62 in.; weight of iron 15·64 cwt. per knot.

2. Red Sea.—Core (*vide* Table III.) covered with 18 bright iron wires (? charcoal), called No. 16, B.W.G.; diameter 0·077. Total diameter 0·56 in.; weight of iron 16 cwt. per knot.

3. Malta-Alexandria.—Core (*vide* Table III.) covered with 18 bright charcoal iron wires, each 0·12 in. diameter, called No. 11 (?). Whole cable 0·85 in. diameter; weight of iron 33·56 cwt. per knot.

4. Persian-Gulf.—Core (*vide* Table III.) covered with 12 galvanized iron wires, 0·18 in. diameter, called No. 7½ (?); diameter of iron cable 0·9 in.; covered with hemp and bituminous compound to 1·25 in. diameter; weight per knot of completed cable 3·7 tons.

5. England-Holland main cable.—10 black wires 0·375 in. diameter, called No. 00; external diameter 1·58 in.; weight per knot 10·4 tons: shore end 15 wires, 0·22, called No. 5; covered with 12 strands made of 3 wires of

same diameter, covered with Bright and Clark's composition to 2½ in. diameter; diameter of iron 2 in., and weight per knot 19·6 tons.

6. Toulon-Algiers.—Core (*vide* Table III.) covered with 10 steel wires, each enveloped in four strands of Russian hemp; diameter of steel wires 0·08, called No. 14; diameter of strands about 0·2 in.; weight of hempen strands (?); diameter of completed cable 0·8 in.; weight in air 1·31 ton.

6. Steel and hemp-coated Gibraltar (proposed).—Core like Malta-Alexandria, covered with 12 steel wires in 4 hemp strands. Diameter of wires 0·08; weight of steel per knot 10·55 cwt., of hemp 6 cwt.; lay of hempen strands 1¼ in.; diameter of completed cable 0·875 in.

7. Iron and hemp-coated Gibraltar (experimental).—Like No. 6, with iron instead of steel.

8. Second Atlantic Cable.—Core (*vide* Table III.) covered with 10 bright steel wires, each in 5 Manilla hemp strands; diameter of each wire 0·095 in., called No. 13; diameter of strand about 0·28 in.; weight of hemp strands per knot about 12·8 cwt.; lay of hemp strands 3 in. Webster and Horsfall's homogeneous steel; diameter of competed cable 1·125 in.; weight of steel per knot about 13·75 cwt., and the serving round core about 2·2 cwt.

9. Siemens's copper-covered cable.—Sample in Table V. Copper conductor 550 lbs. per knot, insulator 420, diameter of core 0·52 in. Stretched hempen strands 440 lbs. per knot; copper armour 675 lbs. per knot; diameter of completed cable 0·75 in.

10. Allan's cable.—Sample in Table V. Solid copper conductor 0·114 in., weighing 240 lbs. per knot, surrounded by 19 steel wires 0·02 in. diameter, weighing 120 lbs. per knot; diameter of steel strand 0·16 in., covered with 300 lbs. of gutta-percha, diameter 0·456, and canvas web: total diameter 0·522.

11. Ratan and stretched hemp (sample in Table V.).—Core 3·63 cwt. per knot; diameter 0·34 in., covered with 15 hempen strands, weighing 1·84 cwt. per knot, and covered with plaited ratan cane, weighing 1·84 cwt.; total diameter 0·625 in.

LECTURE III.

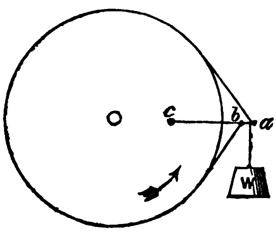
Laying and Repairing Cables.

The lecturer mentioned that he had received a letter from Messrs. Wells and Hall stating that some lengths of their india-rubber cables had been at work for some time under water. This was not doubted, but did not affect the original statement that much india-rubber had decayed, whereas no gutta-percha under water had decayed. Mr. Hooper had also misunderstood the statement in the abstract that pure india-rubber yielded to continued pressure; this was not meant to apply to Mr. Hooper's material, which is always more or less vulcanized. Attention was also drawn to a map of the telegraph lines between Europe and the East, prepared by Messrs. Bright and Clark, and kindly lent by them. The following is an abstract of the lecture arranged under the heads of the syllabus:—

1. *Stowage on board ship.*—The cable is coiled into large circular, or nearly circular, coils, so as to uncoil without receiving a twist, as shown on the last occasion. The coils are now held in iron water-tight tanks, and remain constantly under water. Tanks were first made for the Red-Sea cable, but first

used for the Malta-Alexandria cable. The tanks in the 'Great Eastern' were three in number, from 51 ft. 6 in. to 58 ft. 6 in. diameter, and 20 ft. 6 in. deep. To prevent rolling, their centre of gravity should be only slightly below the water-line. If the water be withdrawn from the tanks before the cable is paid out, the wires rust, and the chemical action heats them injuriously; with galvanized wires or cables covered with Bright and Clark's composition this heating does not occur. The eye of the coil round which the cable lies, generally from 6 to 8 feet in diameter, is filled with a cylinder, to prevent the bight of the cable from falling down, and possibly forming a kink or loop. Mr. Newall uses a cone permanently fixed in the centre of the coil. Messrs. Glass and Elliot lower their solid eye as the uncoiling proceeds. The cone appears to the lecturer to afford the best guarantee against kinking. It was used in the Persian-Gulf expedition. When running out at high speed the cable, if unchecked, would fly out, urged by centrifugal force, so as to be dangerous and unmanageable. This tendency is controlled by rings, lowered as the tanks are emptied, and first forcing the cable to run horizontally towards the centre and then controlling its upward motion. These rings were first used by Messrs. Newall and Co.

2. *Break.*—From the tanks the cable is laid in troughs to the break, by which a restraining force is applied to prevent too rapid egress. The troughs in the 'Great Eastern,' from the fore-hold to the break, measured 450 feet. The cable is wound four or five times round a drum, 6 or 8 feet in diameter, and the rotation of this drum is controlled by friction. The turns round the drum hold the cable securely, and prevent its egress unless the drum itself turns. The riding of the cable is prevented by a simple contrivance, known as a knife or plough, which was exhibited on a model. On the 'Great Eastern' this knife or plough could be adjusted. The simplest manner of applying retarding friction to the break is to hang a weight on to a break-strap, the other end of which is fixed. If the weight is hung from that end of the strap which would be lifted by the friction of the drum as it revolves, the retarding force can never exceed the weight, and a limit may be thus placed to the strain on the cable. But it was found in practice that with a strap making less than one turn round the drum, a weight of, say, four tons had to be applied to give a friction of one ton; the limit due to the position of the weight was, in such a case, of small value, since any heating of the strap or dirt on its surface might rapidly increase the strain fourfold. Mr. Appold's break remedies this defect. The principal on which it is constructed is illustrated by the annexed diagram. The end of the strap *a*, on which the greatest strain comes, is attached to a lever, hinged at *c*. Between the centre of the drum and *a*, the other end is attached to the lever very near *a*; but between *a* and *c* the retarding friction is obviously equal to the difference of the strains on the end *a* and *b* of the break-strap, and the weight *w* is almost exactly equal to that difference. This relation does not depend on the coefficient of friction between the strap and the drum; if the friction increases the weight *w* is raised a little, and the lever *a c*, owing to the eccentrical position of *c*, slightly lengthens the break-strap, reducing the friction. The opposite effect occurs if the coefficient of friction diminishes. The motion of *c a*, required to tighten or loosen the break-strap, is almost infinitely small; so that the angles of the break-strap and lever and the relations of the strains



do not sensibly vary. This arrangement was used on the 'Great Eastern.' It worked admirably, and gives a perfect safeguard against the application of any unforeseen strain by the friction of the break-strap. Strains may, however, occur from other causes, and for their detection a dynamometer is used between the break and the stern. The cable is passed under a weighted pulley, at a somewhat obtuse angle. The weight thus hanging on the cable is raised higher and higher as the strain on the cable increases. A scale is constructed by experiment showing the height corresponding to each strain. By this simple contrivance the actual strain on the cable can be observed at any moment. The following is a convenient formula for calculating the relation between the strains on break-straps and the friction produced. Let Q be the strain on that end of the strap which holds back the wheel, P the strain on the other end, f coefficient of friction, and b the angle embraced by the strap in circular measurement (unit $=57{\cdot}296°$).

Then $Q=e^{fb}P$, where $e=2{\cdot}71828$.

f may be taken for leather on iron .... $=0{\cdot}35$
„ . „ iron on iron, wet ...... $=0{\cdot}15$
„ „ wood on iron, wet, less than $0{\cdot}1$

3. *Theory of Submersion.*—In October 1857, Professor William Thomson published in the 'Engineer' a short sketch of the true mathematical theory of the form assumed by a cable while sinking, and the strains to which it is subjected under various conditions. The consequences of this theory were much more elaborately worked out (independently of Professor Thomson's publication, the lecturer believes) by Messrs. Brook and Longridge, in a paper read before the Institution of Civil Engineers in the spring of 1858. Much of what follows is taken from that paper. If the ship and cable are both at rest in still water, the latter hangs in a catenary curve, the strains on which are known and easily computed. This case actually occurs whenever a ship stops paying out cable, for instance, to cut out a fault: if the cable were suddenly stopped so as to lie at a great angle with the vertical line, a strain would be produced so great as infallibly to break the cable; thus, for a catenary in which the cable at the point of suspension lies at an angle of 9° 30′ with the horizon, the strain at the point of suspension is equal to 72½ times the weight of the cable hanging to the same depth vertically; so that in 2000 fathoms the strain would be equal to the weight of 145 miles of cable; but the Atlantic cable would break as soon as the strain exceeded the weight of 11 miles. From this it will be seen that a cable cannot be immediately stopped whilst being paid out, but must be gradually checked while the ship is backed, so as to keep the cable where entering the water as nearly vertical as possible. Another conclusion which follows is, that the cable while being paid out cannot possibly be hanging in a catenary curve, since the Atlantic cable did lie at an angle of about 9° 30′, and the strain, instead of being 2030 cwt., was only about 12 cwt. The following consideration may help us to perceive how different the case of a body sinking regularly is from the case of a chain at rest. Suppose the ship to drop a number of spheres of the specific gravity of the cable into the water at regular intervals; each of these would, within about two feet of the surface, acquire a definite, sensibly constant velocity $v=\sqrt{\frac{w}{q}}$, where $w=$ the weight and q the resistance to the body moving at one foot per second; these spheres, moving with constant velocity at constant intervals of time, would lie in a straight line from the surface to the bottom, and would be more or less inclined to the horizon as

the speed of the ship was less or greater. If the spheres were joined by an infinitely thin string, to which the water offered no resistance, they would form a cable which could be laid without any tension whatever, and with an amount of slack or waste depending simply on the inclination of the line to the horizon. The practical case of a submarine cable lies between these two extremes of the catenary and the isolated spheres; each short length of the cable lies like an inclined rod in the water, and has, therefore, a tendency to shoot back in a given direction, whereas the isolated spheres tend to fall vertically. Owing to this, cables, or, at least, heavy cables, cannot be laid without tension except at the expense of an enormous waste of cable. It will be unnecessary here to repeat the whole mathematical investigation which is given in Messrs. Brook and Longridge's paper. It will be sufficient to give the results arrived at. Mathematical readers will readily understand that these results are calculable from the data given :—

v = the velocity of the paying-out vessel in feet per second.

v_1 = the velocity of the cable paid out in feet per second.

w = the weight of one foot length of the cable in lbs.

ϕ = the angle which the cable at the surface makes with a horizontal line.

x = the height of any point A from the bottom of the sea.

q = the resistance in lbs. which the water opposes to the motion of each foot of the cable moving perpendicularly to itself, at the speed of one foot per second; q may be called the coefficient of resistance to displacement.

q_1 = the resistance in lbs. which the water opposes to the motion of each foot of the cable drawn through it lengthways, at the speed of one foot per second; q_1 may be called the coefficient of friction.

m = the resistance in lbs. which the water opposes to the motion of each foot of the cable moving perpendicularly to itself, at the speed of v feet per second; m is assumed $=qv^2$.

m_1 = the resistance in lbs. which the water opposes to the motion of each foot of the cable drawn through it lengthwise at the speed v; m_1 is assumed $=q_1 v^2$.

t = tension in lbs. at point A, which in what follows will be assumed as at the surface, where the maximum strain occurs.

Then if the cable be laid without any tension at the bottom, which is now invariably done, the equation to the curve assumed by the cable will become the equation to a straight line inclined at an angle to the horizon such that

$$\cos\phi=\frac{\sqrt{w^2+4m^2}-w}{2m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

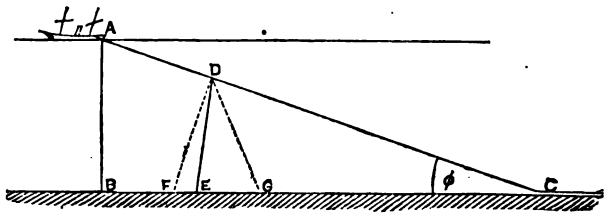
Or, what amounts to the same,

$$\frac{\cos\phi}{\sin^2\phi}=\frac{qv^2}{w} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From this it appears that the angle at which any given cable will be paid out is (when not tight at bottom) independent of the tension t (or of the velocity v_1), and is dependent simply on the velocity v of the ship. Cables which are bulky for their weight, or, in other words, are of light specific gravity, lie at a small angle; but by increasing the ship's speed any cable may be paid out at a small angle. We find further, when no slack is paid out,

$$t=\left(w-m^1\frac{(1-\cos\phi)^2}{\sin\phi}\right)x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$w\,x$ is simply the weight of a length of the cable hanging plumb from the ship to the bottom. This is the maximum tension that can be required to lay any cable without slack. This tension is always slightly diminished by a certain small portion of its amount, constant for a given speed and cable. The annexed diagram may help to explain the results arrived at. The cable A C, lying on an inclined plane of water at the angle ϕ, is carried by a ten-



sion equal to the weight of a length A B of cable, somewhat as a chain would be in equilibrium lying on a frictionless inclined plane A C, and hanging down over a pulley at A to the depth B; but the inclined plane of water is not at rest; it yields under the cable at every instant at every spot: if the cable were pressed through the water in a direction perpendicular to itself, so that the plane of water yielded before the pressure of the cable, and did not slip along it at all, the above analogy would be perfect, and the tension at A would be simply equal to $w\,x$; but since we have supposed the cable to be laid without tension, and without slack at the bottom, any point D must finally come to a point E, such that E C = D C; and it will be easily seen that the point D to do this must move in a straight line E D. Now this line D E is not perpendicular to A C; it falls within that perpendicular, D F; so that if we suppose the plane of water under A C to yield perpendicularly before it, we must also conceive the cable as slipping back a little on the plane by an amount corresponding to the space between D F and D E; but this slip is opposed by the friction of the water, which thus tends to prevent the cable from running back on the inclined plane, and so relieves the tension by a small amount. This amount is so small that it may be practically neglected, and would not have been mentioned did not the same considerations enable us to understand the effect of laying cable with a certain amount of slack. When this is done a point, D, moves in a line, say D G, differing much more in direction from D F than D E does. When much slack is laid, the cable slips back at a considerable velocity on the inclined plane, and meets with a frictional resistance tending considerably to relieve the tension. Thus, when the cable is paid out at a velocity v_1, we have

$$t = \left[w - m^1 \frac{\left(\frac{v^1}{v} \cos\phi \right)^2}{\sin\phi} \right] x. \quad \ldots\ldots \quad (4)$$

When v_1 is considerably in excess of v, the fraction of the whole tension which the friction m^1 subtracts from $w\,x$ is very considerable. It also increases rapidly as the speed of the ship increases, and it further increases if the specific gravity of the cable be small; for then it lies at great length in the water, presenting an immense surface to be gripped by the water. The friction on each foot is but small; but when twelve or thirteen miles of cable lie in the water, presenting a surface of 70,000 or 80,000 square feet, the result is practically very important. Assuming a mean speed of 10·4 feet

per second as that at which the Atlantic cable was paid out when lying at an angle of 9° 30′ in 2000 fathoms of water, with a tension of 12 cwt., we obtain the following values for the constants:—

$$\left.\begin{array}{l} q = 0{\cdot}085 = 0{\cdot}81\ \mathrm{D} \\ q_1 = 0{\cdot}0085 = 0{\cdot}081\ \mathrm{D} \end{array}\right\} \text{where D = diameter of cable in feet.}$$

From some observations it would seem as if the angle had been even less than the above, in which case q would be larger and q_1 smaller. From Beaufoy's experiments we should have expected q to be more nearly 0·65 D; but the roughness of the cable may account for the difference, as it certainly does for the great difference in the coefficient of friction, which is nearly eight times that which a smooth surface would present. It is probable that for a smooth iron cable the value of q_1 would be more nearly equal to 0·001 D, and q = 0·065 D. One reason will now be plain for giving in the foregoing tables the strength of the various cables relatively to the depth in which they are to be laid. The strain required to lay them is always a fraction of that depth; but the strain will not be always the same fraction of the depth, but will be smaller for the lighter cables when laid slack.

4. *Application of Theory.*—The practical results of equation (4) are most important. To lay any cable, however light, quite taut, we require nearly the tension due to a weight of the cable hanging plumb from the surface to the bottom; but by increasing the bulk of any cable, though we do not diminish its actual weight, we may, by laying a little slack, diminish the tension very greatly. With such a cable as the second Atlantic the tension was thus diminished more than half; to lay it taut would have required nearly 28 cwt., and 12 cwt. was the amount actually required when about 15 per cent. slack was paid out. The strain could be maintained constant in all depths by allowing a little more slack to run out in deep water, and even this could be prevented by a slight increase in the speed of the ship. No relief at all comparable to the above is obtained by paying out heavy cables slack; but, on the other hand, still lighter cables can be paid out under still more favourable conditions. The ratan and hemp cable (No. 11, Appendix I.) would, with 12 per cent. slack, be paid out without any strain at all; and if more slack than this were desired the cable would have to be pushed out of the ship. Nearly the same might be said of a bare gutta-percha wire. The Atlantic cable, in the last expedition, was laid in depths varying from 1750 to 2000 fathoms, at speeds for the ship varying from 4¾ to 6¾ knots per hour, while the cable ran out at from 5 to nearly 8 knots per hour. The angle, according to one method of observation, varied from 9° to nearly 12°, but was somewhat less according to other observations; 9° 30′ seems to have been a usual angle. The slack paid out in deep water ranged from 9 per cent. to about 18½ per cent. On the last day the slack was about 14·8 per cent. The strain on the cable was very constant, ranging from 10 cwt. to 14 cwt., and generally being between 11 cwt. and 13 cwt.: the pitching of the ship never caused more than about 2 cwt. difference in the strain; but once, when going slow to change holds, the strain was 17 cwt.; this accords with theory. The lecturer has to thank the engineers of the Telegraph Construction and Maintenance Company for the above information.

5. *Proposed Improvements.*—Reels have been proposed in ships as a substitute for the coils in tanks. Their great mass in motion would be difficult to control. Mr. Siemens actually tried with some success a reel for a light cable and drove it with an engine; he abandoned the plan for the old system. The defects which the reel is supposed to remedy do not exist.

Captain Selwyn has proposed to use a reel floating in the sea, ingeniously retarded by paddles, which would prevent too much slack from being laid. It hardly becomes a landsman to tell a sailor that such a reel would be unmanageable; but the difficulties of coiling in water, of launching the reel if coiled on land, of protecting the surface of the cable against collisions, of testing the cable, of remedying any defect, should any arise, and even of preventing one coil from cutting into those immediately below it seem unavoidable, and the defects the invention is supposed to remedy are imaginary. Buoys have been proposed to relieve the cable from part of its weight; any hollow buoys would be crushed very shortly after leaving the surface. Mere wooden floats would do little, and be difficult of attachment. This invention also labours under the disadvantage of being unnecessary, since cables can be paid out with 12 cwt. strain or less, which will bear 150 cwt. Vanes on a cable, opposing its slipping backwards, would be correct in principle, although probably quite impracticable; the result aimed at is obtained by increasing and roughening the surface of the cable. Most engineers who have had practical experience deprecate any attempt to catch the cable by nippers after it has left the ship. The danger of fouling is more considerable than the extra chance of safety given by the nipper. Lastly, many proposals have been made for some kind of elastic arrangement, to compensate for the change of strain caused by the rise and fall of the ship. When cables are paid out so nearly horizontally as is now the case, these arrangements, even if practicable, are not required; the alteration of the strain caused by the motion of the ship is quite inconsiderable, and there is great difficulty in devising any elastic arrangement which, by the inertia or momentum of its parts, would not aggravate the evil, such as it is. Unless when going very slow, in very bad weather, the best conceivable elastic arrangement would be useless, if not injurious.

6. *Repairs in shallow water.*—So long as the outer wires of a cable remain sound, repairs in shallow water are always easily effected. The cable is caught by a grapnel, lifted to the surface, cut, tested; and if the fault be near at hand, one end of the cable is buoyed, the other end passed round a drum driven by a steam-engine, which gradually hauls in the cable till the fault is found, when it is repaired, the cable again paid out, and spliced at the part buoyed. Bad weather and a rocky bottom are the chief difficulties to be contended with. Sometimes the cable is not cut or hauled on board, but simply underrun, passing over a grapnel or sheave hung outside the bows of the ship; as the ship moves forward the cable rises in front and is again lowered behind the ship. There are many points of practical interest connected with repairs in shallow water, and the lecturer refers those who require further details to Mr. F. C. Webb's paper in the 'Transactions of the Institution of Civil Engineers,' 1857–58. If the bottom be good, *i.e.* sandy or muddy, cables can always be recovered within 100 fathoms, and they are frequently hauled up in much greater depths.

7. *Repairs in deep seas.*—The only method hitherto practised with success has been to commence in shallow water and gradually haul the cable on board as described above. By carefully keeping the cable hanging vertically from the bows the strain on it will not greatly exceed in calm weather the weight of the cable hanging plumb from the ship. Cables have been recovered in this way out of depths of 1000 and 1500 fathoms at the rate of about a mile per hour. Messrs. Newall were very successful in the Mediterranean in recovering many cables by this plan; and the lecturer has seen a cable hang for three days at the bows of a ship where the depth was 800

fathoms, while the ship pitched violently owing to bad weather; the cable did not break, and was relaid with success. Even in this case the rise and fall of the ship did not injure the cable; but the change in the strain on the cable was great, and any good elastic compensation would have been useful; the cable itself, yielding say ¼ per cent. in a mile, gives a certain elasticity. Although this method of recovering deep-sea cables is not hopeless, the risks are great; bad weather or a weak point in the cable entail almost certain failure. A good nipper to catch the cable, should it break inboard, as it frequently does, might be of material service. Few persons will be sanguine enough to expect that a cable could be steadily picked up for 1000 consecutive hours, or say forty days, with about half its theoretical breaking-strain necessarily always upon it. We should, therefore, be grateful to the engineers in charge of the late Atlantic expedition for showing us that even in 2000 fathoms of water the attempt to hook a cable with a grapnel is far from hopeless. The chance of success by this method will now be examined. If a cable were laid absolutely taut along the bottom of the sea, when hooked by the grapnel it would rise a little way in virtue of its elasticity; if it stretched one per cent., by the time ten miles of it were off the ground the apex would be half a mile from the ground (a result few are prepared to expect); but the strain on the cable where caught would be very great, equal to the weight of about 24 miles of the cable, though the weight on the grapnel-rope would be only that of ten miles of cable. The result, therefore,

Table VII.

Giving length of cable lifted with a given slack, and hanging in a catenary curve, &c.

Slack in percentage.	Tension at highest point in terms of the weight of a length of cable hanging vertically from the surface to the bottom.	Length of curve in terms of versine.	Length of versine of span 100.
0	infinite	infinite	0
1	47·6	19·42	5·2
2	20·8	12·75	8·0
3	13·5	10·20	10·1
4	10·0	8·74	11·9
5	8·18	7·84	13·4
6	6·90	7·16	14·8
7	6·01	6·64	16·1
8	5·39	6·24	17·3
9	4·88	5·92	18·4
10	4·52	5·67	19·4
11	4·18	5·43	20·45
12	3·89	5·21	21·5
14	3·49	4·89	23·3
16	3·12	4·58	25·3
18	2·89	4·37	27·0
20	2·67	4·17	28·8
22	2·48	3·98	30·6
24	2·39	3·89	31·9

Table VIII.

Showing the length of a catenary curve of constant span = 100, with various deflections at the centre, and giving strains at highest point in terms of the unit length of chain.

Proportion of versine to span.	Length of versine or dip.	Length of curve.	Strain at highest point in terms of the unit length of chain.
0	0·00	100·0	infinite
$\frac{1}{12}$	8·33	102·1	160·9
$\frac{1}{11}$	9·09	102·6	149·3
$\frac{1}{10}$	10·00	103·0	137·6
$\frac{1}{9}$	11·11	103·4	125·8
$\frac{1}{8}$	12·50	104·3	115·2
$\frac{1}{7}$	14·29	105·4	104·3
$\frac{1}{6\cdot5}$	15·38	106·4	99·7
$\frac{1}{6}$	16·67	107·3	94·5
$\frac{1}{5\cdot5}$	18·18	108·9	90·5
$\frac{1}{5}$	20·00	110·4	86·2
$\frac{1}{4\cdot5}$	22·22	112·4	82·1
$\frac{1}{4}$	25·00	115·4	79·1
$\frac{1}{3}$	33·33	125·4	75·6
$\frac{1}{2}$	50·00	177·3	103·5

of trying to raise a cable such as the Atlantic laid taut would certainly be to break it; but cables are not laid taut in deep water, and the Atlantic is laid with a mean slack of about 12 per cent., and in the last days we may even count on 14 or 15 per cent. slack—that is to say, for every 100 miles passed over, 114 or 115 miles of cable were laid. Lay on the floor 114 inches of chain, between two points 100 inches apart; lift it in the middle on a hook, the two ends will hang down in catenary curves; and when the cable at the extremities is just off the floor, the hook will be 23·3 inches from the floor. Quite similarly a cable laid with 14 per cent. slack will, when caught by the grapnel, hang in two half catenary curves; and by the time 11·4 miles of the cable are off the ground, the grapnel will be 2330 fathoms from from the bottom, *i.e.* at the surface of the Atlantic. The strain on the grapnel-rope will be the weight of the cable lifted, or about 12·4 miles; the strain on the cable itself at the point of suspension will be much less, being only about 3½ times the weight of the cable hanging vertically, or say 8 miles of cable. (Observe that the strain on the cable and the weight of the cable are not synonymous. When the two ends hang plumb, the strain on the cable at top is half the weight of the cable carried. When there is little slack, the strain is much greater than the weight carried.) If the depth were only 2000 fathoms, the strain on the cable when brought to the surface would only be equal to the weight of about 7 miles of cable. Moreover, the actual cable is not held at any point except by its own weight, and there will be a pull at the bottom tending to haul in slack towards the grapnel amounting to several tons: but, even without counting upon slack obtained *in this way*, it is clear that if the cable will bear 11 miles of its own weight,

it could, under favourable circumstances, be hauled to the surface by a single grapnel.

Tables VII. and VIII. give the proportions and strains on catenaries in various convenient practical forms. Thus, from Table VII. we see that if ten per cent. slack be laid, the maximum tension on the catenary lifted in, say one mile, will be the weight of 4·52 miles of cable; in two miles depth the strain would be the weight of 9·4 miles of cable. In the latter case, $2 \times 5 \cdot 67 = 11 \cdot 34$ miles of cable will be off the ground, and the grapnel-rope must be strong enough to bear this. Table VIII. gives similar information, supposing we do not start from a definite percentage of slack, but know the proportion of the dip made by a rope to the strain. But although from these tables it appears that the Atlantic cable might be lifted by sheer pulling, this course is not advisable, owing to the extra strains produced by the heave of the ship, the resistance to displacement by the water, the friction of the water, possible currents of water, the possible drift of the ship to one side of the cable, and the possible existence of a weak point in the cable. Owing to all these elements the practical chances of success by sheer pulling are very small. It has been proposed to lift the cable by a number of ships, acting like so many piers to a suspension-bridge. It is difficult to suppose that they would keep their respective positions accurately, or all haul in at the proper rate. It has also been proposed to catch the cable at one point, then at another nearer the end, then to drop the first grapnel, and catch the cable again nearer the end, and so, working hand over hand, reach a point at last so near the end that the cable could be lifted nearly vertically. This is better than the last plan, but is unnecessarily complicated, and the cable might easily be injured in the attempts to catch it at so many points. The simple plan which at once occurred to all practical men, is to catch the cable with one ship by a holding-grapnel, and then to cut it with a grapnel from a second ship, some three miles to seaward; the loose end held by the first ship could then be hauled on board with little strain. This plan will probably be adopted, with much chance of success. It is certain the cable *was* caught, and probably it can be hooked again; if so, there should be no difficulty in raising it, unless it is rusted to a much greater extent than we have any reason to expect. The grapnel of the first ship should be a holding-grapnel, of which several models were shown, otherwise the loose end might fly back over it if the second ship cut too near the first. The second ship should have a cutting-grapnel, of which models were also shown, lest if the attempt were made to break the cable by brute force it might break at an inconvenient point. Mr. Latimer Clark's grapnel, which would answer either of these purposes, was exhibited. The cable when hooked releases a catch, allowing a block, to which the grapnel-rope is attached, to be hauled up the shank, pulling round two right and left hand screws by two steel bands; the screws close the jaws, which grip the cable or cut it; or one grapnel may be made both to cut and grip the cable. The grapnel can lie in only two positions; and if dragged in the proper direction, cutters placed at two diagonally opposed corners would cut the cable certainly to seaward, and the jaws hold the landward end. A simple form of holding-grapnel, conceived by Mr. Carpmael, jun., was shown; in this the cable is jammed between the prongs and a kind of half bollard. A holdfast or cutting-grapnel, designed by the lecturer, was also shown. Each prong is hinged on a pin projecting beyond the shank, and the prong is so shaped at the root that the cable when on it closes the prong tighter and tighter on itself, whereas the end of the prong when dragging through sand or mud is opened like a Trotman's anchor.

LECTURE IV.

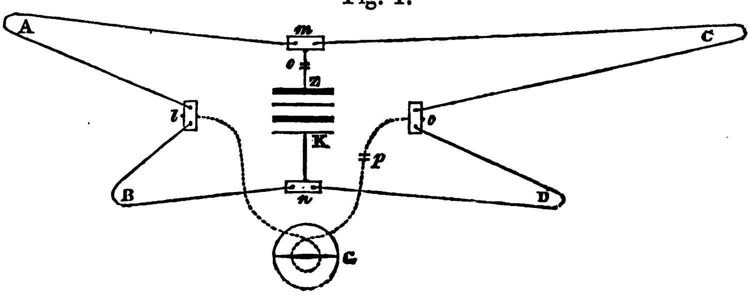
Electrical Tests.

1. *Terms Used.*—In order to understand electrical tests, it is chiefly necessary to have a definite conception of what is meant by electrical resistance. When the two end plates of a voltaic battery are joined by a wire or other conductor, an electric current flows through the conductor, the presence of the current being shown by the power the wire has acquired of deflecting a magnet in its neighbourhood. The magnitude of a current is simply proportionate to the force with which it acts on a magnet (*cœteris paribus*): thus a magnet hung inside a coil of insulated wire is called a galvanometer, or current-measurer, since it may be said to measure the current by the deflection of the magnet. When this deflection is small, as was the case with the instruments exhibited, in which the deflection of the magnet was indicated by the motion of a reflected ray of light, the deviations of that ray of light from its normal position may be considered true relative measurements of the current producing that deviation. The battery may be looked upon as a constant source of power, and the conductor as a kind of pipe conveying the current of electricity. The magnitude of the current depends, with a given battery, on what is called the resistance of the circuit. If the wire be small and long, the current will be feeble, and the resistance of the circuit is said to be great; if the wire be short and thick, the resistance will be small. The resistance of a conductor is the property in virtue of which it prevents a given battery from producing more than a given current, precisely as the resistance of a pipe to the passage of water might be defined as the property in virtue of which it prevents the passage of more than a certain current of water with a given head. The resistance of conductors varies not only with the dimensions, but with the materials of which the conductor is composed; and this resistance can be measured, *i.e.* compared with the resistance of any other given wire, in virtue of Ohm's law, viz. that the current through a given circuit is inversely proportional to the resistance, and directly proportional to the force producing it. That force is constant with a given battery; so that if we find our current halved by the introduction of a certain wire into a circuit, we may be sure that the resistance of the circuit is doubled; but in making that calculation we must take into account the resistance, not only of the wire, but of the measuring instrument and of the battery; when this is done the old distinctions of quantity and intensity currents will be found unnecessary, and indeed false, since a current has but one mensurable property, viz. its magnitude or strength. A current existing in a circuit which already includes a considerable resistance is what used to be called an "intensity current;" a current in a circuit which includes no considerable resistance is what used to be called a "quantity current." The first is little affected by the addition of a resistance which may almost wholly annihilate the second. A convenient method of measuring the resistance of a battery, due to Professor Thomson, is given in Appendix II. By the simple application of Ohm's law we might compare the resistances of two wires by observing the relative effect which they produce in a given circuit; but this is inconvenient, and hardly admits of much accuracy. The battery may vary, both as to force and resistance, during the two tests; and even if constant, the accuracy of the observation will be limited by the accuracy with which a deflection can be observed. More accurate practical tests have therefore been invented to measure and compare the resistance of conductors.

2. *Tests of Conductor.*—Every test used is a test of resistance, and all depend on Ohm's law above cited; the instruments may be much varied, but the

most convenient is probably that known as "Wheatstone's balance, or differential measurer." Let four wires be joined with a galvanometer and battery, as in fig. 1; then, if A, B, C, and D represent the resistances of the four wires, no current whatever will pass through the most sensitive galvanometer when $\frac{A}{B}=\frac{C}{D}$; but if the ratio $\frac{A}{B}$ be a little larger than $\frac{C}{D}$ a current will pass through the galvanometer in one direction; if $\frac{A}{B}$ be smaller than $\frac{C}{D}$, the current will be in the opposite direction. An explanation of this fact will be given in the ensuing Lecture. Four wires thus arranged allow us to

Fig. 1.



measure the resistance of any one of them which is not known, in terms of the three others: if A and B are equal, we may try how great a length of D is exactly equal in resistance to C, a selected standard; and this is precisely the test adopted to choose copper of small resistance or good conducting power. C is, say, 100 inches of copper wire, known to be good. Then the observer tries how great a length of copper wire from a new hank must be inserted at D to bring the galvanometer to zero, or no deflection. If this length be 105 inches the new hank is five per cent. better in quality than the standard; if the length be 95 inches, then the new hank is five per cent. worse in quality than the standard. But this is not all: if we desire to measure a coil of wire having ten times the resistance of C, we may make B exactly ten times A; and then, when we have adjusted the length of the wire D so that the galvanometer is at zero, we may be sure that the resistance of D is ten times C. Hitherto we have spoken of comparing two random wires; but it will clearly be convenient to have some common term of comparison, such as the foot for length, or the pound for weight. With this view the resistance of a certain piece of wire is chosen as the unit, and when other wires are measured, instead of being always directly compared, they are each compared with the unit, and are said to have each so many units of resistance. Several units have been proposed; the lecturer uses that known as the British-Association unit, sometimes called the "Ohm." When a unit has been chosen, whether for length, weight, or electrical resistance, it will always be found convenient to have multiples of the unit for measuring large quantities, and fractions of the unit for comparison with small quantities. With this object separate pieces of wire, equal to 1, 2, 3,.... to 1000 or even 10,000 units, are prepared in cases, and conveniently arranged so that any resistance required can be selected and inserted in the required circuit. These cases of graduated wires are called sets of resistance-coils, and are variously arranged by the different makers. Mr. C. W. Siemens and Messrs.

Elliott, Brothers, both make sets of British-Association coils. If, when possessed of such a set of coils, we receive a wire of which we do not know the resistance, we may arrange a Wheatstone's balance in which two equal coils are connected as at A and B, the new wire at D, and the set of coils at C. We then find by trial the number of units required to bring the galvanometer to zero. If we find D too small to be conveniently measured thus, we may choose two coils equal to 1 and 100 for B and A. When the galvanometer is at rest on completing the circuit, the resistance of D will be the hundredth part of the coils included at C. Similarly, if D be large we may make the coil A 1 and B 100; then the resistance of D will be 100 times that of the coils required at C to bring the galvanometer to zero. A still greater degree of precision in comparing C and D will be obtained if part of the wire between A and B be a uniform wire laid along a measured scale, and if the point l, to which the galvanometer wire is attached, be made movable along this wire, the resistance of which must be known as compared with the other parts of A and B. Now, if A, B, C, and D are as nearly balanced as they can be by the addition and subtraction of units at C, a still more perfect balance (indicated by the absence of deflection in the galvanometer) may be obtained by shifting l a little; then, if its position be observed giving the exact ratio between A and B, the exact value of D can be found in terms of the unit used at C by a simple rule-of-three sum. In fact every change that the rule of three is susceptible of can be worked out effectually by the above arrangement, and measurements can be made without an error of one part in 100,000. Experiments were shown illustrating the above statements. It will now be seen that we have the means of comparing the resistance of wires very accurately, and of comparing all wires with a common unit: but it is also convenient to be able to calculate beforehand what the resistance of a given wire will be or ought to be; and for this purpose it will be sufficient to know the resistance of some one wire of known dimensions of each material; the resistance of all other wires of that material can then be simply calculated, since that resistance is directly proportional to the length, and inversely proportional to the section, of the wire. Table IX. is a table of "specific resistances," defined in various ways. The first column contains the numbers which will probably be found most useful. The following is an example of its use:—Let it be required to know the resistance at 0° of a conductor of pure hard copper, weighing 400 lbs. per knot. This is equivalent to 460 grains per foot. The resistance of a wire weighing one grain per foot is 0·2106; therefore the resistance of a foot of a wire weighing 460 grains will be $\frac{0{\cdot}2106}{460}$; but the resistance of one knot will be 6087 times that of one foot, hence the resistance required will be $\frac{6087 \times 0{\cdot}2106}{460} = 2{\cdot}79$ units. If the diameter of the wire be given instead of its weight per knot, the calculation is still simpler, and the constant for English measures would be taken from the third column of the Table. Thus the resistance at 0° of a knot of pure hard-drawn copper wire 0·1 inch diameter would be $\frac{6087 \times 9{\cdot}94}{100^2} = 6{\cdot}05$. It will be seen that annealing wires materially alters their resistance, though it leaves their chemical composition quite unaltered. A rise in temperature increases the resistance of all the metals; and Dr. Matthiessen discovered that for all pure metals the increase of resistance between 0° and 100° C. is sensibly the same except for iron. Table X. gives

Table IX.

Specific resistance in B. A. units of metals and alloys at 0° Centigrade, from Dr. Matthiessen's experiments.

Name of Metals.	Resistance of a wire 1 foot long, weighing 1 grain.	Resistance of a wire 1 metre long, weighing 1 gramme.	Resistance of a wire 1 foot long, $\frac{1}{1000}$ inch in diameter.	Resistance of a wire 1 metre long, 1 millimetre in diameter.	Approximate percentage of variation in resistance per degree of temperature at 20°.
Silver, annealed	0·2214	0·1544	9·151	0·01937	0·377
„ hard-drawn	0·2415	0·1680	9·936	0·02103	..
Copper, annealed	0·2064	0·1440	9·718	0·02057	0·388
„ hard-drawn	0·2106	0·1469	9·940	0·02104	..
Gold, annealed	0·5849	0·4080	12·52	0·02650	0·365
„ hard-drawn	0·5950	0·4150	12·74	0·02697	..
Aluminium, annealed	0·1085	0·0757	17·72	0·03751	..
Zinc, pressed	0·5831	0·4067	34·22	0·07244	0·365
Platinum, annealed	2·810	1·96	55·09	0·1166	..
Iron, annealed	1·097	0·7654	59·10	0·1251	..
Nickel, annealed	1·535	1·071	75·78	0·1604	..
Tin, pressed	1·396	0·9738	80·36	0·1701	0·365
Lead, pressed	3·236	2·257	119·39	0·2527	0·387
Antimony, pressed	3·456	2·411	216·0	0·4571	0·389
Bismuth, pressed	18·64	13·03	798·0	1·689	0·354
Mercury, liquid	18·72	13·06	578·6	1·2247	0·072
*Platinum-silver alloy, hard or annealed	4·243	2·959	148·35	0·3140	0·031
†German Silver, hard or annealed	2·652	1·850	127·32	0·2695	0·044
‡Gold-Silver alloy, hard or annealed	2·391	1·668	66·10	0·1399	0·065

Note to Table IX.—This Table has been considerably modified, both on account of some clerical errors, and on account of the use of different specific gravities.

To convert resistance of wire 1 foot long, weighing 1 grain, into resistance per knot, multiply by 5293·1, and divide by weight of copper per knot in lbs.

Table X.

Constants, for metals or alloys, by which to calculate the resistance R at temperature t from the resistance r at zero :—$R = r\,(1 + a\,t \pm b\,t^2)$.

	a.		b.
§Pure metals	0·003824	+	0·00000126
Mercury	0·0007485	—	0·000000398
German silver	0·0004433	+	0·000000152
Platinum silver	0·00031	..	..
Gold silver	0·0006999	—	0·000000062

* The alloy used for B. A. resistance units, 2 parts platinum, 1 part silver, by weight.
† The alloy commonly used for resistance-coils.
‡ 2 parts gold, 1 part silver, by weight.
§ Approximate or mean formula.

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the formula and constants by which the resistance of any wire between those limits may be calculated. Roughly, all pure metals increase from 0·37 to 0·39 per cent. for each degree of temperature within the limits usually occurring in rooms. Table XI. gives the specific resistance of the more important metals at various temperatures. The resistance of most alloys is very much greater than the mean of the metals comprising them; indeed a singularly small mixture of a foreign metal reduces the resistance of the pure metals very largely—so much so, that in commerce copper cannot be obtained which is equal or even nearly equal to that of pure copper. The figures and constants given in the above Tables are only applicable with

TABLE XI.

Resistance in B. A. units of wire 1 foot long, weighing 1 grain.

Temperature Centigrade.	Soft copper.	Hard copper.	German silver*.	Platinum silver†.
0	0·2064	0·2106	2·652	4·24
5	0·2102	0·2147	2·657	4·25
10	0·2144	0·2188	2·661	4·25
11	0·2153	0·2197	..	..
12	0·2161	0·2205	..	..
13	0·2170	0·2214	..	..
14	0·2178	0·2222	..	..
15	0·2186	0·2231	2·666	4·26
16	0·2194	0·2239	..	..
17	0·2203	0·2248	..	..
18	0·2211	0·2256	..	..
19	0·2220	0·2265	..	..
20	0·2228	0·2272	2·67	4·27
21	0·2237	0·2283	..	..
22	0·2242	0·2288	..	..
23	0·2253	0·2299	..	..
24	0·2262	0·2308	..	..
25	0·2271	0·2317	2·675	4·27
26	0·2279	0·2325	..	..
27	0·2287	0·2334	..	..
28	0·2296	0·2343	..	..
29	0·2305	0·2352	..	..
30	0·2313	0·2360	2·68	4·28
31	0·2322	0·2369	..	..
32	0·2328	0·2375	..	..
33	0·2340	0·2388	..	..
34	0·2348	0·2396	..	..
35	0·2357	0·2405	2·684	4·28
36	0·2365	0·2413	..	..
37	0·2376	0·2424	..	..
38	0·2383	0·2432	..	..
39	0·2391	0·2440	..	..
40	0·2400	0·2449	2·689	4·29

* Calculated from specific gravity 8·47.
† Calculated from specific gravity 12·0. (Approximate values only.)

accuracy to pure metals. In old cables the quality sometimes was very bad; but lately the resistance of cable copper has usually been only about 10 per cent. more than that of pure copper. Table XII. gives the resistance of the copper of various cables at 24° Centigrade, also the specific resistance at the same temperature. Although alloys cannot be used for cables, owing to their high resistance, they are very useful in the construction of resistance-coils, since not only are coils of great resistance made of small bulk by their use, but these coils are much less altered by a change of temperature than if made of simple metals. The Tables contain the resistances of the chief alloys now in use, with the coefficients for temperature corrections. There are many points of great practical importance in measuring the resistance of conductors, which cannot be here fully treated of. Thus all resistance-coils should be wound double, so that the current may pass both ways round the

Table XII.

Resistance per knot and specific resistance in B. A. units of conductors and insulators of various cables at 24° C.

Name of Cable.	Resistance per knot of conductor at 24° C.	Specific resistance of foot-grain at 24° C.	Resistance per knot of insulator at 24° C. after 1 minute's electrification.	Specific resistance of insulator, or resistance of 1-foot cube, at 24° C., after 1 minute's electrification.
Red Sea	7·94	·2700	28×10^{6} to 38×10^{6}	$0{\cdot}875\times10^{12}$ to $1{\cdot}187\times10^{12}$
Malta-Alexandria, mean ..	3·49	·2637	115×10^{6}	$4{\cdot}06\times10^{12}$
Persian Gulf, mean	6·284	·2469	193×10^{6}	$5{\cdot}910\times10^{12}$
Second Atlantic, mean ..	4·272	·2421	349×10^{6}	$11{\cdot}22\times10^{12}$
Hooper's Persian-Gulf core, mean	..	..	8000×10^{6}	245×10^{12}

coil equally; this prevents self-induction—a disturbing element. Care must generally be taken in using the Wheatstone balance to connect first the battery at *o* (fig. 1), and then the galvanometer at *p*. The battery must be left connected for the shortest possible time, to avoid heating the wires; special precautions must be taken to avoid resistances at connexions, which are often considerable. The resistance of the wires composing the balance should not differ too greatly from that to be measured: short wire galvanometers answer best for short wires; long wire galvanometers for long wires; one cell of large surface generally gives better results than large batteries. The temperature of the wire to be measured and that of the resistance-coils should be accurately observed. These and many other points could only be fully developed in a treatise on testing. Practically, the copper of a cable is tested before it is used, to ascertain whether its quality is equal to that specified; when a knot of wire is covered it is again tested for resistance, to ensure that the proper quantity and quality of wire has been used; finally, after the cable is covered, the resistance test serves to check the length of the cable in circuit, to ensure that the conductor is at no point interrupted, and that the temperature in the tank is not higher than it should be.

3. *Tests of Insulator.*—A material is said to insulate well if it offers great resistance to the passage of a current of electricity. The word resistance is here used precisely in the sense in which it was applied to conductors: conductors and insulators both resist the passage of a current—the former allowing a considerable current to be produced by a small battery, the latter allowing but a feeble current to be produced by a powerful battery. The object of surrounding a conductor with a good insulator is to prevent any serious proportion of the current from being diverted to the sea or earth near the conductor: the insulator acts the part of the pipe, directing and containing the current; the copper acts more nearly the part of the vacant space, allowing the current to pass, and retarding it only by friction. A pipe to contain water can be made so that it shall not leak; but no material known, except dry air, will perfectly contain electricity: some leakage, indicated by a current, always occurs; and the simplest test of the soundness of the insulator is to connect one end of the conductor A (fig. 2) with one pole of a battery, Z, the other pole of which is joined to the water surrounding the insulated wire in the tank T (fig. 2). If a galvanometer, G, be placed between the battery and the conductor, and the other end of the conductor insulated, any current producing a deflection in the galvanometer must pass through the sheath from the copper to the water: such a current is often called a leakage. With a battery of known strength, and a galvanometer with which the observer is already well acquainted, the greater or less deflection of the galvanometer-needle will often be sufficient to show whether this leakage is so excessive as to indicate a flaw in the insulator connecting the water and the copper: this was the earliest insulation test; but it is clearly far from giving a measurement of the accurate kind which has been described for conductors. No two galvanometers are alike, nor is any one instrument constant in its indications; moreover an instrument of suitable delicacy for one length of insulated wire is unsuitable for another: the test is, therefore, a very rude and imperfect one; but a slight modification allows us to use it for the purpose of expressing with some accuracy the resistance of the insulator in the same units as those used for the conductor. Immediately after observing the deflection with the connexions in fig. 2, remove the insulated wire A B, and join the galvanometer and battery, as in fig. 3, inserting a set of resistance-coils at R, and joining a b by a short coil S, with a resistance bearing a certain definite ratio to that of the galvanometer—for instance, with $\frac{1}{999}$ part of that resistance. The current from the battery will, at a and b, divide itself between the two branches S and G, in the ratio of 999 to 1: now adjust the resistance-coils R until the same deflection be obtained as before; then, if we call x the whole resistance of the circuit when connected as in fig. 3, the resistance of the circuit in fig. 2, which is sensibly equal to that of the insulating sheath, will be 1000 x, since the current in the first case must have been 1000 times less than in the second case, when only $\frac{1}{1000}$ part of the current flowed through the galvanometer. It will be obvious that we might

Fig. 2.

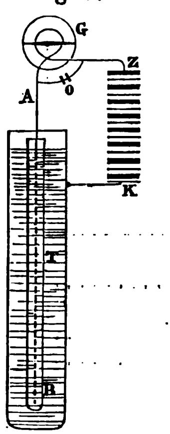
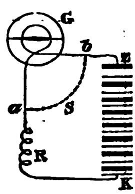


Fig. 3.



use for the first connexion a battery of 100 times greater electromotive force than is used in the second case; then the resistance of the insulator would be 100,000 x. In many cases x may be taken as equal to R, neglecting the other parts of the circuit. This can always be safely done if R be large, S very small, and K Z be a large single cell; but it is not difficult to calculate the whole resistance x when the above conditions cannot be obtained. The resistance between a and b is made up of two wires joined in what is called multiple arc: call S the one resistance, G the other, then the resistance of the two is $\frac{1}{\frac{1}{G}+\frac{1}{S}}$. To obtain the total resistance of the circuit, add the resistance of the battery and the resistance R. It will be further obvious that the resistance R need not be adjusted so as to give exactly the deflection obtained with the connexion in fig. 2. If any convenient deflection be observed with a given resistance R_1, the resistance R is given by a simple proportion. Moreover, it is unnecessary to repeat the test of fig. 3 every time an insulation test is made: we may often assume that unless some accident has happened to the instrument it will remain constant for some hours; in that case, having found the resistance which corresponds to one deflection (fig. 2), the resistance corresponding to other deflections results from a simple proportion. It is assumed, of course, that a galvanometer is used in which the deflections are proportional to the magnitude of the current, as in the case with reflecting galvanometers. In making the test (fig. 2) care must be taken to prevent the first shock of the current from passing through the galvanometer; for this purpose a connexion of very small resistance may be placed, as at o. This connexion must be broken immediately after the battery has been applied. The reason why this precaution is necessary will be mentioned in the next Lecture. An astatic reflecting galvanometer, with coils round both magnets, of the form designed by Professor Thomson, and with long fine wire, will be found well adapted for this test: to ensure accuracy care must be taken to make the coil S (frequently called a shunt) of thick wire and of such form as not easily to be heated, to maintain this strand as nearly at the same temperature as the galvanometer coil as may be, to let the battery remain in circuit as short a time as possible, to let the insulated coil remain in the water for such a time as may ensure its being at a known temperature throughout, to practise extreme cleanliness in the keys used, and to cut the ends of the wires tested to avoid loss by surface conduction. When these and other precautions have been taken, tolerably uniform results can be obtained; but they do not approach in accuracy those obtained in measuring conductors: for instance, the accuracy cannot be greater than that with which a deflection can be observed, or, say, 1 part in 200. In long cables the resistance across the insulator can be measured with the Wheatstone balance, by using the insulator as one of the four conductors A, B, C, or D (fig. 1). It will be seen that as the length of an insulated wire increases, the resistance to conduction across the insulator decreases; for there is continually a larger and larger area of material to conduct the current, and the distance across the insulator from the copper to the water remains the same. Thus the insulating sheath of 1000 miles of Malta-Alexandria cable is nearly equal in conducting power (or resistance) to a sheet of gutta-percha one acre in area and one tenth of an inch in thickness separating a copper plate from a sheet of water. The resistance at 24° C. of this insulating sheet of enormous section and very small thickness would be about 115,000 B. A. units;

the resistance at 24° C. of the long copper conductor would be about 3,490 B. A. units. These resistances are not so dissimilar as to be incomparable, even directly, by the Wheatstone balance; resistance-coils of German silver of 10,000 units, or even 100,000 units, nearly equal to the above insulation resistance, as it is sometimes called, are not uncommon. It will now be seen how it is that bodies of which the specific properties differ so enormously as copper and gutta-percha can yet be directly compared. The specific resistance of insulators can be given just as the specific resistance of conductors has been given; but it is customary to use a different definition, and call the specific resistance of an insulator the resistance of a foot cube electrified on the two opposite faces. Table XII. gives the resistance of the gutta-percha of the most important cables per knot, and their specific resistance as above defined. The following equation allows the resistance R of a core of known dimensions to be calculated from its specific resistance S:—

$$R = S \frac{\log_e \frac{D}{d}}{2\pi L}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

here $\log_e \frac{D}{d}$ = the hyperbolic log of the ratio of the diameter of the insulator to that of the conductor (given in Table III. above), $\pi = 3{\cdot}1416$, and L= the length of the core in feet. To convert the specific resistance, as above defined, into that of a wire or rod 1 foot long, weighing one grain, the figures given in the Table for gutta-percha must be multiplied by about 443,000. The fashion observed of writing 10^{12}, or ten at the power twelve, simply means that the number given must be multiplied by 1,000,000,000,000, or by one followed by twelve zeros. This plan of writing large numbers saves space, and is convenient in multiplication for those acquainted with the simpler properties of exponents. Hitherto that quality only has been spoken of in which insulators resemble conductors, viz. that of possessing a measurable resistance; but there are marked differences in the behaviour of an insulator and a conductor when a current is passing through them. The resistance of the conductor, if prevented from heating, remains perfectly constant; but the resistance of an insulator is apparently much greater during the second minute after the battery is applied than during the first; it increases again, but not so much, during the third minute, and continues to increase by smaller and smaller amounts for at least half an hour. It will be shown in the final Lecture that this apparent change of resistance is probably due to a kind of absorption; but, whatever be the cause of the phenomenon, it entails great inconvenience in testing. When the current is reversed, the apparent resistance falls as low as ever, or lower, and then again increases for half an hour. To meet this continual change, due to what is called electrification, tests are always made at definite times, generally one minute after the battery has been applied; but even with this precaution the residual effects of previous electrification are often embarrassing. All insulating substances, except air, present the same phenomenon, but in a greater or less degree. It is very marked, frequently producing a change of 50 per cent. in the apparent resistance of gutta-percha, and its effect is greater in cables which are thickly covered with the insulator. Neither pressure nor change of temperature greatly affect the proportionate effect of electrification. With Mr. Hooper's material, as supplied to the Indian Government, the change produced is extraordinary; at the end of ten minutes the resistance seemed to have increased nearly fourfold, and at the

end of about nineteen hours the resistance was twenty-three times greater than at the end of one minute*. This singular property of insulators is one of the chief difficulties to be met in any attempt to obtain strictly accurate measurements. A change of temperature also causes a much greater alteration in insulators than in conductors, and a rise of temperature causes a fall in resistance instead of an increase; thus the specific resistance of gutta-percha is about twenty times as great at freezing-point as at 24° Centigrade. Gutta-percha, as now supplied, behaves very uniformly in this respect, as is shown by the independent experiments of Mr. Siemens, Messrs. Bright and Clark, and the lecturer. The following equation, due to Messrs. Bright and Clark, will allow the resistance R of any core at a temperature $T+t$ (in degrees Centigrade) to be calculated from the resistance r at T° Centigrade.

$$R=r \times 0{\cdot}8878^{t} \quad \ldots \ldots \ldots \quad (6)$$

The number 0·8878 is not quite constant, but seems to vary between that figure and 0·9. The effect of temperature on india-rubber is not nearly so great; but the lecturer is not in possession of experiments the whole circumstances of which, including the preparation of the india-rubber, are known to him, and he prefers not to give results which might be misapplied and mislead. Pressure improves the insulation resistance of gutta-percha 2·3 per cent. for each 100 lbs. per square inch, according to experiments on the Malta-Alexandria cable, and 2·6 per cent., according to experiments on the Persian-Gulf cable. When it is remembered that the pressure in 2000 fathoms is about 2 tons per square inch, it will be seen that the improvement due to this cause is not to be despised. India-rubber behaves differently. Mr. Siemens published some curious experiments in the British Association Report for 1863, showing that pure india-rubber slightly fell off in resistance as the pressure increased. No sensible effect on the resistance of gutta-percha has yet been observed due to the absorption of water. The objects of the tests described are:—first, to ensure the use of a proper material, the quality of which may be specified in the contract; secondly, to detect any serious flaw in the outer coating, which would at once be shown by the diminution it would cause in the resistance. The more accurately the resistance of the insulator can be observed and calculated, the more certain we can feel of detecting even the smallest irregularity.

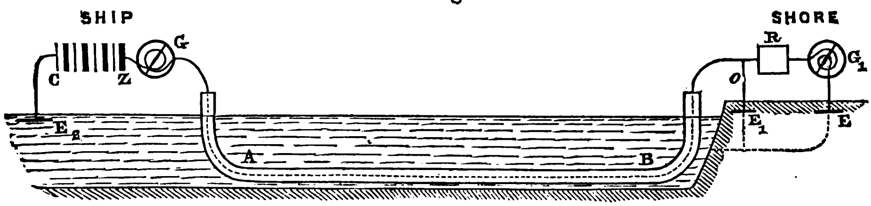
4. *Tests at Sea.*—Whereas tests on land are, in great measure, directed to secure a fit quality of material, the tests at sea should have but one object—the detection of a fault as soon as it occurs, and the determination of its nature and position. Common forms of galvanometer cannot be used at sea; but Professor Thomson's well-known marine galvanometer allows every test hitherto described to be applied as rigorously at sea as on shore. It consists of a very light magnet and mirror, strung on a fine tight fibre, and so perfectly balanced as not to deflect from its normal position relatively to the suspending frame and coils, however they may be inclined in any direction. A powerful magnet, in a fixed position relatively to the suspending frame and coils, directs the suspended magnet, and overcomes the influence of the earth, from which the magnet is still further screened by a thick hollow iron case, which wholly surrounds the coils, except where a glass window allows a ray of light from a lamp to enter and return so as to fall upon a scale after reflection from the mirror. These instruments are made by Mr. White, of Glasgow, and Messrs. Elliott, Brothers, of London, and are now almost

* This information was kindly supplied by Mr. Laws.

exclusively used for tests at sea, which may so far be considered quite unaffected by the direction of the ship or its motion. Two disturbing agencies are found in the currents induced in the coils of cable as the ship rolls and the so-called earth-currents, depending on the different electric potential or tension of the earth which may occur between the earth plate on shore and the connexion with the sea at the ship. These disturbances are readily overcome by the use of sufficiently strong batteries, so that no details or explanation of their action need be given here. It may be granted that on board ship all tests can be accurately made, and it only remains to consider what system shall be followed to ensure immediate detection of a rupture in the copper conductor or an injury to the insulator. Hitherto it has been a common practice to arrange a succession of tests recurring in a constant order at definite intervals of time. During the first twenty minutes of each hour an insulation test may be used; and the simple test (fig. 2) of watching that the spot of light in the galvanometer does not quit its proper place on the scale, as it certainly will do the instant any flaw in the insulator allows a connexion between the copper and the water, is probably the best as well as the simplest. During the next twenty minutes the resistance of the copper may be measured, showing that it is unbroken, and indicating the temperature of the bottom. During the last twenty minutes speaking instruments may be connected with the cable, and intelligence given and received; then *da capo*. But this system has great defects. We may wish to send or receive intelligence when it is impossible to do so; the clerk, or clock, or shore may not keep time with the ship, and cause needless alarm or confusion; special emergencies may require special tests, and then the routine plan either prevents these or causes confusion; but worse, much worse than all this, a fatal injury to the insulation may altogether escape detection during the periods allotted for continuity tests and speaking; it may pass over into the sea, and, when finally discovered, may be some miles from the ship. It would be better to maintain constantly a simple insulation test, and let the shore end remain insulated and unwatched. No fault could then occur in the insulator without being instantly detected; and even a break in the copper, inside the insulator, would be shown by a sudden fall in the leakage, owing to the shorter length of cable which would then be under the action of the test. A simultaneous injury to insulator and conductor would be still more obviously indicated; but such a plan as this would result in voluntarily throwing away the assistance to be derived from intelligent observations on shore, which, it will be seen in the next Lecture, may give important assistance in determining the position of a fault when it does occur. To meet this dilemma plans have lately been devised by which an insulation test on the ship and a simultaneous insulation test on shore can be nearly constantly maintained; speaking can be practised at any moment by ship or shore, and even during the transmission of messages the insulation test need not be wholly suspended. The first of these plans, in order of publication if not of conception, is due to Mr. Willoughby Smith. The connexions required are shown in fig. 4. C Z is the ship battery; E_2 the ship earth plate, or sea connexion; G the marine galvanometer; A B the cable, connected at the shore end with a great resistance R, equal to, say, the insulation resistance of four or five knots of cable. G_1 is a very delicate galvanometer on shore, placed between the resistance R and the earth plate. When these connexions are made, a slight deflection on the ship galvanometer, G, will indicate the normal leakage of current through the gutta-percha. *Almost* the full tension of the battery will act on R, and cause a feeble cur-

rent to pass through this resistance, causing a moderate deflection on G_1. This feeble current will, of course, add to the leakage indicated by G; but if R be equal to the gutta-percha of, say, five knots, and A B be 1000 knots long, the leakage through R will only add $\frac{1}{210}$ part to the deflection on G, and this may be neglected. A fault of insulation occurring in A B will

Fig. 4.



instantly increase the deflection on G, will lower the tension of the battery acting on R, and so diminish the deflection on G_1. Ship and shore will both be advised of the misfortune. A break of continuity in the copper of A B, without loss of insulation, will diminish the deflection on G, and wholly stop the deflection at G_1 after a little while; thirdly, if the cable breaks altogether, there will be a great increase in the deflection of G, and a total cessation of all deflection on G_1. The shore can, without altering the connexions, communicate with the ship by making shorter or longer contacts between the earth and the cable at *o*. This will cause corresponding deflections on G; but if a resistance be inserted between *o* and the earth, the deflection on G will be small, so that any considerable fault of insulation would still show on G, by causing a sudden and permanent alteration in the mean deflection even during the signals. The ship can signal to the shore by reversing its battery or by simply increasing and diminishing its tension. The insulation test on board would, the lecturer presumes, be wanting during these signals It is to be hoped that this or an equivalent system will be adopted in future. It gives perfect freedom from routine, and a greatly increased chance of detecting any fault the instant it occurs. This is the more important, as faults always do occur on board ship, and either in the top flake of the coils or in the machinery. In the next lecture Professor Thomson's plan of attaining the same object will be described.

Appendix II.—*Method of Measuring the Resistance of a Battery.*

First make the connexions shown in fig. 5, where C Z represents the battery to be tested, G the galvanometer, and A B a short wire. The resistance of the conducting wire of the galvanometer must be known, and let it be called equal to *a* units. Then adjust the resistance of A B so that

Fig. 5.

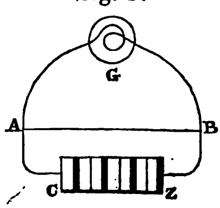
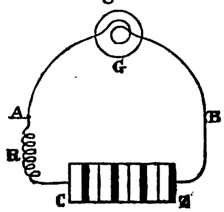


Fig. 6.



it shall be equal to one unit, and observe the deflection of the galvanometer. Next break the connexion at A B, and introduce a resistance R, as shown in fig. 6. Adjust this resistance till the deflection is the same as before; let the resistance at R, when thus adjusted, be called b, then the resistance of the battery, or, more strictly, the part of the circuit A C Z B, will be equal to $\frac{b}{a}$. If it be not convenient to make A B exactly equal to *one*, any other convenient resistance c may be taken, and then the resistance of the battery will be equal to $\frac{c\,b}{a}$.

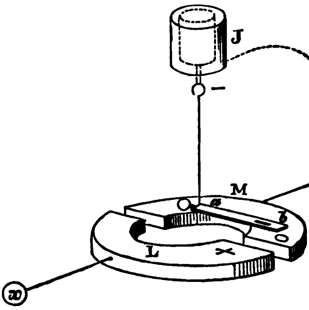
LECTURE V.

Electrical Tests (*continued*).

1. *Testing Short Lengths.*—It was shown in the last lecture that whenever a current could be observed traversing a conductor or an insulator, the resistance of the conductor or the insulator could be measured in definite units; but when it is desirable to test a very short length of insulated wire, the methods hitherto described are not available: the resistance which is opposed by the insulator is then so great as to prevent the passage of any current which can be detected even on the most sensitive galvanometer. A distinct class of tests must then be applied, which, as will presently be shown, give the same information as the tests by the direct observation of currents, and even indirectly give the resistance of the insulator in the same units. All are probably familiar with some of the properties acquired by a body charged with statical electricity by the electrical machine, such as the power of at first attracting light bodies and then repelling them when they are similarly electrified, &c.: few will have attempted to repeat these simple experiments without discovering that there is much difficulty in retaining the charge of electricity, as it is called; no sooner has the pith ball or the brass knob acquired the desired properties than they are lost by leakage along the insulating glass stems or silk threads. The insulation required for these experiments is of exactly the same nature as that required for submarine cables; the leakage down the glass stem is due to conduction, just as the leakage from a submarine cable is due to conduction through the insulator; and the current or quantity of electricity conveyed away is often in both cases too small to be shown by a galvanometer. It is not difficult, however, to observe how long the body charged keeps its peculiar properties; and this time is itself a measure of the goodness of the insulation, or, in other words, of the magnitude of the resistance which the insulator offers to the passage of the current. We may, therefore, charge a short length of cable with statical electricity, and observe the time required to allow that charge to fall to, say, half its original amount. The gradual fall of the charge may be observed by an electrometer, an instrument specially adapted to show the tension or potential, as it is called, of an electrified body. The rudest of all these instruments is the common gold-leaf electroscope; and roughly we might test a short insulated wire by charging it while the conductor is connected to the gold leaves, and the insulator is dipped into an uninsulated basin of water: if the gold leaves at once collapse the insulation is bad; if they remain long divergent the insulation is good. But no two gold-leaf electrometers are alike, nor are they in any way adapted for exact measurement. Peltier's electrometer would give somewhat more accurate indications;

the needle repelled by the brass knob would deflect to a given number of degrees, and its gradual return could equally be observed in degrees and fractions. The deviations are not, however, proportional to the potentials producing them, and most Peltier's electrometers are constructed without any expectation that they should be used for accurate measurements. They require, moreover, a very high tension to show any effect whatever. Professor William Thomson has, to obviate these defects, designed various electrometers, of which the divided-ring electrometer is, perhaps, the most convenient for testing cables. It is constructed on the following principle:— A light flat aluminium needle *a b*, fig. 1, balanced by a counterpoise, is suspended by a platinum wire from a point connected with the interior coating of a Leyden jar. Under the needle two half rings, L and M, are placed, with the division on one side directly under the aluminium needle in its position of rest. The whole is placed inside a metal case, not shown in the drawing. Suppose the needle *a b* not to be charged, then, if L be connected with *x*, an electrified body, while M is connected with the earth, the needle will turn slightly towards L; and this will be the case whether the electricity of *x* be positive or negative. If we now charge the Leyden jar with, say, negative electricity the needle will be brought to the same potential as the inner coating; it will be much more strongly attracted than before by L if the electricity of *x* be positive, and would be powerfully repelled if *x* were negative. If *x* loses its electricity and returns to the potential of the earth, the needle *a b* will return to its original indifferent position between L and M, being equally attracted by both. One object of connecting the needle with a Leyden jar is to provide a considerable supply of electricity for the needle, so that the unavoidable slight leakage which must occur may not affect one test or even a series of tests. A loss of one unit of electricity per minute will matter little if the whole store be one thousand, such as may be held by the jar; but if the store be only one or two units, such as would be received by the needle, such a loss would be fatal. The deflections will also be greater and the instrument will be more sensitive the higher the potential to which the jar is charged; but the indications will only be constant so long as the jar is charged to the same degree. In the instrument as made the deflections are shown by a spot of light reflected from a mirror hung above the needle, as in the reflecting galvanometers. The Leyden jar is placed in an atmosphere dried by sulphuric acid, and will hold a sensibly constant charge for days at a time. Finally, the metal case screens the needle from all attraction or repulsion by bodies electrified outside, owing to a well-known law. The deflections, being angularly very small, are proportional to the potentials of the bodies to be tested, which are connected with L; while M is kept permanently in connexion with the earth. With this instrument, nothing is easier than to compare accurately the times occupied by the charged conductor of a piece of cable covered with water in falling from the first tension to half or any other fraction; and the times thus occupied are relative measures of the insulation resistance of the insulating cover. No very high tension is required, and the test by this

Fig. 1.



instrument gives one direct proof of the identity of electricity given by friction and that from the voltaic battery. In making the test the cable may be charged by a spark or two from a machine or electrophorus, or it may be charged by simple contact for an instant with a wire joined to one pole of a voltaic battery of, say, 50 or 100 elements. It will readily be understood that when we know how fast a reservoir of given capacity empties itself by a given pipe, we may calculate the resistance which the pipe has opposed to the passage of the water. Such a calculation would, with water, be much more complicated than with electricity; and the following formula gives the means of calculating, in B.A. units, the resistance of the insulator. When the potential P at the beginning, p at the end of a time t, measured in seconds, are known—

$$R = \frac{t}{S \log_e \frac{P}{p}}; \quad \ldots \ldots \ldots \quad (7)$$

or,

$$R = \frac{0{\cdot}4343\, t}{S \log \frac{P}{p}}. \quad \ldots \ldots \ldots \quad (8)$$

In the first of these equations the hyperbolic, and in the second the ordinary logarithm of $\frac{P}{p}$ is used; but in both we have a quantity S, called the capacity of the cable, the meaning and measurement of which will be presently explained.

2. *Testing Joints.*—One use of the test described is to test the joints of insulated wires. When the conductor of a cable is joined in the ordinary way to a battery or otherwise electrified, any leakage which may be observed may be due to the whole or any part of the insulator, and no test of this kind proves a joint separate from the rest of the cable, and the general leakage from a long cable is comparatively so great, that it may entirely mask a very slight flaw at some one point, such as a joint. A joint may be tested by dipping it into an insulated trough connected with one pole of a battery, while the other pole and the cable conductor are joined or connected with the earth. Any current which is then observed must pass through the insulator in the trough, and the test becomes a test of one spot only; but a joint may not be as good as the rest of a cable, and yet have so high a resistance as to show no current in this way. A more searching test is given by the use of the electrometer. Dip the joint in the insulated trough as before, and connect the trough with the test-plate L (fig. 1); electrify the trough by a machine or electrophorus, and watch the gradual loss by leakage into the cable at that point. The conductor should be connected with the centre. It is of course essential that the trough itself should be very perfectly insulated during this experiment; and a similar remark applies to all tests of short lengths of insulated wire. In unpractised hands the loss by moisture on the surface of supports, dirty keys, and other connexions will generally be much greater than the loss which it is desired to measure. Conduction along the surface of ebonite used for the instruments, and along the surface of the gutta-percha or other insulator under test, can be partly prevented by extreme cleanliness; but an artificially dried atmosphere is necessary in all cases where extremely high insulation is required—as, for *instance*, for the Leyden jar of Professor Thomson's electrometer or the

trough to test joints. The ends of a short wire to be tested should also be freshly cut. Ebonite, after being in use for instruments for some time, often requires to be freshly polished. As an example of what can be done by selecting proper materials and by drying the atmosphere, so as to prevent a moist film from being deposited on the surface, the lecturer can, from his own experience, state that Leyden jars, in Professor Thomson's electrometers, can be made to hold their charge so well that not one half per cent. will escape in 24 hours. Such a cable as the Atlantic falls from charge to half charge in about fifteen minutes. Some wires, covered by Hooper's material, fall from charge to half charge in from seven hours to two days.

3. *Induction Tests.*—Hitherto the word charge has been used as having a sense with which all are familiar, and the indications of an electrometer, which really measures tension or difference of potential from the earth, have been received as evidence of a greater or smaller charge; it is time to justify these expressions. The charge of electricity which an insulated body will receive really means a definite quantity of electricity. This quantity, when escaping to the earth, which is assumed to be at zero tension, produces a current equal to the quantity divided by the time occupied in its escape. Bodies are said to contain equal charges when these charges will produce in their discharge equal currents. A charge held by a body is said to be at a certain tension or potential, meaning the quality measured by the electrometer above described. The total charge which a given body in given circumstances will receive is proportional to the potential or tension to which it is raised. Thus the charge produced by contact with the pole of a battery of fifty cells is fifty times that produced by a single cell. The tension or potential produced by a frictional machine is of exactly the same nature as that produced by the voltaic battery; it is simply greater in amount. Thus a body may be charged equally by sparks from an electrophorus and by a voltaic battery; then if this charge be allowed to escape through a galvanometer, the current in each case will be equal, and produce an equal deflection; and, again, if two bodies are charged to the same potential, say by contact with one pole of the same battery, then if the current produced by the discharge from the two be equal the charges on both were equal, and the *capacity* of both bodies was equal. The capacity of a body for receiving a charge depends on many elements; it increases as the external surface of the body increases, and it increases as the surrounding bodies in connexion with the earth are brought near to the insulated electrified body. Thus the capacity of a Leyden jar, where the inner electrified surface is large and close to the outer unelectrified surface, is much greater than that of a sphere of equal surface in a large room. The general laws regulating capacity and potential are too complex to be here explained; the capacity of a cable increases directly as its length, just as the capacity of two equal Leyden jars is double that of one. The gutta-percha acts the part of the glass; the copper that of the inner tinfoil; the water or moisture outside represents the outer uninsulated coating. Owing to the large surface and slight thickness of the insulation, the capacity of a long cable is very great, and its discharge can be shown on almost any galvanometer; the discharge from a yard can be seen on a sensitive instrument if charged by, say, 100 Daniell's cells. Fig. 2 shows the connexions required to show the discharge. M is a common key, by which the conductor of the cable A B can be first placed in connexion with the battery Z C by a contact at *p*, and then removed from the battery, and immediately connected at *o* with one terminal of a galvanometer, G, the other terminal of which is in connexion with earth at E_1.

The contact at *p* charges the cable, and that at *o* discharges it through the galvanometer. It will readily be seen that as the quantity which goes into the cable must be equal to that which leaves it, if the galvanometer G were placed between Z and *p*, it would be affected to the same extent by the entrance of the charge as by its exit at *o*. With a well insulated cable, deflections due to the rush in or out of the charge are far greater than that due to leakage across the sheath; and it is to avoid the disturbance due to this momentary current that in making insulation tests no sensible part of the current is at first allowed to run through the galvanometer, but is conducted through a short circuit as at *o*, fig. 2, Lecture IV. The deflection due to the charge or discharge of a short cable is the result of a single very short impulse; and this deflection may be used to measure the charge in two ways. First we may make a standard knot of cable or Leyden jar, or condenser as it is sometimes called; we may take the discharge from that, as in fig. 3,

Fig. 2.

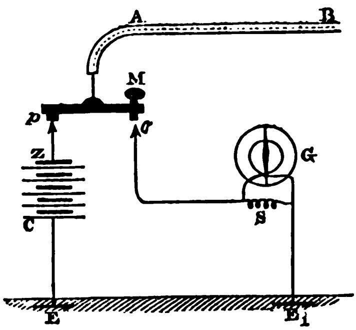
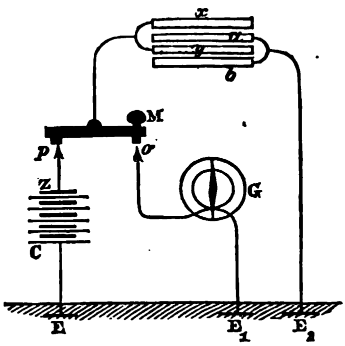


Fig. 3.



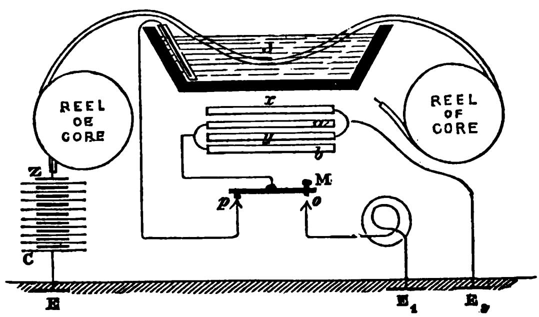
where *a b* represent plates in connexion with the earth, and *x y* insulated plates separated from them by mica, gutta-percha, paraffin, glass, or air, the other connexions being the same as in fig. 2. Next we may charge the cable from the same battery, and by trial bring the galvanometer to the same deflection by shunting part of the current through coils, which can be adjusted; if $\frac{1}{100}$ part of the current pass through G, then the capacity of the cable is one hundred times that of the condenser: the relative charges, if not differing much, may be taken as proportional to the deflections on a reflecting galvanometer, or, more strictly, to the sines of half the angles on any instrument. A galvanometer with a comparatively heavy needle is better for this purpose than a reflecting instrument with mirror and light magnets, owing to the resistance of the air. The comparison of capacities may be made in a much more accurate manner by various less well-known devices, as:—by the transfer of a charge from one condenser to another, and the measurement of the potential before and after the transfer; by the relative effect of two discharges in opposite directions through a differential galvanometer; by balancing a succession of discharges through one coil of a differential galvanometer against a permanent current adjusted with the aid of resistance-coils; and by balancing the discharges against a permanent current in an arrangement resembling a Wheatstone balance. These arrangements have, it is believed, not yet been used as practical tests; but it will be useful to give here a formula by which we may compare the results of any two or more observers, who have not got galvanometers which

they have compared, or condensers of known relative capacities. Let S be the capacity of a conductor measured in the units required for equation (8); let t = half the time of a complete oscillation of the needle of the galvanometer under the influence of terrestrial magnetism alone, i = the angle to which the needle is thrown by the momentary current, R_1 = the resistance in B. A. units of the circuit through which the battery used to charge the cable would produce the unit deflection on the galvanometer, then

$$S = 2\,\frac{t \sin \frac{1}{2} i}{\pi\, R_1} \qquad (9)$$

The accuracy with which this measurement can be made is not very great, owing to the difficulty of measuring t on most instruments. The charge is said to be due to induction, and these tests are called induction tests; it now remains to show their practical application. The discharge may be used to measure insulation, thus: charge the cable by contact at p (fig. 2), and then break contact at p without making contact at o; the charge which is, as it were, bottled up inside the cable, leaks gradually through the gutta-percha. After, say, one minute, make contact at o, and observe the difference between the deflection thus obtained and that obtained when the cable is discharged immediately after being charged. The difference measures the loss in one minute. The galvanometer can in this way, by successive trials, be used to ascertain the rate at which the charge is lost for all the tests described above as measured with the electrometer, though less conveniently. A similar test is applied by Messrs. Bright and Clark to the testing of joints. A joint (fig. 4) is placed in an insulated trough of water connected with a condenser, the battery is applied to one end of the cable, and any slight leakage which may occur at the joint gradually accumulates in the condenser. After a minute or more the condenser is discharged through a galvanometer, which may then show the result of a minute's accumulation even when the permanent current passing at any moment would not have been sensible. But these are only the indirect consequences of the induction test. Its main

Fig. 4.



object is to compare the capacities of various cables and the inductive properties of various materials; and the reason why these points are important is, that the number of words which can be transmitted per minute through long submarine cables is, *cæteris paribus*, inversely proportional to their capacity; so that a long cable A, each knot of which will, from a given battery, receive only half the charge received by cable B (equal to A in other respects), will transmit double the number of words per minute. The cause

of this cannot here be explained, but the fact is experimentally and theoretically proved. By theory, the charge on equal lengths of two wires, covered with the same insulator, should be inversely proportional to log $\frac{D}{d}$, where D and d, as before, indicate the diameter of the insulator and conductor: but the capacity also varies with the insulator used; thus the charge on a knot of the Persian-Gulf cable, insulated with gutta-percha, was 35 per cent. greater than a knot of similar dimensions insulated by Hooper's material, and was somewhat more than four times greater than the charge would have been had it been possible to find an insulator with the properties of air instead of gutta-percha. The property of a material in virtue of which it affects the charge is called its inductive capacity, and the ratio of the charge induced when the solid material is used to that which would be induced if air were the insulator is called the specific inductive capacity of the material. It will be seen that equation (9) gives the means of expressing the capacity of a knot of cable in certain units. Table XIII. gives the calculated capacity of some cables and materials in these units, with the specific inductive capacity of gutta-percha, india-rubber, and Hooper's material. These numbers are very much less well ascertained than the resistance measurements given in previous tables. Nevertheless improvement in this quality is of very much greater importance than improvement in insulation resistance. Neither temperature nor pressure seems to affect the charge or capacity of cables very materially.

4. *Tests to detect Faults.*—Faults in cables may be classed as follows:—1. A fracture or interruption in the copper conductor, which, nevertheless, remains insulated inside the gutta-percha covering. 2. A fracture of the copper conductor and gutta-percha, in which a considerable length of copper wire remains exposed to the water. 3. A fault intermediate between the first two, with copper and gutta-percha both broken, but little copper exposed. 4. A connexion between the iron covering and the copper by a nail or wire driven in. 5. A hole or imperfection in the gutta-percha sheath, establishing a connexion of considerable resistance between the conductor and the sea. The first of these faults is of course followed by a total cessation of all electrical communication between the two ends of the cable. Its position may be detected in two ways. The charge which the cable will contain may be measured as above described; and if the charge per knot is known, the charge observed will directly give the distance of the break; and the accuracy with which the position of the fault can be determined is limited only by the accuracy with which the relative charges can be compared: the cable is an insulated Leyden jar, the capacity of which is simply proportional to the length of the conductor from the shore to the fault; so that if the discharge from a knot of the cable, with a given battery and reflecting galvanometer, is represented by a deflection of ten divisions, and the discharge from a cable containing a broken copper conductor is 100 divisions, we may feel certain that the fault is about ten miles from shore. By the more accurate modes of comparing discharges, the distance of a fault of this kind, even on a long cable, might be accurately found. The method by the throw or deflection of a needle is not applicable to a very long cable, because of the time occupied by the discharge; the theory of the formula given above supposes that the needle moves under a sudden impulse, very short compared with the time of oscillation of the needle. A second plan of determining the position of this kind of fault is to measure the resistance of the insulating sheath. Thus, if we know by the

TABLE XIII.—INDUCTION TESTS.

Name of Cable.	Material.	S=Electro-magnetic capacity per knot, multiplied by 10^{12}.	s=Electrostatical capacity per knot, in absolute measure (French).	Specific inductive capacity $I=3{\cdot}281 \times \frac{2s}{6087} \log_\epsilon \frac{D}{d}$ (English).	Source of Information.
Malta-Alexandria	Gutta-percha	0·0399	3583	4·18	Calculated from value of I given by F. Jenkin. Report on Electrical Instruments, Class XIII., International Exhibition, 1862.
Atlantic cable	Gutta-percha	0·0345	3340	4·3	Condenser adjusted by Mr. Willoughby Smith. Experiments by F. Jenkin and Chas. Hockin.
Persian-Gulf cable	Gutta-percha	0·0323	3120	4·2	Condenser adjusted by J. C. Laws. Experiments as above.
Hooper's cable, Persian-Gulf pattern	Partly vulcanized India-rubber	0·0239	2310	3·11	Compared by Prof. W. Thomson, and independently by Messrs. Bright and Clark, with Persian-Gulf gutta-percha cable.
	Masticated India-rubber	....	....	2·82	Jurors' Report, International Exhibition, as above.

discharge-test that the cable is insulated where broken, and find the insulation resistance to be 1000 units, whereas the insulation resistance of one knot is 1,000,000 units, we may conclude that the fault is 1000 miles off, as it will require one thousand miles of sound core to give so small a resistance as 1000 units. Faults of this kind are very rare where strands of copper properly jointed are used. The second kind of fault enumerated also wholly stops communication between the two ends of the cable, and almost invariably occurs when a cable is broken with violence. The copper and gutta-percha are then both stretched, and the gutta-percha springs back when the copper breaks, and leaves the latter exposed: but sometimes the copper breaks some little way from where the gutta-percha yields and inside it; then the third kind of fault occurs, intermediate between the two former. When some inches of copper are exposed, a connexion of small resistance is formed with the sea. In this case the resistance of the copper conductor, measured from the shore, measures the distance of the fault; we know the resistance per knot, and if we observe 500 times this resistance, the fault is 500 miles off, the resistance of the earth itself being *nil*. A small correction ought of course to be made for conduction through the insulator when sound, but in good cables this may be neglected. It is by this test that the operators at Valencia are able to tell that they have still the full length of the cable between them and the spot where the cable was first broken. There is little difficulty in determining whether a fault of this nature has occurred; for the comparatively small resistance of the cable shows that it is connected with the sea where it ought not to be, and the constancy of that resistance shows the connexion to be complete. This brings us to the third class of fault, where the connexion between the sea and the copper exists, but is imperfect, or due to only a small area of exposed copper. The fault itself then possesses considerable resistance, sometimes more than that of all the copper conductor of the cable; and, what is worse, this resistance is inconstant, varying rapidly and capriciously between extremely wide limits. The test for resistance in that case simply tells us that the fault cannot be beyond the distance corresponding to the smallest resistance observed. The fourth kind of fault corresponds almost exactly in behaviour to the second, but the connexion with the sea is still more perfect; the resistance will vary still less, and there will be a total absence of the feeble currents which result from the copper and iron of a cable when broken and separated by salt water. Earth currents, due to a difference of potential between the shore and sea at the fault, may of course, in both cases, be observed. The fifth kind of fault is easily detected: there is a considerable fall in the insulation resistance, and a slight or moderate fall in the apparent resistance of the copper conductor between the two stations; but messages can still be transmitted, as a portion only of the whole current, inversely proportional to the resistance of the fault, escapes into the sea. If one station insulates the cable and the other measures the resistance, the fault behaves like a fault of the third class, and this test will not detect its position. If, however, the resistance of the fault remain constant, and the two measurements of resistance, R and r, be made from station A, when station B respectively insulates the end of the cable and connects it with the earth, we obtain two equations concerning the resistances, in which there is only one unknown quantity, viz. the resistance of the fault. When this is eliminated, the following equation is obtained:—

$$D = r - \sqrt{(R - r)(L - r)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

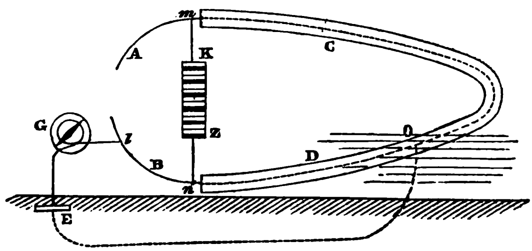
where D = the resistance of the conductor between the fault and the observer,

and L = the resistance of the whole conductor between the stations. On very long cables a correction for the effect of the uniform leakage through the insulator would be necessary; but this correction fades into insignificance in comparison with the error introduced by the supposition that the resistance of the fault will remain constant during the two tests. Successive tests from the two stations, the distant end being insulated in each case, will also give two equations, by which, on the same supposition that the resistance of the fault remains constant, its position can be determined. Then calling R and R_1 the resistance in the two cases, we have—

$$D = \frac{L + (R - R_1)}{2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where D is the resistance of the conductor between the station which observed the resistance R and the fault. This test labours under the same defect as the previous one. When a return insulated wire can be substituted for the earth, so that the observer has both ends of a complete metallic circuit before him, the position of a fault such as is described, even of varying resistance, can be accurately determined by more than one method. Mr. Varley uses a diffential galvanometer, a well-known instrument, to ascertain when an equal current runs into both ends of the metallic circuit and out at the fault. This will only be the case when the resistance between the galvanometer and the fault is the same by both roads; this condition is easily fulfilled by adding resistance-coils between one coil of the galvanometer and the defective wire. The resistance which must thus be added to bring the galvanometer to zero is obviously equal to twice the resistance of the metallic conductor between the fault and the distant station. Prof. Wheatstone's balance may be so arranged as to give another method, by making the connexions as in fig. 5, where the fault, supposed to be at *o*, forms, as it were, part of the galvanometer wire. In this case, as in the preceding, a variation in the resistance of the fault does not affect the result; it will cause a greater or less deflection in the galvanometer until the desired balance is effected; but it will not alter the relative resistances of the several parts of the main circuit required to reduce the deflection to zero. The test in fig. 5 is made

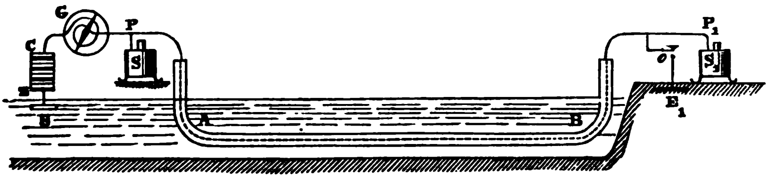
Fig. 5.



by adjusting the relative resistances of A and B until no deflection is obtained; then the fault will be at a point such that $\frac{A}{B} = \frac{C}{D}$, where C and D represent the resistance of the conductor separating *m* from the fault and *n* from the fault. When the total resistance of the conductor is known, this will give the position of the fault very accurately. Mr. John Murray, of Glasgow, is said by Professor Thomson to have first applied this test on board the 'Niagara' during the first Atlantic expedition. Is was reinvented by the

lecturer, and may be used to detect very small faults even on short lengths. It is now only necessary to describe one more plan of determining the position of a fault of this nature by a simultaneous test at both stations, or on board ship and on shore. This plan is Professor Thomson's, but it was also re-invented by the lecturer after seeing Mr. Smith's test, described in Lecture IV. This reinvention is not very remarkable, as the lecturer owes the chief part of his theoretical instruction in electrical science to Professor Thomson, and is familiar with his instruments and methods; but as this is, he believes, the first publication of the plan, he thinks it well to state these circumstances. The connexions required are shown in fig. 6, where G is a galvanometer;

Fig. 6.



S an electrometer at the same station; S_1 an electrometer at a distant station, where the end of the submerged cable A B is insulated. The battery C Z is connected with the other end of the cable. Then let C= the current observed on the galvanometer, V the potential at the same station, U the potential at the distant station, l= the length of the cable, K the resistance of the unit length of the conductor, n the resistance of the unit length of insulator to conduction across the sheath, and let $\sqrt{\frac{K}{n}} = a$. All these quantities may be known, and should be measured in the so-called absolute units or other equally coherent system. Let λ be the distance of the fault from the ship or galvanometer station, then:—

$$\lambda = \frac{1}{2a} \log_{\epsilon} \frac{F}{D}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where

$$F = V + \frac{K}{a} C - U \epsilon^{a l},$$

And

$$D = U \epsilon^{-a l} + \frac{K}{a} C - V.$$

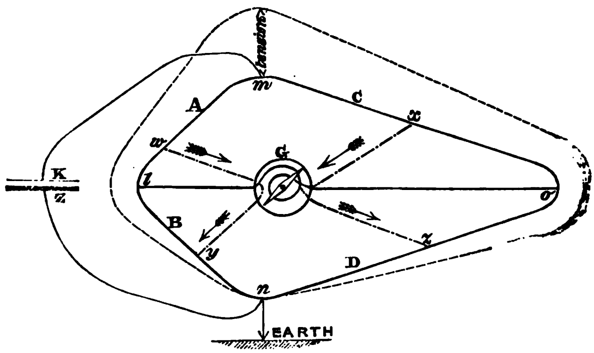
Undoubtedly this test is not of so simple a nature that it could be executed by an ordinary clerk; but it is interesting to know that a test does exist by which even a fault of this description, which has hitherto baffled electricians, can have its position fixed with mathematical certainty. This is the more important as the connexions shown in fig. 6 are precisely those which are, in the lecturer's opinion, the best adapted for tests during the submersion of a cable. The marine galvanometer G would give one test of insulation, the electrometer S a second one, the electrometer S_1 a third test on shore. The shore would speak to the ship without causing a suspension of the insulation test either on S or G; and even when the ship speaks to the shore the electrometer S will maintain the insulation test, as it is not affected, like the galvanometer, by the rush of the current in and out of the cable as it is partly discharged or additionally charged by the withdrawal or addition of part of

the battery power. The electrometers have, on the same grounds, a superiority over the galvanometers in their behaviour, under the influence of earth currents or the rolling of the ship. But in favour of Mr. Willoughby Smith's plan it must be conceded that more operators can at present be found who are familiar with galvanometers than with electrometers; so that clerical errors would not be so likely to occur with his plan as with that just described. The lecturer concluded by saying that he might have shown many pretty experiments with powerful magnets, long sparks, coloured tubes, &c., but he had been anxious to show experiments by which real work was done rather than the more amusing or striking features of electrical science; and he begged to thank the audience for the kindness with which they had listened to his endeavours.

APPENDIX III.—*Explanation of Wheatstone's Balance.*

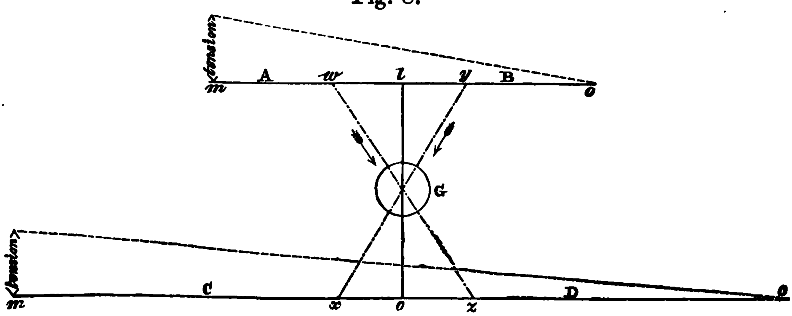
Fig. 7 shows the connexion of this arrangement, in which the letters are

FIG. 7.



similarly arranged and have the same meaning as in fig. 1, Lecture IV.; but a dotted line has been added surrounding the four conductors A, B, C, and D, and these have been joined to earth at *n*. Let us suppose the galvanometer wires to be removed from *l* and *n*; the ball K of the battery will produce a maximum tension at *m*, and this tension will gradually decrease round both conductors A B and C D to nothing at *n*, as would be shown by an

Fig. 8.



electrometer such as is above described. Moreover this tension decreases

precisely in proportion to the decrease of resistance between any given point and earth; so that if the wires AB, CD were straightened out as in fig. 8, and the resistance represented by their length, the fall of tension along each wire would be exactly represented by the height of the dotted straight line shown above each conductor, the maximum height being supposed to correspond to the full tension of the battery. The current in the two wires would of course be very different, but halfway along each the tension would be equal to half the maximum; at two thirds the distance, it would in each wire be two thirds of the maximum, and so on. Now if two points of the conductors, at equal potentials or tensions, are joined by a wire, no current will pass along that wire, for a current is always due to a difference of tension acting like a head of water: the case is analogous to that of a pipe joining two reservoirs; if the water in each be at the same level no current will flow through the pipe, no matter how deep the reservoirs may be; so that if the galvanometer wires are applied to two points so chosen that $\frac{A}{B}=\frac{C}{D}$, the tensions at those points being equal, no current will pass through the galvanometer. But if the wires are applied as at y and x, a current will pass through in the direction of the arrow; and if the wires are applied as at z and w, a current in an opposite direction will pass; so that by trial we may always ascertain the exact points at which the condition $\frac{A}{B}=\frac{C}{D}$ is fulfilled. The dotted line round A B C D in fig. 7 corresponds to the dotted line in fig. 8, and represents the gradual fall of tensions. When the galvanometer wires are applied as at y and x or w and z, this dotted line should be slightly modified; but the modification will not affect the truth of the above reasoning.

Full size

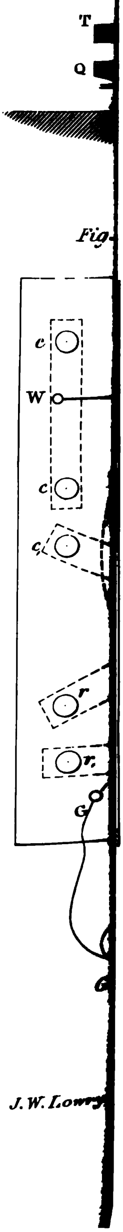


Fig. 7. Diagram of connexions when commutator is in position drawn. (Fig. 1.) d connected with d, & f with f,

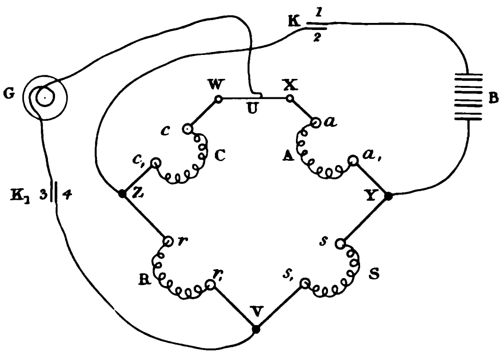


Fig. 8. Diagram of connexions with commutator D placed across board, d connected with f & d, with f,

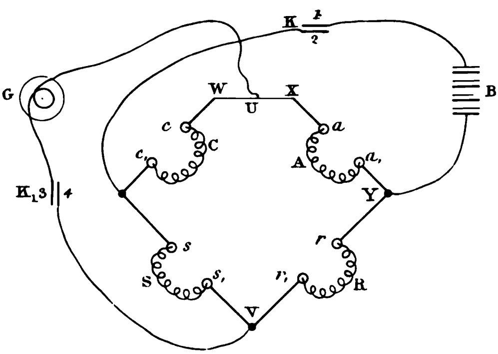
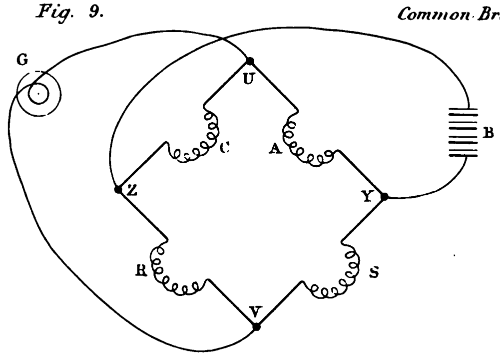


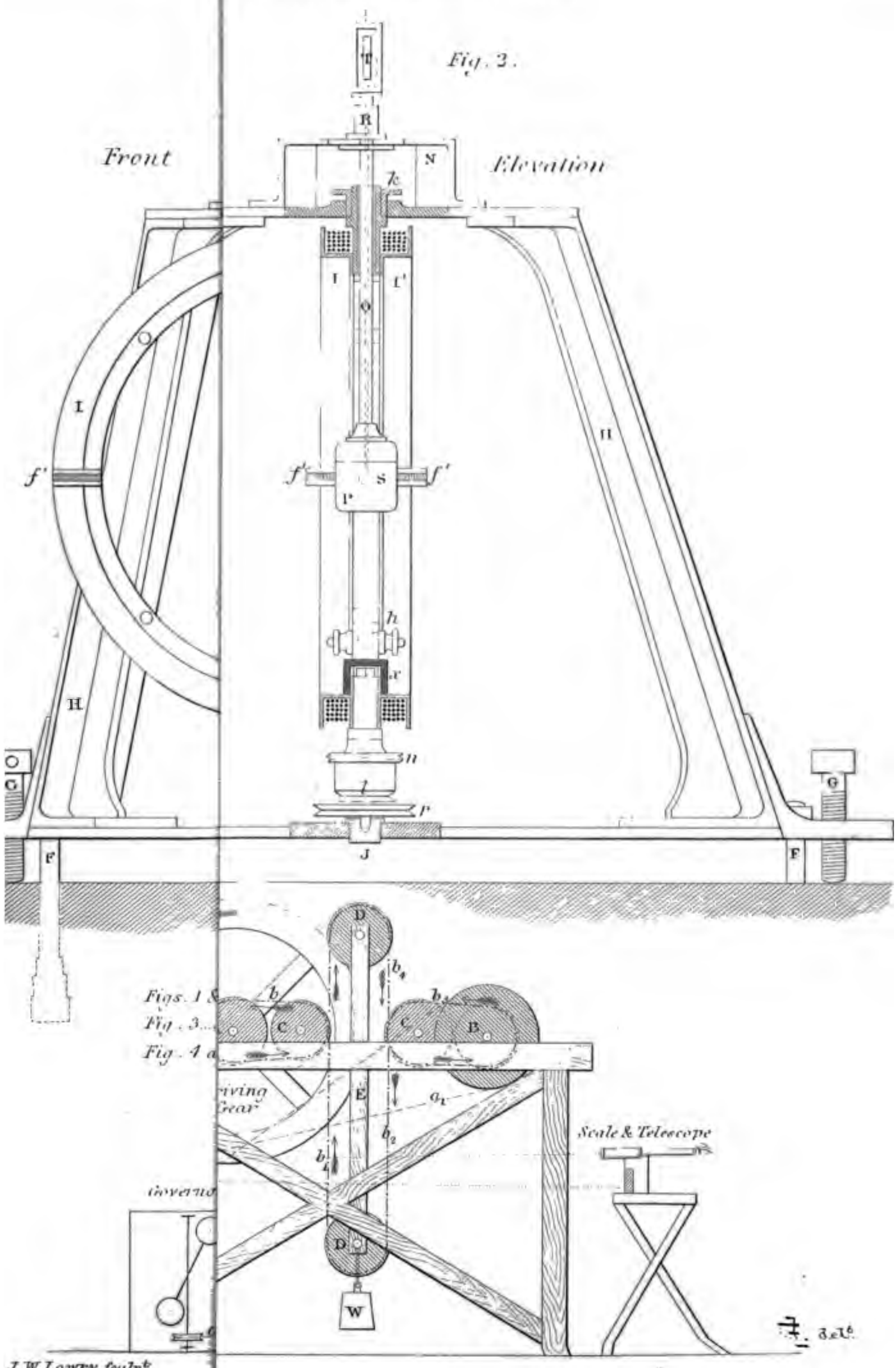
Fig. 9. *Common Bridge*



UCTION APPARATUS,

termination of Electrical Resistance in

UTE ELECTRO-MAGNETIC UNITS.



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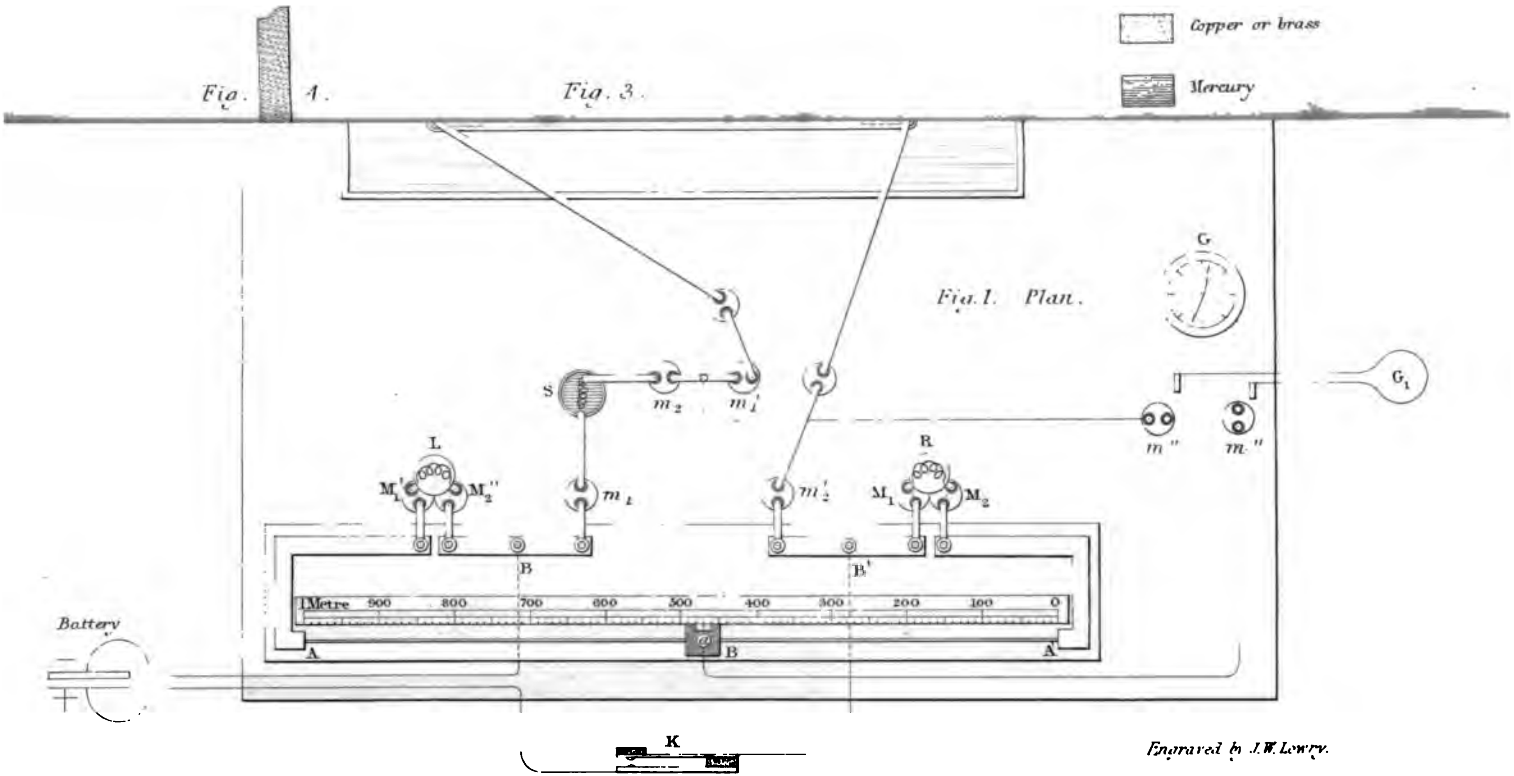

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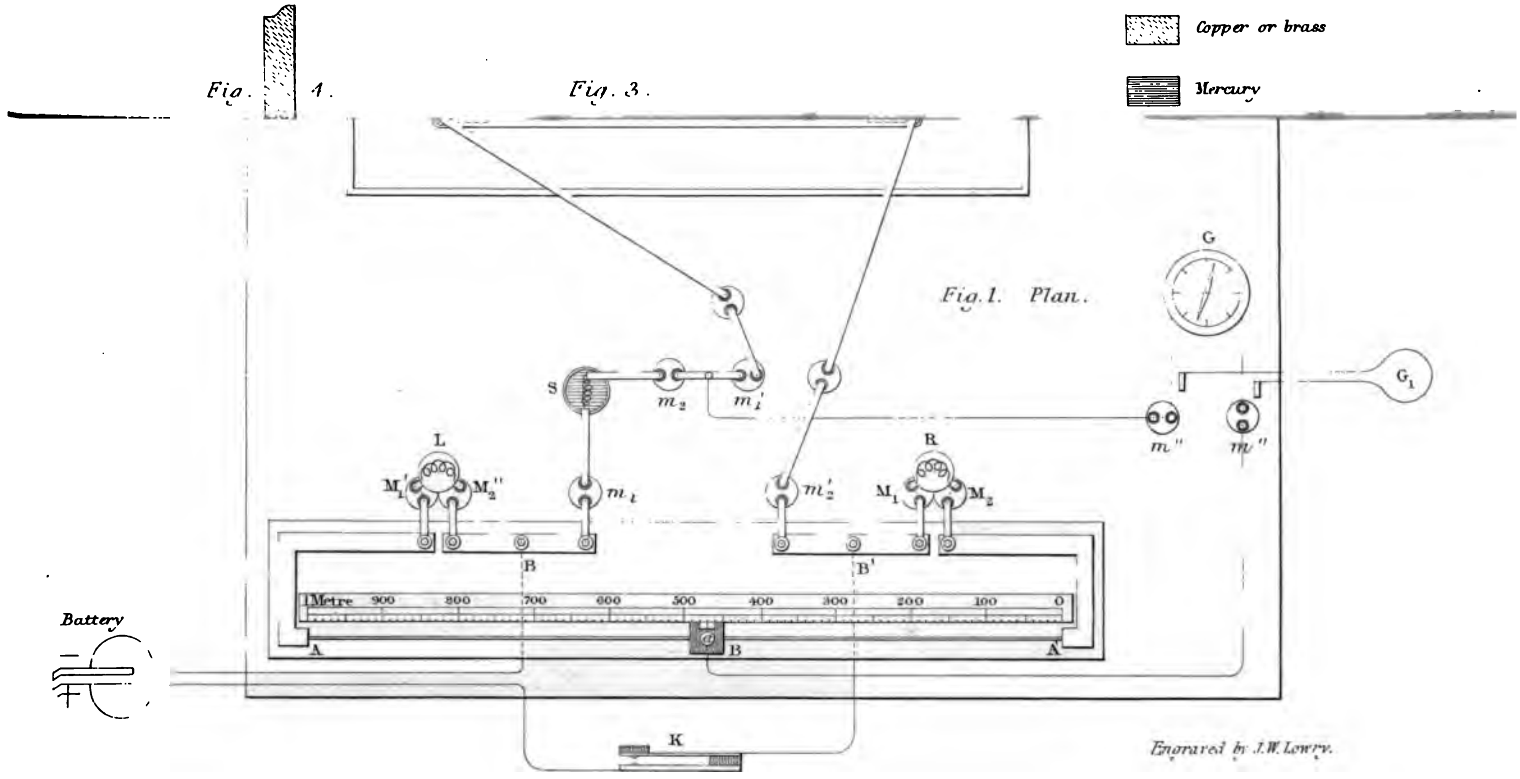
Fig. 4. Fig. 3.



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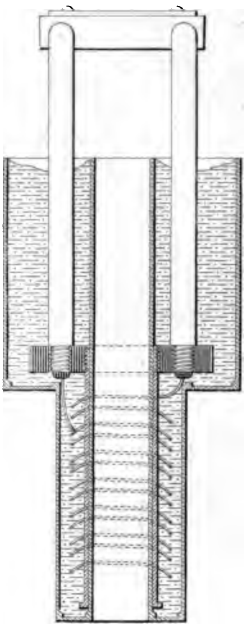
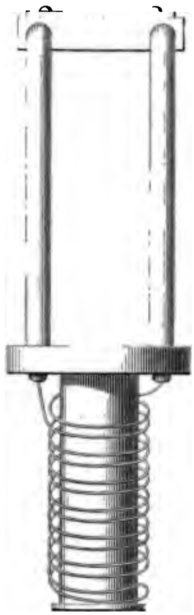
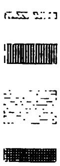
Standard Unit of Electrical Resistance.



SECTION.



ELEVATION.



Metal.

Ebonite.

Paraffine.

Solder.

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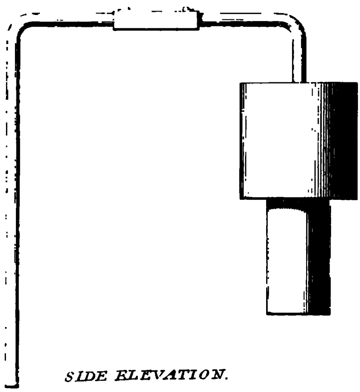
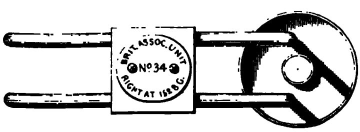


SIDE ELEVATION.



PLAN.

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Report

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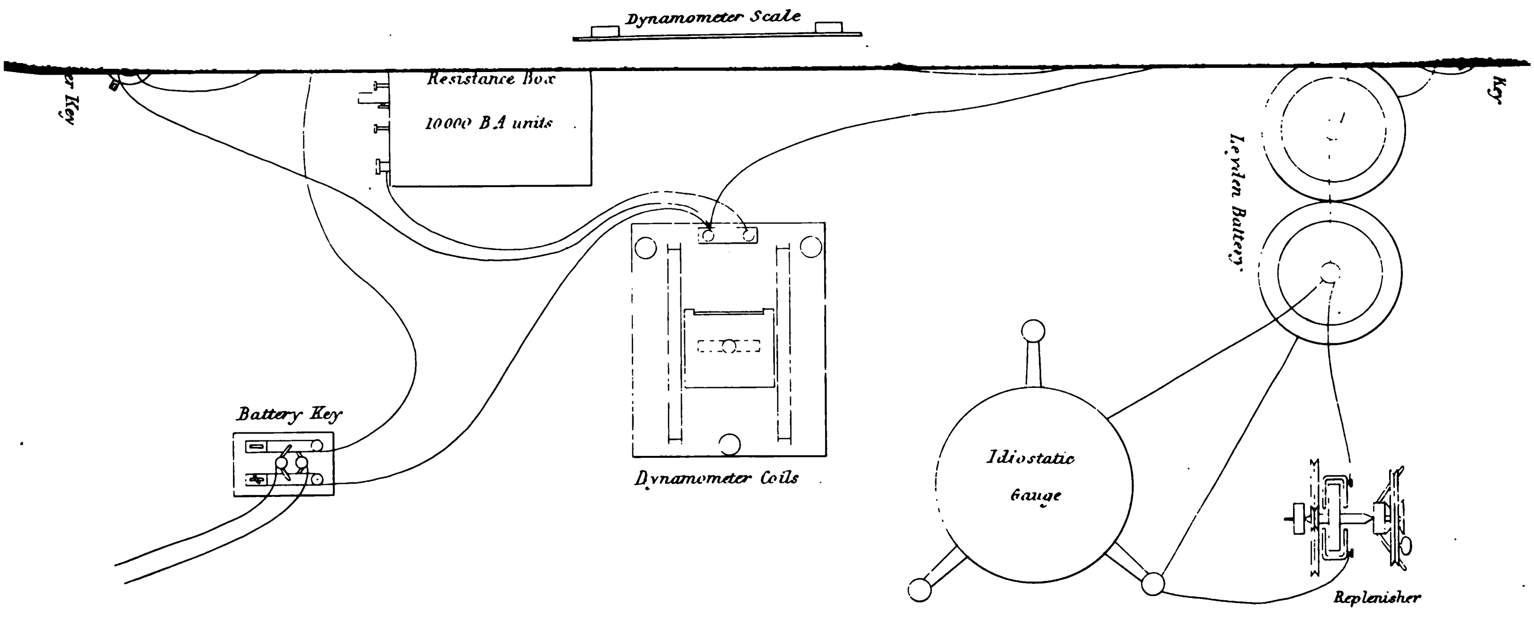
J.W. Lowry Sculp.t

7 Report

J.W. Lowry sculpt



J.W. Lowry sculpt.